07-01-19
07-03-19

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|---|---|---|
| **Student:** ____________ | **Instructor:** Alfredo Alvarez | **Assignment:** |
| **Date:** ____________ | **Course:** 2413 Cal I | CAL2413ANSWERSI150FIESTA |

1. The function s(t) represents the position of an object at time t moving along a line. Suppose s(1) = 120 and s(5) = 220. Find the average velocity of the object over the interval of time [1,5].

The average velocity over the interval [1,5] is $v_{av}$ = ______. (Simplify your answer.)

average velocity on [a, b]

$\frac{f(b)-f(a)}{b-a}$ formula

Answer: 25

$\frac{s(5)-s(1)}{(5)-(1)} = \frac{220-120}{5-1} = \frac{100}{4} = 25 =$

ID: 2.1.3

2. The position of an object moving along a line is given by the function $s(t) = -8t^2 + 64t$. Find the average velocity of the object over the following intervals.

formula $\frac{f(b)-f(a)}{b-a}$ on [a, b]

(a) [1, 7]
(b) [1, 6]
(c) [1, 5]
(d) [1, 1 + h] where h > 0 is any real number.

(a) The average velocity of the object over the interval [1, 7] is ______.

(b) The average velocity of the object over the interval [1, 6] is ______.

(c) The average velocity of the object over the interval [1, 5] is ______.

(d) The average velocity of the object over the interval [1, 1 + h] is ______.

$s(t) = -8t^2 + 64t$

Answers 0

8

16

$-8h+48$

ID: 2.1.13

$\frac{s(7)-s(1)}{(7)-(1)} =$

$\frac{(-8(7)^2+64(7))-(-8(1)^2+64(1))}{(7)-(1)} =$

$\frac{(-8(7)(7)+64(7))-(-8(1)(1)+64(1))}{(7)-(1)} =$

$\frac{(-392+448)-(-8+64)}{(7)-(1)} =$

$\frac{(56)-(56)}{(7)-(1)} =$

$\frac{56-56}{7-1} =$

$\frac{0}{6} =$

$0 =$

$s(t) = -8t^2 + 64t$

$\frac{s(6) - s(1)}{(6)-(1)} =$

$\frac{(-8(6)^2 + 64(6)) - (-8(1)^2 + 64(1))}{(6)-(1)} =$

$\frac{(-8(6)(6) + 64(6)) - (-8(1)(1) + 64(1))}{(6)-(1)} =$

$\frac{(-288 + 384) - (-8 + 64)}{(6)-(1)} =$

$\frac{(96) - (56)}{(6)-(1)} =$

$\frac{96 - 56}{6 - 1} =$

$\frac{40}{5} =$

$8 =$

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$\frac{s(5) - s(1)}{(5)-(1)} =$

$\frac{(-8(5)^2 + 64(5)) - (-8(1)^2 + 64(1))}{(5)-(1)} =$

$\frac{(-8(5)(5) + 64(5)) - (-8(1)(1) + 64(1))}{(5)-(1)} =$

$\frac{(-200 + 320) - (-8 + 64)}{(5)-(1)} =$

$\frac{(120) - (56)}{(5)-(1)} =$

$\frac{64}{4} =$

$16 =$

$s(t) = -8t^2 + 64t$

$$\frac{s(1+h) - s(1)}{(1+h) - (1)} =$$

$$\frac{(-8(1+h)^2 + 64(1+h)) - (-8(1)^2 + 64(1))}{(1+h) - (1)} =$$

$$\frac{(-8(1+h)(1+h) + 64 + 64h) - (-8(1)(1) + 64(1))}{(1+h) - (1)} =$$

$$\frac{(-8(1 + 1h + 1h + h^2) + 64 + 64h) - (-8 + 64)}{(1+h) - (1)} =$$

$$\frac{(-8(1 + 2h + h^2) + 64 + 64h) - (56)}{(1+h) - 1} =$$

$$\frac{-8 - 16h - 8h^2 + 64 + 64h - 56}{1 + h - 1} =$$

$$\frac{-8h^2 + 48h}{h} =$$

$$\frac{-8h^2}{h} + \frac{48h}{h} =$$

$$-8h + 48$$

3. For the position function $s(t) = -16t^2 + 106t$, complete the following table with the appropriate average velocities. Then make a conjecture about the value of the instantaneous velocity at $t = 1$.

| Time Interval | [1, 2] | [1, 1.5] | [1, 1.1] | [1, 1.01] | [1, 1.001] |
|---|---|---|---|---|---|
| Average Velocity | ___ | ___ | ___ | ___ | ___ |

Complete the following table.

| Time Interval | [1, 2] | [1, 1.5] | [1, 1.1] | [1, 1.01] | [1, 1.001] |
|---|---|---|---|---|---|
| Average Velocity | | | | | |

(Type exact answers. Type integers or decimals.)

The value of the instantaneous velocity at t = 1 is ____.
(Round to the nearest integer as needed.)

$s(t) = -16t^2 + 106t$

Answers 58
66
72.4
73.84
73.984
74

$$\frac{s(2) - s(1)}{(2) - (1)} =$$

$$\frac{(-16(2)^2 + 106(2)) - (-16(1)^2 + 106(1))}{(2) - (1)} =$$

$$\frac{(-16(2)(2) + 106(2)) - (-16(1)(1) + 106(1))}{(2) - (1)}$$

ID: 2.1.17

$$\frac{(-64 + 212) - (-16 + 106)}{(2) - (1)} =$$

$$\frac{(148) - (90)}{(2) - (1)} =$$

$$\frac{148 - 90}{2 - 1} =$$

$$\frac{48}{1} =$$

58 =

$$s(t) = -16t^2 + 106t$$

$$\frac{s(1.5) - s(1)}{(1.5) - (1)} =$$

$$\frac{(-16(1.5)^2 + 106(1.5)) - (-16(1)^2 + 106(1))}{(1.5) - (1)} =$$

$$\frac{(-16(1.5)(1.5) + 106(1.5)) - (-16(1)(1) + 106(1))}{(1.5) - (1)} =$$

$$\frac{(-36 + 159) - (-16 + 106)}{(1.5) - (1)} =$$

$$\frac{(123) - (90)}{(1.5) - (1)} =$$

$$\frac{123 - 90}{1.5 - 1} =$$

$$\frac{33}{.5} =$$

$$66 =$$

$$s(t) = -16t^2 + 106t$$

$$\frac{s(1.1) - s(1)}{(1.1) - (1)} =$$

$$\frac{(-16(1.1)^2 + 106(1.1)) - (-16(1)^2 + 106(1))}{(1.1) - (1)} =$$

$$\frac{(-16(1.1)(1.1) + 106(1.1)) - (-16(1)(1) + 106(1))}{(1.1) - (1)} =$$

$$\frac{(-19.36 + 116.6) - (-16 + 106)}{(1.1) - (1)} =$$

$$\frac{(97.24) - (90)}{(1.1) - (1)} =$$

$$\frac{97.24 - 90}{1.1 - 1} =$$

$$\frac{7.24}{.1} =$$

$$72.4 =$$

$$s(t) = -16t^2 + 106t$$

$$\frac{s(1.01) - s(1)}{(1.01) - (1)} =$$

$$\frac{(-16(1.01)^2 + 106(1.01)) - (-16(1)^2 + 106(1))}{(1.01) - (1)} =$$

$$\frac{(-16(1.01)(1.01) + 106(1.01)) - (-16(1)(1) + 106(1))}{(1.01) - (1)} =$$

$$\frac{(-16.3216 + 107.06) - (-16 + 106)}{(1.01) - (1)} =$$

$$\frac{(90.7384) - (90)}{(1.01) - (1)} =$$

$$\frac{90.7384 - 90}{1.01 - 1} =$$

$$\frac{.7384}{.01} =$$

$$73.84 =$$

$$S(t) = -16t^2 + 106t$$

$$\frac{S(1.001) - S(1)}{(1.001) - (1)} =$$

$$\frac{(-16(1.001)^2 + 106(1.001)) - (-16(1)^2 + 106(1))}{(1.001) - (1)} =$$

$$\frac{(-16(1.001)(1.001) + 106(1.001)) - (-16(1)(1) + 106(1))}{(1.001) - (1)} =$$

$$\frac{(-16.032016 + 106.106) - (-16 + 106)}{(1.001) - (1)} =$$

$$\frac{(90.073984) - (90)}{(1.001) - (1)} =$$

$$\frac{90.073984 - 90}{(1.001) - (1)}$$

$$\frac{90.073984 - 90}{1.001 - 1} =$$

$$\frac{.073984}{.001} =$$

$$73.984 =$$

the value of the instantaneous velocity at $X = 1$ is 74

4. For the function $f(x) = 12x^3 - x$, make a table of slopes of secant lines and make a conjecture about the slope of the tangent line at $x = 1$.

Complete the table.

(Round the final answer to three decimal places as needed. Round all intermediate values to four decimal places as needed.)

| Interval | Slope of secant line |
|---|---|
| [1, 2] | |
| [1, 1.5] | |
| [1, 1.1] | |
| [1, 1.01] | |
| [1, 1.001] | |

$f(x) = 12x^3 - x$

An accurate conjecture for the slope of the tangent line at $x = 1$ is ______.
(Round to the nearest integer as needed.)

Answers 83.000
56.000
38.720
35.360
35.000
35

$$\frac{f(2) - f(1)}{(2) - (1)} =$$

$$\frac{(12(2)^3 - (2)) - (12(1)^3 - (1))}{(2) - (1)} =$$

ID: 2.1.28

$$\frac{(12(2)(2)(2) - (2)) - (12(1)(1)(1) - (1))}{(2) - (1)} =$$

$$\frac{(96 - 2) - (12 - 1)}{(2) - (1)} =$$

$$\frac{(94) - (11)}{(2) - (1)} =$$

$$\frac{94 - 11}{2 - 1} =$$

$$\frac{83}{1} =$$

$83 =$

$$\frac{f(1.5)-f(1)}{(1.5)-(1)} =$$

$f(x) = 12x^3 - x$

$$\frac{(12(1.5)^3 - (1.5)) - (12(1)^3 - (1))}{(1.5)-(1)} =$$

$$\frac{(12(1.5)(1.5)(1.5) - (1.5)) - (12(1)(1)(1) - (1))}{(1.5)-(1)} =$$

$$\frac{(40.5 - 1.5) - (12-1)}{(1.5)-(1)} =$$

$$\frac{(39) - (11)}{(1.5)-(1)} =$$

$$\frac{39-11}{1.5-1} =$$

$$\frac{28}{.5} =$$

$56 =$

$$f(x)=12x^3-x$$

$$\frac{f(1.1)-f(1)}{(1.1)-(1)}=$$

$$\frac{(12(1.1)^3-(1.1))-(12(1)^3-(1))}{(1.1)-(1)}=$$

$$\frac{(12(1.1)(1.1)(1.1)-(1.1))-(12(1)(1)(1)-(1))}{(1.1)-(1)}=$$

$$\frac{(15.972-1.1)-(12-1)}{(1.1)-(1)}=$$

$$\frac{(14.872)-(11)}{(1.1)-(1)}=$$

$$\frac{14.872-11}{1.1-1}=$$

$$\frac{3.872}{.1}=$$

$$38.720=$$

$$f(x) = 12x^3 - x$$

$$\frac{f(1.01) - f(1)}{(1.01) - (1)} =$$

$$\frac{(12(1.01)^3 - (1.01)) - (12(1)^3 - (1))}{(1.01) - (1)} =$$

$$\frac{(12(1.01)(1.01)(1.01) - (1.01)) - (12(1)(1)(1) - (1))}{(1.01) - (1)} =$$

$$\frac{(12.363612 - 1.01) - (12 - 1)}{(1.01) - (1)} =$$

$$\frac{(11.353612) - (11)}{(1.01) - (1)} =$$

$$\frac{11.353612 - 11}{1.01 - 1} =$$

$$\frac{.353612}{.01} =$$

$$35.3612 =$$

Or Round

$$35.360$$

$$\frac{f(1.001) - f(1)}{(1.001) - (1)} =$$

$f(x) = 12x^3 - x$

$$\frac{(12(1.001)^3 - (1.001)) - (12(1)^3 - (1))}{(1.001) - (1)} =$$

$$\frac{(12(1.001)(1.001)(1.001) - (1.001)) - (12(1)(1)(1) - (1))}{(1.001) - (1)} =$$

$$\frac{(12.03603601 - 1.001) - (12 - 1)}{(1.001) - (1)} =$$

$$\frac{(11.03503601) - (11)}{(1.001) - (1)} =$$

$$\frac{11.03503601 - 11}{1.001 - 1} =$$

$$\frac{.03503601}{.001} =$$

$35.03601 =$

or Round

$35.000$

an accurate ~~conjecture~~ for the Slope of the tangent line at $X = 1$ is $35$

5. Let $f(x) = \frac{x^2-4}{x-2}$. **(a)** Calculate f(x) for each value of x in the following table. **(b)** Make a conjecture about the value of $\lim_{x\to 2} \frac{x^2-4}{x-2}$.

**(a)** Calculate f(x) for each value of x in the following table.

| x | 1.9 | 1.99 | 1.999 | 1.9999 |
|---|---|---|---|---|
| $f(x) = \frac{x^2-4}{x-2}$ | | | | |
| x | 2.1 | 2.01 | 2.001 | 2.0001 |
| $f(x) = \frac{x^2-4}{x-2}$ | | | | |

(Type an integer or decimal rounded to four decimal places as needed.)

**(b)** Make a conjecture about the value of $\lim_{x\to 2} \frac{x^2-4}{x-2}$.

$\lim_{x\to 2} \frac{x^2-4}{x-2} =$ ____ (Type an integer or a decimal.)

$f(x) = \frac{x^2-4}{x-2}$

Answers 3.9

3.99

3.999

3.9999

4.1

4.01

4.001

4.0001

4

$f(1.9) = \frac{(1.9)^2-4}{(1.9)-2}$

$f(1.9) = \frac{(1.9)(1.9)-4}{(1.9)-2}$

$f(1.9) = \frac{3.61-4}{1.9-2}$

ID: 2.2.7

$f(1.9) = \frac{-.39}{-.1}$

$f(1.9) = 3.9$

$$f(x) = \frac{x^2-4}{x-2}$$

$$f(1.99) = \frac{(1.99)^2-4}{(1.99)-2}$$

$$f(1.99) = \frac{(1.99)(1.99)-4}{(1.99)-2}$$

$$f(1.99) = \frac{3.9601-4}{1.99-2}$$

$$f(1.99) = \frac{-.0399}{-.01}$$

$$f(1.99) = 3.99$$

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$$f(1.999) = \frac{(1.999)^2-4}{(1.999)-2}$$

$$f(1.999) = \frac{(1.999)(1.999)-4}{(1.999)-2}$$

$$f(1.999) = \frac{3.996001-4}{1.999-2}$$

$$f(1.999) = \frac{-.003999}{-.001}$$

$$f(1.999) = 3.999$$

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$$f(x) = \frac{x^2-4}{x-2}$$

$$f(1.9999) = \frac{(1.9999)^2-4}{(1.9999)-2}$$

$$f(1.9999) = \frac{(1.9999)(1.9999)-4}{(1.9999)-2}$$

$$f(1.9999) = \frac{3.99960001-4}{1.9999-2}$$

$$f(1.9999) = 3.9999$$

$$f(2.1) = \frac{(2.1)^2-4}{(2.1)-2}$$

$$f(2.1) = \frac{(2.1)(2.1)-4}{(2.1)-2}$$

$$f(2.1) = \frac{4.41-4}{2.1-2}$$

$$f(2.1) = \frac{.41}{.1}$$

$$f(2.1) = 4.1$$

$$f(x) = \frac{x^2-4}{x-2}$$

$$f(2.01) = \frac{(2.01)^2-4}{(2.01)-2}$$

$$f(2.01) = \frac{(2.01)(2.01)-4}{(2.01)-2}$$

$$f(2.01) = \frac{4.0401-4}{2.01-2}$$

$$f(2.01) = \frac{.0401}{-.01}$$

$$f(2.01) = 4.01$$

$$f(2.001) = \frac{(2.001)^2-4}{(2.001)-2}$$

$$f(2.001) = \frac{(2.001)(2.001)-4}{(2.001)-2}$$

$$f(2.001) = \frac{4.004001-4}{2.001-2}$$

$$f(2.001) = \frac{.004001}{.001}$$

$$f(2.001) = 4.001$$

$$f(x) = \frac{x^2-4}{x-2}$$

$$f(2.0001) = \frac{(2.0001)^2-4}{(2.0001)-2}$$

$$f(2.0001) = \frac{(2.0001)(2.0001)-4}{(2.0001)-2}$$

$$f(2.0001) = \frac{4.00040001-4}{2.0001-2}$$

$$f(2.0001) = 4.0001$$

or

$$\lim_{x \to 2} \frac{x^2-4}{x-2} =$$

$$\lim_{x \to 2} \frac{(x)^2-(2)^2}{x-2} =$$

$$\lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} =$$

$$\lim_{x \to 2} (x+2) =$$

$$2+2 =$$

$$4 =$$

a conjuctive about

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = 4$$

6. Let $g(t) = \dfrac{t-64}{\sqrt{t}-8}$.

**a.** Make two tables, one showing the values of g for t = 63.9, 63.99, and 63.999 and one showing values of g for t = 64.1, 64.01, and 64.001.

**b.** Make a conjecture about the value of $\lim\limits_{t\to 64} \dfrac{t-64}{\sqrt{t}-8}$.

**a.** Make a table showing the values of g for t = 63.9, 63.99, and 63.999.

| t | 63.9 | 63.99 | 63.999 |
|---|---|---|---|
| g(t) | | | |

(Round to four decimal places.)

Make a table showing the values of g for t = 64.1, 64.01, and 64.001.

| t | 64.1 | 64.01 | 64.001 |
|---|---|---|---|
| g(t) | | | |

(Round to four decimal places.)

**b.** Make a conjecture about the value of $\lim\limits_{t\to 64} \dfrac{t-64}{\sqrt{t}-8}$. Select the correct choice below and fill in any answer boxes in your choice.

$$g(t) = \frac{t-64}{\sqrt{t}-8}$$

○ **A.** $\lim\limits_{t\to 64} \dfrac{t-64}{\sqrt{t}-8} =$ ______ (Simplify your answer.)

○ **B.** The limit does not exist.

$$g(63.9) = \frac{(63.9)-(64)}{\sqrt{63.9}-8}$$

$$g(63.9) = \frac{63.9-64}{7.993747557-8}$$

$$g(63.9) = \frac{-.1}{-.006252443}$$

Answers 15.9937

15.9994

15.9999

16.0062

16.0006

16.0001

A. $\lim\limits_{t\to 64} \dfrac{t-64}{\sqrt{t}-8} = \boxed{16}$ (Simplify your answer.)

$$g(63.9) = 15.99374836$$

ID: 2.2.9

OR Round

15.9937

$$g(t) = \frac{t - 64}{\sqrt{t} - 8}$$

$$g(63.99) = \frac{(63.99) - 64}{\sqrt{63.99} - 8}$$

$$g(63.99) = \frac{63.99 - 64}{7.999374976 - 8}$$

$$g(63.99) = \frac{-.01}{7.999374976 - 8}$$

$$g(63.99) = 15.9994$$

$$g(63.999) = \frac{(63.999) - (64)}{\sqrt{63.999} - 8}$$

$$g(63.999) = \frac{63.999 - 64}{7.9999375 - 8}$$

$$g(63.999) = 15.9999$$

$$g(t) = \frac{t-64}{\sqrt{t}-8}$$

$$g(64.1) = \frac{(64.1)-(64)}{\sqrt{64.1}-8}$$

$$g(64.1) = \frac{64.1-64}{8.00624756-8}$$

$$g(64.1) = 16.0062$$

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$$g(64.01) = \frac{(64.01)-(64)}{\sqrt{64.01}-8}$$

$$g(64.01) = \frac{64.01-64}{8.000624976-8}$$

$$g(64.01) = 16.0006$$

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$$g(t) = \frac{t-64}{\sqrt{t}-8}$$

$$g(64.001) = \frac{(64.001)-(64)}{\sqrt{64.001}-8}$$

$$g(64.001) = \frac{64.001-64}{8.0000625-8}$$

$$g(64.001) = 16.0001$$

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$$\lim_{t\to 64} \left(\frac{t-64}{\sqrt{t}-8}\right)\left(\frac{\sqrt{t}+8}{\sqrt{t}+8}\right) =$$

$$\lim_{t\to 64} \frac{(t-64)(\sqrt{t}+8)}{(\sqrt{t})^2+\cancel{8\sqrt{t}}-\cancel{8\sqrt{t}}-64} =$$

$$\lim_{t\to 64} \frac{(t-64)(\sqrt{t}+8)}{(\sqrt{t})^2-64} =$$

$$\lim_{t\to 64} \frac{\cancel{(t-64)}(\sqrt{t}+8)}{\cancel{(t-64)}} =$$

$$\lim_{t\to 64} (\sqrt{t}+8) =$$

$$\sqrt{64}+8 =$$

$$8+8 =$$

$$16 =$$

$$\lim_{t\to 64} \frac{t-64}{\sqrt{t}-8} = 16$$

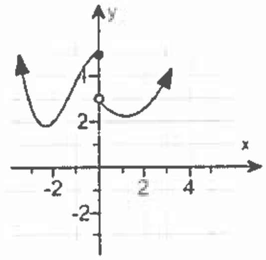
7. Use the graph to find the following limits and function value.

a. $\lim_{x \to 0^-} f(x)$

b. $\lim_{x \to 0^+} f(x)$

c. $\lim_{x \to 0} f(x)$

d. $f(0)$




a. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 0^-} f(x) =$ 5 (Type an integer.)

B. The limit does not exist.

b. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 0^+} f(x) =$ 3 (Type an integer.)

B. The limit does not exist.

c. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 0} f(x) =$ ______ (Type an integer.)

B. The limit does not exist.

d. Find the function value. Select the correct choice below and fill in any answer boxes in your choice.

A. $f(0) =$ 5 (Type an integer.)

B. The answer is undefined.

Answers A. $\lim_{x \to 0^-} f(x) =$ 5 (Type an integer.)

A. $\lim_{x \to 0^+} f(x) =$ 3 (Type an integer.)

B. The limit does not exist.

A. $f(0) =$ 5 (Type an integer.)

ID: 2.2.15

8. Explain why $\lim_{x \to -5} \frac{x^2-2x-35}{x+5} = \lim_{x \to -5} (x-7)$, and then evaluate $\lim_{x \to -5} \frac{x^2-2x-35}{x+5}$.

Choose the correct answer below.

- A. The numerator of the expression $\frac{x^2-2x-35}{x+5}$ simplifies to $x-7$ for all x, so the limits are equal.
- B. Since $\frac{x^2-2x-35}{x+5} = x-7$ whenever $x \neq -5$, it follows that the two expressions evaluate to the same number as x approaches $-5$.
- C. Since each limit approaches $-5$, it follows that the limits are equal.
- D. The limits $\lim_{x \to -5} \frac{x^2-2x-35}{x+5}$ and $\lim_{x \to -5} (x-7)$ equal the same number when evaluated using direct substitution.

Now evaluate the limit.

$\lim_{x \to -5} \frac{x^2-2x-35}{x+5} =$ ______ (Simplify your answer.)

Answers B.

Since $\frac{x^2-2x-35}{x+5} = x-7$ whenever $x \neq -5$, it follows that the two expressions evaluate to the same number as x approaches $-5$.

$-12$

$$\lim_{x \to -5} \frac{x^2-2x-35}{x+5} =$$

$$\lim_{x \to -5} \frac{\cancel{(x+5)}(x-7)}{\cancel{(x+5)}} =$$

$$\lim_{x \to -5} (x-7) =$$

$$-5-7 =$$

$$-12 =$$

ID: 2.3.5

9. Assume $\lim_{x\to 9} f(x) = 9$ and $\lim_{x\to 9} h(x) = 3$. Compute the following limit and state the limit laws used to justify the computation.

$$\lim_{x\to 9}\frac{f(x)}{h(x)}$$

$\lim_{x\to 9}\frac{f(x)}{h(x)} =$ ☐ (Simplify your answer.)

Select each limit law used to justify the computation.

- ☐ **A.** Quotient
- ☐ **B.** Difference
- ☐ **C.** Root
- ☐ **D.** Sum
- ☐ **E.** Constant multiple
- ☐ **F.** Power
- ☐ **G.** Product

$\lim_{x\to 9}\frac{f(x)}{h(x)} = \frac{\lim_{x\to 9} f(x)}{\lim_{x\to 9} h(x)} = \frac{9}{3} = 3 =$

Answers 3

A. Quotient

ID: 2.3.8

10. Find the following limit or state that it does not exist.

$$\lim_{x\to 441}\frac{\sqrt{x}-21}{x-441}$$

Simplify the given limit.

$\lim_{x\to 441}\frac{\sqrt{x}-21}{x-441} = \lim_{x\to 441}$ ☐ (Simplify your answer.)

$\lim_{x\to 441}\left(\frac{\sqrt{x}-21}{x-441}\right)\left(\frac{\sqrt{x}+21}{\sqrt{x}+21}\right) =$

$\lim_{x\to 441}\frac{(\sqrt{x})^2+\cancel{21\sqrt{x}}-\cancel{21\sqrt{x}}-441}{(x-441)(\sqrt{x}+21)} =$

$\lim_{x\to 441}\frac{\cancel{(x-441)}(1)}{\cancel{(x-441)}(\sqrt{x}+21)} =$

Evaluate the limit, if possible. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

◯ **A.** $\lim_{x\to 441}\frac{\sqrt{x}-21}{x-441} =$ ______ (Type an exact answer.)

◯ **B.** The limit does not exist.

$\lim_{x\to 441}\frac{1}{\sqrt{x}+21} = \frac{1}{\sqrt{441}+21} = \frac{1}{21+21} = \frac{1}{42} =$

Answers $\frac{1}{\sqrt{x}+21}$

A. $\lim_{x\to 441}\frac{\sqrt{x}-21}{x-441} = \boxed{\frac{1}{42}}$ (Type an exact answer.)

ID: 2.3.41-Setup & Solve

11. Determine the following limit at infinity.

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} \text{ if } f(x)\to 900{,}000 \text{ and } g(x)\to\infty \text{ as } x\to\infty$$

Find the limit. Choose the correct answer below.

- A. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$
- B. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$
- C. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 900{,}000$
- D. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{1}{900{,}000}$

Answer: A. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$

ID: 2.5.11

$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{\lim_{x\to\infty} f(x)}{\lim_{x\to\infty} g(x)} = \frac{900000}{\infty} = 0 =$

formula

$\lim_{x\to\infty} \frac{1}{x^n} = 0$

$\lim_{x\to\infty} \frac{1}{x} = 0$

12. Determine the following limit at infinity.

$$\lim_{x\to\infty} \frac{8+9x+7x^3}{x^3} =$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $\lim_{x\to\infty} \frac{8+9x+7x^3}{x^3} =$ ______
- B. The limit does not exist and is neither $-\infty$ nor $\infty$.

Answer: A. $\lim_{x\to\infty} \frac{8+9x+7x^3}{x^3} = \boxed{7}$

$\lim_{x\to\infty} \left(\frac{8}{x^3} + \frac{9x}{x^3} + \frac{7x^3}{x^3}\right) =$

$\lim_{x\to\infty} \left(\frac{8}{x^3} + \frac{9}{x^2} + 7\right) =$

$0 + 0 + 7 =$

$7 =$

ID: 2.5.12

formula

$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to\infty} \frac{1}{x^n} = 0$

13. Determine the following limit.

$$\left(\lim_{w\to\infty} \frac{10w^2+5w+3}{\sqrt{4w^4+w^3}}\right) \frac{\sqrt{\frac{1}{w^4}}}{\sqrt{\frac{1}{w^4}}} = \lim_{w\to\infty} \frac{(10w^2+5w+3)\left(\frac{1}{w^2}\right)}{\sqrt{(4w^4+w^3)\left(\frac{1}{w^4}\right)}} = \lim_{w\to\infty} \frac{\frac{10w^2}{w^2}+\frac{5w}{w^2}+\frac{3}{w^2}}{\sqrt{\frac{4w^4}{w^4}+\frac{w^3}{w^4}}}$$

Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

$$\lim_{w\to\infty} \frac{10+\frac{5}{w}+\frac{3}{w^2}}{\sqrt{4+\frac{1}{w}}} = \frac{10+0+0}{\sqrt{4+0}} = \frac{10}{\sqrt{4}} = \frac{10}{2} = 5$$

- A. $\lim_{w\to\infty} \dfrac{10w^2+5w+3}{\sqrt{4w^4+w^3}} =$ _______ (Simplify your answer.)
- B. The limit does not exist and is neither $\infty$ nor $-\infty$.

Answer: A. $\lim_{w\to\infty} \dfrac{10w^2+5w+3}{\sqrt{4w^4+w^3}} =$ 5 (Simplify your answer.)

Formula

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

ID: 2.5.29

14. Determine the following limit.

$$\left(\lim_{x\to-\infty} \frac{\sqrt{169x^2+x}}{x}\right) \frac{\sqrt{\frac{1}{x^2}}}{\sqrt{\frac{1}{x^2}}} = \lim_{x\to-\infty} \frac{\sqrt{(169x^2+x)\frac{1}{x^2}}}{x\left(\frac{1}{x}\right)} = \lim_{x\to-\infty} \frac{\sqrt{\frac{169x^2}{x^2}+\frac{x}{x^2}}}{x\left(\frac{1}{x}\right)} =$$

Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

$$\lim_{x\to-\infty} \frac{\sqrt{169+\frac{1}{x}}}{\frac{x}{x}} = \lim_{x\to-\infty} \frac{\sqrt{169+\frac{1}{x}}}{1} = -\frac{\sqrt{169+0}}{1} = -\sqrt{169} = -13$$

- A. $\lim_{x\to-\infty} \dfrac{\sqrt{169x^2+x}}{x} =$ _______ (Simplify your answer.)
- B. The limit does not exist and is neither $\infty$ nor $-\infty$.

Answer: A. $\lim_{x\to-\infty} \dfrac{\sqrt{169x^2+x}}{x} =$ $-13$ (Simplify your answer.)

ID: 2.5.31

Formula

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

15. Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for the following function. Then give the horizontal asymptotes of f, if any.

$$f(x) = \frac{4x}{8x+5}$$

$\lim_{x\to\infty} \left(\frac{4x}{8x+5}\right)\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\frac{4x}{x}}{\frac{8x}{x}+\frac{5}{x}}$

$= \lim_{x\to\infty} \frac{4}{8+\frac{5}{x}}$

$\frac{4}{8+0} = \frac{4}{8} = \frac{4(1)}{4(2)} = \frac{1}{2}$

Evaluate $\lim_{x\to\infty} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $\lim_{x\to\infty} \frac{4x}{8x+5} =$ ______ (Simplify your answer.)
- B. The limit does not exist and is neither $\infty$ nor $-\infty$.

Evaluate $\lim_{x\to-\infty} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $\lim_{x\to-\infty} \frac{4x}{8x+5} =$ ______ (Simplify your answer.)
- B. The limit does not exist and is neither $\infty$ nor $-\infty$.

Give the horizontal asymptotes of f, if any. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- A. The function has one horizontal asymptote, ______.
  (Type an equation.)
- B. The function has two horizontal asymptotes. The top asymptote is ______ and the bottom asymptote is ______.
  (Type equations.)
- C. The function has no horizontal asymptotes.

formula

$\lim_{x\to\infty} \frac{1}{x^n} = 0$

Answers A. $\lim_{x\to\infty} \frac{4x}{8x+5} = \boxed{\frac{1}{2}}$ (Simplify your answer.)

A. $\lim_{x\to-\infty} \frac{4x}{8x+5} = \boxed{\frac{1}{2}}$ (Simplify your answer.)

A. The function has one horizontal asymptote, $\boxed{y = \frac{1}{2}}$. (Type an equation.)

Same as above

ID: 2.5.37

$\lim_{x\to\infty} \left(\frac{4x}{8x+5}\right)\left(\frac{\frac{1}{x}}{\frac{1}{x}}\right) =$ mult

$\lim_{x\to\infty} \frac{\frac{4x}{x}}{\frac{8x}{x}+\frac{5}{x}} =$

$\lim_{x\to\infty} \frac{4}{8+\frac{5}{x}} =$

$\frac{4}{8+0} = \frac{4}{8} = \frac{\cancel{4}(1)}{\cancel{4}(2)} = \frac{1}{2}$

16. Evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for the following rational function. Use $\infty$ or $-\infty$ where appropriate. Then give the horizontal asymptote of f (if any).

$$f(x) = \frac{4x^2 - 7x + 8}{2x^2 + 3}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $\lim_{x\to\infty} f(x) =$ ______ (Simplify your answer.)
- B. The limit does not exist and is neither $\infty$ nor $-\infty$.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $\lim_{x\to-\infty} f(x) =$ ______ (Simplify your answer.)
- B. The limit does not exist and is neither $\infty$ nor $-\infty$.

Identify the horizontal asymptote. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The horizontal asymptote is y = ______.
- B. There are no horizontal asymptotes.

formula
$\lim_{x\to\infty} \frac{1}{x^n} = 0$

Answers A. $\lim_{x\to\infty} f(x) = \boxed{2}$ (Simplify your answer.)

A. $\lim_{x\to-\infty} f(x) = \boxed{2}$ (Simplify your answer.)

A. The horizontal asymptote is y = $\boxed{2}$.

ID: 2.5.39

$$\lim_{x\to\infty} \left(\frac{4x^2-7x+8}{2x^2+3}\right) \frac{\frac{1}{x^2}}{\frac{1}{x^2}} =$$

$$\lim_{x\to\infty} \frac{\frac{4\cancel{x^2}}{\cancel{x^2}} - \frac{7x}{x^2} + \frac{8}{x^2}}{\frac{2\cancel{x^2}}{\cancel{x^2}} + \frac{3}{x^2}} =$$

$$\lim_{x\to\infty} \frac{4 - \frac{7}{x} + \frac{8}{x^2}}{2 + \frac{3}{x^2}} =$$

$$\frac{4-0+0}{2+0} = \frac{4}{2} = 2 =$$

17. Evaluate $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ for the rational function. Then give the horizontal asymptote of f (if any).

$$f(x) = \frac{20x^6 - 8}{4x^6 - 9x^5}$$

Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

- A. $\lim_{x \to -\infty} \frac{20x^6 - 8}{4x^6 - 9x^5} =$ ______ (Simplify your answer.)
- B. The limit does not exist and is neither $\infty$ or $-\infty$.

Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

- A. $\lim_{x \to \infty} \frac{20x^6 - 8}{4x^6 - 9x^5} =$ ______ (Simplify your answer.)
- B. The limit does not exist and is neither $\infty$ or $-\infty$.

Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

- A. The function has a horizontal asymptote at y = ______. (Simplify your answer.)
- B. The function has no horizontal asymptote.

formula

$\lim_{x \to \infty} \frac{1}{x^n} = 0$

Answers A. $\lim_{x \to -\infty} \frac{20x^6 - 8}{4x^6 - 9x^5} =$ 5 (Simplify your answer.)

A. $\lim_{x \to \infty} \frac{20x^6 - 8}{4x^6 - 9x^5} =$ 5 (Simplify your answer.)

A. The function has a horizontal asymptote at y = 5. (Simplify your answer.)

ID: 2.5.40

$\lim_{x \to \infty} \left(\frac{20x^6 - 8}{4x^6 - 9x^5}\right)\left(\frac{\frac{1}{x^6}}{\frac{1}{x^6}}\right)$

$\lim_{x \to \infty} \frac{\frac{20x^6}{x^6} - \frac{8}{x^6}}{\frac{4x^6}{x^6} - \frac{9x^5}{x^6}} =$

$\lim_{x \to \infty} \frac{20 - \frac{8}{x^6}}{4 - \frac{9}{x}} =$

$\frac{20 - 0}{4 - 0} =$

$\frac{20}{4} =$

$5 =$

18. Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for the following function. Then give the horizontal asymptotes of f, if any.

$$f(x) = \frac{3x^3 - 7}{x^4 + 5x^2}$$

Evaluate $\lim_{x\to\infty} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ **A.** $\lim_{x\to\infty} \frac{3x^3 - 7}{x^4 + 5x^2} =$ ________ (Simplify your answer.)

○ **B.** The limit does not exist and is neither $\infty$ nor $-\infty$.

Evaluate $\lim_{x\to-\infty} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ **A.** $\lim_{x\to-\infty} \frac{3x^3 - 7}{x^4 + 5x^2} =$ ________ (Simplify your answer.)

○ **B.** The limit does not exist and is neither $\infty$ nor $-\infty$.

Identify the horizontal asymptotes. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

○ **A.** The function has one horizontal asymptote, ________.
(Type an equation.)

○ **B.** The function has two horizontal asymptotes. The top asymptote is ________ and the bottom asymptote is ________.
(Type equations.)

○ **C.** The function has no horizontal asymptotes.

Formula

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

Answers A. $\lim_{x\to\infty} \frac{3x^3 - 7}{x^4 + 5x^2} = \boxed{0}$ (Simplify your answer.)

A. $\lim_{x\to-\infty} \frac{3x^3 - 7}{x^4 + 5x^2} = \boxed{0}$ (Simplify your answer.)

A. The function has one horizontal asymptote, $\boxed{y = 0}$. (Type an equation.)

ID: 2.5.41

$$\lim_{x\to\infty} \left(\frac{3x^3 - 7}{x^4 + 5x^2}\right)\left(\frac{\frac{1}{x^4}}{\frac{1}{x^4}}\right)$$

$$\lim_{x\to\infty} \frac{\frac{3x^3}{x^4} - \frac{7}{x^4}}{\frac{x^4}{x^4} + \frac{5x^2}{x^4}} = \lim_{x\to\infty} \frac{\frac{3}{x} - \frac{7}{x^4}}{1 + \frac{5}{x^2}} = \frac{0 - 0}{1 + 0} = \frac{0}{1} = 0$$

19. Find all the asymptotes of the function.

$$f(x) = \frac{8x^2+24}{2x^2+3x-2}$$

$\left(\lim_{x\to\infty} \frac{8x^2+24}{2x^2+3x-2}\right)\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right) =$

formula $\lim_{x\to\infty} \frac{1}{x^n} = 0$

$\lim_{x\to\infty} \frac{\frac{8x^2}{x^2}+\frac{24}{x^2}}{\frac{2x^2}{x^2}+\frac{3x}{x^2}-\frac{2}{x^2}} =$

Find the horizontal asymptotes. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

$\lim_{x\to\infty} \frac{8+\frac{24}{x^2}}{2+\frac{3}{x}-\frac{2}{x^2}} =$

- A. The function has one horizontal asymptote, ______.
  (Type an equation using y as the variable.)
- B. The function has two horizontal asymptotes. The top asymptote is ______ and the bottom asymptote is ______.
  (Type equations using y as the variable.)
- C. The function has no horizontal asymptotes.

$\frac{8+0}{2+0-0} =$

Find the vertical asymptotes. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

$\frac{8}{2} =$

$4 =$ horizontal asymptote

- A. The function has one vertical asymptote, ______.
  (Type an equation using x as the variable.)
- B. The function has two vertical asymptotes. The leftmost asymptote is ______ and the rightmost asymptote is ______.
  (Type equations using x as the variable.)
- C. The function has no vertical asymptotes.

Find the slant asymptote(s). Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- A. The function has one slant asymptote, ______.
  (Type an equation using x and y as the variables.)
- B. The function has two slant asymptotes. The asymptote with the larger slope is ______ and the asymptote with the smaller slope is ______.
  (Type equations using x and y as the variables.)
- C. The function has no slant asymptotes.

Answers A. The function has one horizontal asymptote, $\boxed{y=4}$. (Type an equation using y as the variable.)

B.
The function has two vertical asymptotes. The leftmost asymptote is $\boxed{x=-2}$ and the rightmost asymptote is $\boxed{x=\frac{1}{2}}$.

set $2x^2+3x-2=0$

(Type equations using x as the variable.)

$(2x-1)(x+2)=0$

C. The function has no slant asymptotes.

$2x-1=0$ or $x+2=0$

ID: EXTRA 2.66

$2x-1+1=0+1$ or $x+2-2=0-2$

$2x=1$

$\frac{2x}{2}=\frac{1}{2}$ or $x=-2$

$x=\frac{1}{2}$

vertical asymptote

20. Determine whether the following function is continuous at a. Use the continuity checklist to justify your answer.

$$f(x) = \frac{4x^2 + 25x + 25}{x^2 - 8x}, \; a = 8$$

Select all that apply.

- [ ] **A.** The function is continuous at a = 8.
- [x] **B.** The function is not continuous at a = 8 because f(8) is undefined.
- [x] **C.** The function is not continuous at a = 8 because $\lim_{x \to 8} f(x)$ does not exist.
- [x] **D.** The function is not continuous at a = 8 because $\lim_{x \to 8} f(x) \neq f(8)$.

Answer: B. The function is not continuous at a = 8 because f(8) is undefined. , C. The function is not continuous at a = 8 because $\lim_{x \to 8} f(x)$ does not exist. , D. The function is not continuous at a = 8 because $\lim_{x \to 8} f(x) \neq f(8)$.

ID: 2.6.17

21. Determine whether the following function is continuous at a. Use the continuity checklist to justify your answer.

$$f(x) = \begin{cases} \dfrac{x^2 - 144}{x - 12} & \text{if } x \neq 12 \\ 3 & \text{if } x = 12 \end{cases} ; \; a = 12$$

Select all that apply.

- [ ] **A.** The function is continuous at a = 12.
- [ ] **B.** The function is not continuous at a = 12 because f(12) is undefined.
- [ ] **C.** The function is not continuous at a = 12 because $\lim_{x \to 12} f(x)$ does not exist.
- [x] **D.** The function is not continuous at a = 12 because $\lim_{x \to 12} f(x) \neq f(12)$.

Answer: D. The function is not continuous at a = 12 because $\lim_{x \to 12} f(x) \neq f(12)$.

ID: 2.6.21

22. Determine the intervals on which the following function is continuous.

$$f(x) = \frac{x^2 - 5x + 6}{x^2 - 4}$$

Let $x^2 - 4 = 0$

$(x)^2 - (2)^2 = 0$

On what interval(s) is f continuous?

$(x+2)(x-2) = 0$

$x+2=0$ or $x-2=0$

(Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)

Answer: $(-\infty,-2),(-2,2),(2,\infty)$

$x+2-2=0-2$ or $x-2+2=0+2$

$x=-2$ or $x=2$

ID: 2.6.28

or $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

23. Evaluate the following limit.

$$\lim_{x \to 5} \sqrt{x^2 + 24} = \sqrt{(5)^2+24} = \sqrt{25+24} = \sqrt{49} = 7$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $\lim_{x \to 5} \sqrt{x^2 + 24} =$ 7, because $x^2 + 24$ is continuous for all x and the square root function is continuous for all $x \geq 0$.
(Type an integer or a fraction.)
- B. The limit does not exist and is neither $\infty$ nor $-\infty$.

Answer: A.

$\lim_{x \to 5} \sqrt{x^2 + 24} =$ 7, because $x^2 + 24$ is continuous for all x and the square root function is continuous for all $x \geq 0$.
(Type an integer or a fraction.)

ID: 2.6.53

24. Suppose x lies in the interval (1,3) with $x \neq 2$. Find the smallest positive value of $\delta$ such that the inequality $0 < |x-2| < \delta$ is true for all possible values of x.

The smallest positive value of $\delta$ is ____. (Type an integer or a fraction.)

$-\delta = -1$ or $\delta = 1$

Answer: 1

$0 < |x-2| < \delta$

$|x-2| < \delta$

$\frac{-\delta}{-1} = \frac{-1}{-1}$

ID: 2.7.1

$\delta = 1$

$-\delta < x-2 < \delta$

$-\delta+2 < x-2+2 < \delta+2$

$-\delta+2 < x < \delta+2$

$\delta = 1$

$-\delta+2 = 1$ or $\delta+2 = 3$

$-\delta+2-2 = 1-2$ or $\delta+2-2 = 3-2$

25. Suppose $|f(x) - 7| < 0.3$ whenever $0 < x < 7$. Find all values of $\delta > 0$ such that $|f(x) - 7| < 0.3$ whenever $0 < |x - 2| < \delta$.

The values of $\delta$ are $0 < \delta \le$ [ 2 ]. (Type an integer or a fraction.)

Answer: 2

ID: 2.7.7

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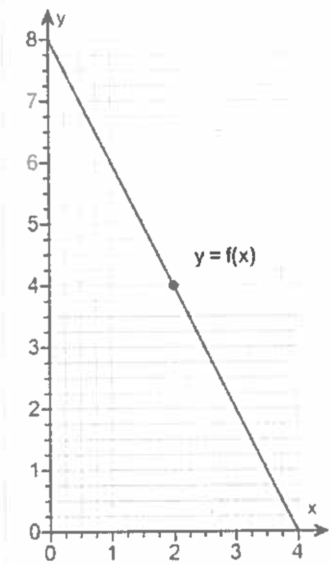
26. The function f in the figure satisfies $\lim\limits_{x \to 2} f(x) = 4$. Determine the largest value of $\delta > 0$ satisfying each statement.

**a.** If $0 < |x - 2| < \delta$, then $|f(x) - 4| < 2$.
**b.** If $0 < |x - 2| < \delta$, then $|f(x) - 4| < 1$.




**a.** $\delta$ = [ 1 ] (Simplify your answer.)

**b.** $\delta$ = [ 1/2 ] (Simplify your answer.)

Answers 1

$\frac{1}{2}$

ID: 2.7.9

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27. Use the precise definition of a limit to prove the following limit. Specify a relationship between ε and δ that guarantees the limit exists.

$$\lim_{x \to 0} (6x + 9) = 9$$

Let $\varepsilon > 0$ be given. Choose the correct proof below.

- A. Choose $\delta = \varepsilon$. Then, $|(6x+9) - 9| < \varepsilon$ whenever $0 < |x - 0| < \delta$.
- B. Choose $\delta = 6\varepsilon$. Then, $|(6x+9) - 9| < \varepsilon$ whenever $0 < |x - 0| < \delta$.
- C. Choose $\delta = \frac{\varepsilon}{9}$. Then, $|(6x+9) - 9| < \varepsilon$ whenever $0 < |x - 0| < \delta$.
- D. Choose $\delta = \frac{\varepsilon}{6}$. Then, $|(6x+9) - 9| < \varepsilon$ whenever $0 < |x - 0| < \delta$.
- E. None of the above proofs is correct.

$|(6x+9) - 9| < e$

$|6x + 9 - 9| < e$

$|6x| < e$

$|6| \cdot |x| < e$

$6|x| < e$

$\frac{\cancel{6}|x|}{\cancel{6}} < \frac{e}{6}$

$|x| < \frac{e}{6}$

$|x - 0| < \frac{e}{6}$ rewrite

Answer: D. Choose $\delta = \frac{\varepsilon}{6}$. Then, $|(6x+9) - 9| < \varepsilon$ whenever $0 < |x - 0| < \delta$.

let $\delta = \frac{e}{6}$

ID: 2.7.19

28. Find the average velocity of the function over the given interval.

$y = \frac{3}{x-2}$, $[4,7]$
a b

- A. 2
- B. 7
- C. $\frac{1}{3}$
- D. $-\frac{3}{10}$

Answer: D. $-\frac{3}{10}$

ID: 2.1-3

$\frac{f(b) - f(a)}{b - a} =$

$\frac{f(7) - f(4)}{7 - 4} =$

$\frac{\left(\frac{3}{7-2}\right) - \left(\frac{3}{4-2}\right)}{7-4} =$

$\frac{\frac{3}{5} - \frac{3}{2}}{3} =$

$\frac{\frac{3}{5}\left(\frac{2}{2}\right) - \frac{3}{2}\left(\frac{5}{5}\right)}{3} =$

$\frac{\frac{6}{10} - \frac{15}{10}}{3} =$

$\frac{\frac{6-15}{10}}{3} =$

$\frac{\frac{-9}{10}}{\frac{3}{1}}$

$\frac{-9}{10} \cdot \frac{1}{3} =$

$\frac{-3(\cancel{3})}{10} \cdot \frac{1}{(\cancel{3})} =$

$\frac{-3}{10} =$

29. Find all vertical asymptotes of the given function.

$g(x) = \frac{x+11}{x^2+16x}$

Set $x^2 - 16x = 0$

$x(x-16) = 0$

$x = 0$ or $x - 16 = 0$

or $x - \cancel{16} + \cancel{16} = 0 + 16$

$x = 16$

- A. $x = 0, x = -16$
- B. $x = -16, x = -11$
- C. $x = 0, x = -4, x = 4$
- D. $x = -4, x = 4$

Answer: A. $x = 0, x = -16$

ID: 2.4-19

30. Divide numerator and denominator by the highest power of x in the denominator to find the limit at infinity.

$\lim_{x\to\infty} \frac{\sqrt[3]{x} - 3x - 5}{-6x + x^{2/3} - 7}$

$\lim_{x\to\infty} \frac{x^{1/3} - 3x^1 - 5}{-6x^1 + x^{2/3} - 7} =$

$\lim_{x\to\infty} \frac{x^{1/3} - 3x^{3/3} - 5}{-6x^{3/3} + x^{2/3} - 7} =$

$\lim_{x\to\infty} \left(\frac{x^{1/3} - 3x^{3/3} - 5}{-6x^{3/3} + x^{2/3} - 7}\right) \frac{\frac{1}{x^{3/3}}}{\frac{1}{x^{3/3}}}$

- A. $\frac{1}{2}$
- B. $-\infty$
- C. 2
- D. 0

Answer: A. $\frac{1}{2}$

ID: 2.5-12

$\lim_{x\to\infty} \frac{\frac{x^{1/3}}{x^{3/3}} - \frac{3\cancel{x}^{3/3}}{x^{3/3}} - \frac{5}{x^{3/3}}}{\frac{-6\cancel{x}^{3/3}}{\cancel{x}^{3/3}} + \frac{x^{2/3}}{x^{3/3}} - \frac{7}{x^{3/3}}}$

formula $\lim_{x\to\infty} \frac{1}{x^n} = 0$

$\lim_{x\to\infty} \frac{\frac{1}{x^{2/3}} - 3 - \frac{5}{x}}{-6 + \frac{1}{x^{1/3}} - \frac{7}{x}} =$

$\frac{0 - 3 - 0}{-6 + 0 - 0} = \frac{-3}{-6} = \frac{-\cancel{3}(1)}{-\cancel{3}(2)} = \frac{1}{2} =$

31. Use the graph to find a $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta$ and $|f(x) - L| < \varepsilon$. Use the following information:
$f(x) = 2x + 3$, $\varepsilon = 0.2$, $x_0 = 1$, $L = 5$.

$|(2x+3) - (5)| < 0.2$
$|2x+3-5| < 0.2$
$|2x-2| < 0.2$
$2|x-1| < 0.2$
$\frac{2|x-1|}{2} < \frac{0.2}{2}$

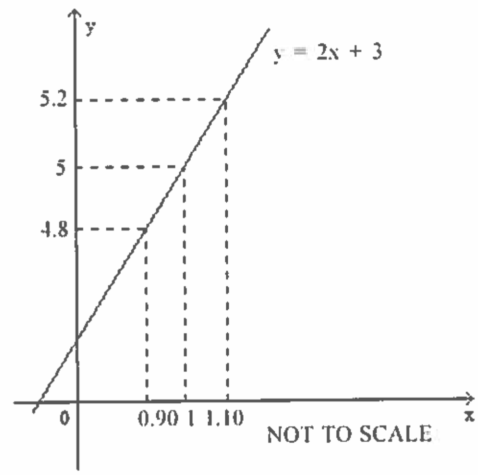
$|x-1| < 0.1$

$\delta = 0.1$

[1] Click the icon to view the graph.

- A. 4
- B. 0.2
- C. 0.4
- D. 0.1

1: Graph




Answer: D. 0.1

ID: 2.7-1

32. Find the value of the derivative of the function at the given point.

$f(x) = 4x^2 - 5x$; $(-1, 9)$

$f'(-1) =$ ______ (Type an integer or a simplified fraction.)

Answer: $-13$

$f(x) = 4x^2 - 5x$

ID: 3.1.1

$f'(x) = 8x - 5$

$f'(-1) = 8(-1) - 5$

$f'(-1) = -8 - 5$

$f'(-1) = -13$

$f'(a) = 2a =$ Slope $m$

33.

a. Use the definition $m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ to find the slope of the line tangent to the graph of f at P.

b. Determine an equation of the tangent line at P.

$f(x) = x^2 - 3$, $P(-5, 22)$

a. $m_{tan} =$ ☐

b. $y =$ ☐

Answers $-10$

$-10x - 28$

ID: 3.1.25

$f'(a) =$

$\lim_{h \to 0} \frac{((a+h)^2 - 3) - (a^2 - 3)}{h} =$

$\lim_{h \to 0} \frac{(a+h)(a+h) - 3 - a^2 + 3}{h} =$

$\lim_{h \to 0} \frac{\cancel{a^2} + ah + ah + h^2 \cancel{-3} \cancel{-a^2} \cancel{+3}}{h}$

$\lim_{h \to 0} \frac{2ah + h^2}{h} =$

$\lim_{h \to 0} (2a + h) =$

$2a + 0 = 2a$

$f'(-5) = 2(-5) = -10 = m$

$m = -10 \quad (-5, 22)$

$y - y_1 = m(x - x_1)$

$y - (22) = -10(x - (-5))$

$y - 22 = -10(x + 5)$

$y - 22 = -10x - 50$

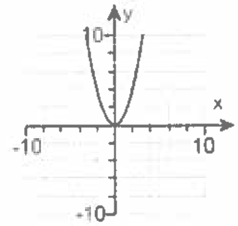
$y - 22 + 22 = -10x - 50 + 22$

$y = -10x - 28$

34. Match the graph of the function on the right with the graph of its derivative.

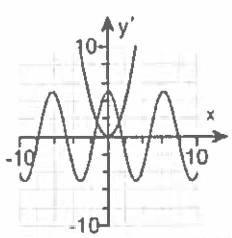
Example if $y = x^2$

then $y' = 2x$

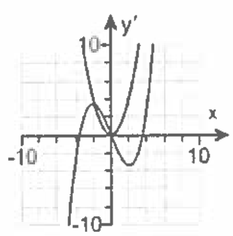



Choose the correct graph of the function (in blue) and its derivative (in red) below.

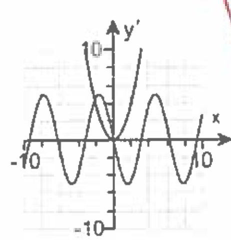
○ A.



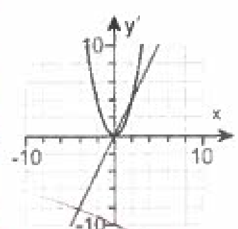

○ B.



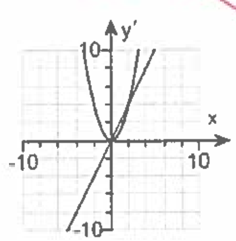

○ C.




✓ D.




Answer:




D.

ID: 3.2.49

$y = x \quad y' = 1$

$y = x^2 \quad y' = 2x$

$y = x^3 \quad y' = 3x^2$

$y = x^4 \quad y' = 4x^3$

Examples

39. **a.** Use the Product Rule to find the derivative of the given function.
**b.** Find the derivative by expanding the product first.

$f(x) = (x-3)(4x+2)$

**a.** Use the product rule to find the derivative of the function. Select the correct answer below and fill in the answer box(es) to complete your choice.

- ○ **A.** The derivative is $(x-3)(\_\_\_) + (4x+2)(\_\_\_)$.
- ○ **B.** The derivative is $(x-3)(4x+2)(\_\_\_)$.
- ○ **C.** The derivative is $(\_\_\_)(x-3)$.
- ○ **D.** The derivative is $(\_\_\_)x(4x+2)$.
- ○ **E.** The derivative is $(x-3)(4x+2) + (\_\_\_)$.

**b.** Expand the product.

$(x-3)(4x+2) = \_\_\_$ (Simplify your answer.)

Using either approach, $\frac{d}{dx}(x-3)(4x+2) = \_\_\_$.

Answers A. The derivative is $(x-3)(\boxed{4}) + (4x+2)(\boxed{1})$.

$4x^2 - 10x - 6$

$8x - 10$

$f(x) = (x-3)(4x+2)$

ID: 3.4.9

$f'(x) = (x-3)'(4x+2) + (x-3)(4x+2)'$

$f'(x) = (1-0)(4x+2) + (x-3)(4+0)$

$f'(x) = (1)(4x+2) + (x-3)(4)$

$f'(x) = 4x+2 + 4x-12$

$f'(x) = 8x-10$ OR

$f(x) = (x-3)(4x+2)$

$f(x) = 4x^2 + 2x - 12x - 6$

$f(x) = 4x^2 - 10x - 6$

$f'(x) = 8x - 10 - 0$

$f'(x) = 8x - 10$

35. A line perpendicular to another line or to a tangent line is called a normal line. Find an equation of the line perpendicular to the line that is tangent to the following curve at the given point P.

$y = 7x - 15$; $P(2, -1)$

$m = -\frac{1}{7}$

$(2, -1)$ $x_1$ $y_1$

The equation of the normal line at $P(2, -1)$ is ______.

Answer: $y = -\frac{1}{7}x + \left(-\frac{5}{7}\right)$

$y - y_1 = m(x - x_1)$
$y - (-1) = -\frac{1}{7}(x - (2))$
$y + 1 = -\frac{1}{7}(x - 2)$
$y + 1 = -\frac{1}{7}x + \frac{2}{7}$
$y + 1 - 1 = -\frac{1}{7}x + \frac{2}{7} - 1$
$y = -\frac{1}{7}x + \frac{2}{7} - \frac{7}{7}$
$y = -\frac{1}{7}x - \frac{5}{7}$

ID: 3.2.63

36. Evaluate the derivative of the function given below using a limit definition of the derivative.

$f(x) = x^2 + 6x - 8$

$f'(x) =$ ______

Answer: $2x + 6$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$

$\lim_{h \to 0} \frac{((x+h)^2 + 6(x+h) - 8) - (x^2 + 6x - 8)}{h}$

$\lim_{h \to 0} \frac{(x+h)(x+h) + 6x + 6h - 8 - x^2 - 6x + 8}{h}$

$\lim_{h \to 0} \frac{x^2 + xh + xh + h^2 + 6x + 6h - 8 - x^2 - 6x + 8}{h}$

$\frac{2xh + h^2 + 6h}{h} =$

$\lim_{h \to 0} 2x + h + 6 =$

$2x + 0 + 6 =$

$2x + 6 =$

ID: EXTRA 3.2

37. Use the Quotient Rule to evaluate and simplify $\frac{d}{dx}\left(\frac{x-4}{2x-5}\right)$.

$\frac{d}{dx}\left(\frac{x-4}{2x-5}\right) =$ ______

Answer: $\frac{3}{(2x-5)^2}$

$\frac{(x-4)'(2x-5) - (x-4)(2x-5)'}{(2x-5)^2}$

$\frac{(1-0)(2x-5) - (x-4)(2-0)}{(2x-5)^2}$

$\frac{(1)(2x-5) - (x-4)(2)}{(2x-5)^2} =$

$\frac{2x - 5 - 2x + 8}{(2x-5)^2} = \frac{3}{(2x-5)^2}$

ID: 3.4.5

38. Use the Quotient Rule to find $g'(1)$ given that $g(x) = \frac{2x^2}{6x+1}$.

$g'(1) =$ ______
(Simplify your answer.)

Answer: $\frac{16}{49}$

$\frac{(2x^2)'(6x+1) - (2x^2)(6x+1)'}{(6x+1)^2} =$

$\frac{(4x)(6x+1) - (2x^2)(6+0)}{(6x+1)^2} =$

ID: 3.4.6

$g'(x) = \frac{24x^2 + 4x - 12x^2}{(6x+1)^2}$

$g'(x) = \frac{12x^2 + 4x}{(6x+1)^2}$

$g'(1) = \frac{12(1)^2 + 4(1)}{(6(1)+1)^2}$

$g'(1) = \frac{12 + 4}{(6+1)^2}$

$g'(1) = \frac{16}{(7)^2}$

$g'(1) = \frac{16}{49}$

40. Use the quotient rule to find the derivative of the given function. Then find the derivative by first simplifying the function. Are the results the same?

$$h(w) = \frac{5w^4 - w}{w}$$

$h'(w) = \frac{(5w^4-w)'(w) - (5w^4-w)(w)'}{(w)^2}$

What is the immediate result of applying the quotient rule? Select the correct answer below.

$h'(w) = \frac{(20w^3-1)(w) - (5w^4-w)(1)}{w^2}$

- A. $(20w^3 - 1)(w) + (5w^4 - w)(1)$
- B. $15w^2$
- C. $5w^3 - 1$
- D. $\frac{w(20w^3 - 1) - (5w^4 - w)(1)}{w^2}$

$h'(w) = \frac{20w^4 - \cancel{w} - 5w^4 + \cancel{w}}{w^2}$

$h'(w) = \frac{15w^4}{w^2}$

What is the fully simplified result of applying the quotient rule?

$h'(w) = 15w^2$

What is the result of first simplifying the function, then taking the derivative? Select the correct answer below.

OR

$h(w) = \frac{5w^4 - w}{w}$

- A. $15w^2$
- B. $5w^3 - 1$
- C. $(20w^3 - 1)(w) + (5w^4 - w)(1)$
- D. $\frac{w(20w^3 - 1) - (5w^4 - w)(1)}{w^2}$

$h(w) = \frac{5w^4}{w^1} - \frac{w}{w}$

Are the two results the same?

$h(w) = 5w^3 - 1$

- Yes
- No

$h'(w) = 15w^2 - 0$

$h'(w) = 15w^2$

Answers D. $\frac{w(20w^3 - 1) - (5w^4 - w)(1)}{w^2}$

$15w^2$

A. $15w^2$

Yes

ID: 3.4.11

41. If $f(x) = -\cos x$, then what is the value of $f'(\pi)$?

$f(x) = -\cos x$

$f'(\pi) =$ (Simplify your answer.)

$f'(x) = -(-\sin x)$

$f'(\pi) = 0$

Answer: 0

$f'(x) = \sin x$

ID: 3.5.5

$f'(\pi) = \sin(\pi)$

42. Evaluate the limit.

$$\lim_{x\to 0}\frac{\sin 3x}{\sin 2x}$$

$\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to 0} \frac{\frac{\sin(3x)}{x}}{\frac{\sin(2x)}{x}} = \lim_{x\to 0} \frac{\frac{\sin(3x)}{x}\cdot\frac{3}{3}}{\frac{\sin(2x)}{x}\cdot\frac{2}{2}} = \lim_{x\to 0}\frac{3\cdot\frac{\sin(3x)}{3x}}{2\cdot\frac{\sin(2x)}{2x}}$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$\frac{3}{2}\lim_{x\to 0}\frac{\frac{\sin(3x)}{3x}}{\frac{\sin(2x)}{2x}} = \frac{3}{2}\cdot\frac{(1)}{(1)} = \frac{3}{2}$

○ A. $\lim_{x\to 0}\frac{\sin 3x}{\sin 2x} =$ ______

○ B. The limit is undefined.

$\frac{3}{2}\,\frac{\lim_{x\to 0}\frac{\sin(3x)}{3x}}{\lim_{x\to 0}\frac{\sin(2x)}{2x}} =$

Answer: A. $\lim_{x\to 0}\frac{\sin 3x}{\sin 2x} = \boxed{\frac{3}{2}}$

ID: 3.5.13

43. Find $\frac{dy}{dx}$ for the following function.

$y = 7\sin x + 9\cos x$

$\frac{dy}{dx} =$ ______

$y = 7\sin(x) + 9\cos(x)$

$y' = 7\cos(x)(x)' - 9\sin(x)(x)'$

$y' = 7\cos(x)(1) - 9\sin(x)(1)$

$y' = 7\cos(x) - 9\sin(x)$

Answer: $7\cos x - 9\sin x$

ID: 3.5.23

44. Find the derivative of the following function.

$y = e^{-x}\sin x$

$\frac{dy}{dx} =$ ______

$y' = (e^{-x})'(\sin x) + (e^{-x})(\sin(x))'$

$y' = (e^{-x}(-x)')(\sin(x)) + (e^{-x})(\cos(x)(x)')$

$y' = e^{-x}(-1)(\sin(x)) + (e^{-x})(\cos(x)(1))$

$y' = -e^{-x}\sin(x) + e^{-x}\cos(x)$

$y' = e^{-x}\cos(x) - e^{-x}\sin x$

$y' = e^{-x}(\cos(x) - \sin(x))$

Answer: $e^{-x}(\cos x - \sin x)$

ID: 3.5.25

45. Find an equation of the line tangent to the following curve at the given point.

$y = 2x^2 + 3\sin x;\ P(0,0)$

The equation for the tangent line is ______.

$y' = 4x + 3\cos(x)$

$y'(0) = 4(0) + 3\cos(0)$

$y'(0) = 0 + 3(1)$

$y'(0) = 0 + 3$

$y'(0) = 3 = m$ slope

$m = 3$

$(x_1, y_1) = (0, 0)$ $x_1$ $y_1$

$y - y_1 = m(x - x_1)$

$y - (0) = 3(x - 0)$

$y - 0 = 3(x - 0)$

$y = 3(x)$

$y = 3x$

Answer: $y = 3x$

ID: EXTRA 3.73

46. Let $h(x) = f(g(x))$ and $p(x) = g(f(x))$. Use the table below to compute the following derivatives.

a. $h'(1)$

b. $p'(4)$

$h'(1) =$ ☐ (Simplify your answer.)

$p'(4) =$ ☐ (Simplify your answer.)

$h'(x) = f'(g(x)) \cdot g'(x)$

$p'(x) = g'(f(x)) \cdot f'(x)$

| x | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| f(x) | 3 | 2 | 4 | 1 |
| f′(x) | −2 | −5 | −4 | −6 |
| g(x) | 4 | 1 | 2 | 3 |
| g′(x) | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{3}{5}$ | $\frac{4}{5}$ |

$h'(1) = f'(g(1)) \cdot g'(1)$

$= f'(4) \cdot \frac{1}{5}$

$= \frac{-6}{1} \cdot \frac{1}{5}$

$= -\frac{6}{5}$

$p'(4) = g'(f(4)) \cdot f'(4)$

$p'(4) = g'(1) \cdot (-6)$

$= \frac{1}{5} \cdot (-6)$

$= \frac{1}{5} \cdot \frac{-6}{1}$

$= \frac{-6}{5}$

Answers $-\frac{6}{5}$

$-\frac{6}{5}$

ID: 3.7.25

47. Calculate the derivative of the following function.

$y = (5x + 1)^7$

$\frac{dy}{dx} =$ ☐

$y' = 7(5x+1)^{7-1}(5x+1)'$

$y' = 7(5x+1)^6(5+0)$

$y' = 7(5x+1)^6(5)$

$y' = 35(5x+1)^6$

Answer: $35(5x+1)^6$

ID: 3.7.27

48. Calculate the derivative of the following function.

$y = 3\left(6x^5 + 7\right)^{-4}$

$\frac{dy}{dx} =$ ☐

$y = 3(6x^5+7)^{-4}$

$y' = 3(-4)(6x^5+7)^{-4-1} \cdot (6x^5+7)'$

$y' = -12(6x^5+7)^{-5} \cdot (30x^4+0)$

$y' = -12(6x^5+7)^{-5} \cdot (30x^4)$

$y' = -360x^4(6x^5+7)^{-5}$

$y' = \frac{-360x^4}{(6x^5+7)^5}$

Answer: $\frac{-360x^4}{\left(6x^5+7\right)^5}$

ID: 3.7.31

49. Calculate the derivative of the following function.

$y = \cos(3t + 20)$

$\frac{dy}{dt} =$ ☐

Answer: $-3\sin(3t + 20)$

ID: 3.7.32

$y = \cos(3t+20)$

$y' = -\sin(3t+20)\,(3t+20)'$

$y' = -\sin(3t+20)\,(3+0)$

$y' = -\sin(3t+20)\,(3)$

$y' = -3\sin(3t+20)$

50. Calculate the derivative of the following function.

$y = \tan(e^x)$

$\frac{dy}{dx} =$ ☐

Answer: $e^x \sec^2 e^x$

ID: 3.7.35

$y = \tan(e^x)$

$y' = \sec^2(e^x)\cdot(e^x)'$

$y' = \sec^2(e^x)\cdot(e^x(x)')$

$y' = \sec^2(e^x)\cdot(e^x(1))$

$y' = \sec^2(e^x)\cdot(e^x)$

$y' = e^x\sec^2(e^x)$

51. Calculate the derivative of the following function.

$y = \sin(6\cos x)$

$\frac{dy}{dx} =$ ☐

Answer: $-6\cos(6\cos x)\cdot\sin x$

ID: 3.7.44

$y = \sin(6\cos(x))$

$y' = \cos(6\cos(x))\cdot(6\cos(x))'$

$y' = \cos(6\cos(x))\cdot(-6\sin(x)\,(x)')$

$y' = \cos(6\cos(x))\cdot(-6\sin(x)(1))$

$y' = \cos(6\cos(x))\cdot(-6\sin(x))$

$y' = -6\cos(6\cos(x))\cdot\sin(x)$

52. Calculate $\frac{dy}{dx}$ using implicit differentiation.

$16x = y^4$

$\frac{dy}{dx} =$ ☐

Answer: $\frac{4}{y^3}$

ID: 3.8.5

$16x = y^4$

$16(1) = 4y^3 y'$

$16 = 4y^3 y'$

$\frac{16}{4y^3} = \frac{\cancel{4y^3} y'}{\cancel{4y^3}}$

$\frac{16}{4y^3} = y'$

$\frac{4\cdot 4}{4y^3} = y'$

$\frac{4}{y^3} = y'$

OR

$\frac{4}{y^3} = \frac{dy}{dx}$

53. Calculate $\frac{dy}{dx}$ using implicit differentiation.

$\cos(y) + 8 = x$

$\frac{dy}{dx} =$ ______

Answer: $-\csc y$

ID: 3.8.7

$\cos(y) + 8 = x$

$-\sin(y)\, y' + 0 = 1$

$-\sin(y)\, y' = 1$

$\frac{\cancel{-\sin(y)}\, y'}{\cancel{-\sin(y)}} = \frac{1}{-\sin(y)}$

$y' = \frac{1}{-\sin(y)}$

$y' = -\frac{1}{\sin(y)}$

$y' = -\csc(y)$

OR

$\frac{dy}{dx} = -\csc(y)$

54. Consider the curve $x = y^3$. Use implicit differentiation to verify that $\frac{dy}{dx} = \frac{1}{3y^2}$ and then find $\frac{d^2y}{dx^2}$.

Use implicit differentiation to find the derivative of each side of the equation.

$\frac{d}{dx}x =$ ______ and $\frac{d}{dx}y^3 =$ ______ $\frac{dy}{dx}$

Solve for $\frac{dy}{dx}$.

$\frac{dy}{dx} =$ ______

Find $\frac{d^2y}{dx^2}$.

$\frac{d^2y}{dx^2} =$ ______

Answers 1

$3y^2$

$\frac{1}{3y^2}$

$-\frac{2}{9y^5}$

ID: 3.8.11

$x = y^3$

$1 = 3y^2 y'$

$\frac{1}{3y^2} = \frac{3y^2 y'}{3y^2}$

$\frac{1}{3y^2} = y'$

$\frac{dy}{dx} = \frac{1}{3y^2}$

$y' = \frac{1}{3y^2}$

$y' = \frac{1}{3}y^{-2}$

$y'' = \frac{1}{3}(-2y^{-2-1}) \cdot y'$

$y'' = \frac{1}{3}(-2y^{-3})(y')$

$y'' = \frac{1}{3}\left(\frac{-2}{y^3}\right)(y')$

$y'' = \frac{-2}{3y^3} \cdot y'$

$y'' = \frac{-2}{3y^3} \cdot \left(\frac{1}{3y^2}\right)$ subsx

$y'' = \frac{-2}{9y^{3+2}}$

$y'' = \frac{-2}{9y^5}$

OR

$\frac{d^2y}{dx^2} = \frac{-2}{9y^5}$

55. Carry out the following steps for the given curve.

**a.** Use implicit differentiation to find $\frac{dy}{dx}$.

**b.** Find the slope of the curve at the given point.

$x^3 + y^3 = -63; (-4, 1)$

**a.** Use implicit differentiation to find $\frac{dy}{dx}$.

$\frac{dy}{dx} = $ ______

**b.** Find the slope of the curve at the given point.

The slope of $x^3 + y^3 = -63$ at $(-4, 1)$ is ______.
(Simplify your answer.)

Answers $\frac{-x^2}{y^2}$

$-16$

ID: 3.8.13

$x^3 + y^3 = -63$

$3x^2 + 3y^2y' = 0$

$3y^2y' = -3x^2$

$\frac{\cancel{3y^2}y'}{\cancel{3y^2}} = \frac{-3x^2}{3y^2}$

$y' = \frac{-1x^2}{y^2}$

$y'(-4, 1) = \frac{-1(-4)^2}{(1)^2}$

$y'(-4, 1) = \frac{-1(-4)(-4)}{(1)(1)}$

$y'(-4, 1) = \frac{-16}{1}$

$y'(-4, 1) = -16$

56. Use implicit differentiation to find $\frac{dy}{dx}$.

$\cos(y) + \sin(x) = 2y$

$\frac{dy}{dx} = $ ______

Answer: $\frac{\cos x}{2 + \sin y}$

ID: 3.8.27

$\cos(y) + \sin(x) = 2y$

$-\sin(y)y' + \cos(x)(x)' = 2y'$

$-\sin(y)y' + \cos(x)(1) = 2y'$

$-\sin(y)y' + \cos(x) = 2y'$

$\cos(x) = 2y' + \sin(y)y'$

$\cos(x) = y'(2 + \sin(y))$

$\frac{\cos(x)}{2 + \sin(y)} = \frac{y'(\cancel{2 + \sin(y)})}{\cancel{(2 + \sin(y))}}$

$\frac{\cos(x)}{2 + \sin(y)} = y'$

OR

$\frac{\cos(x)}{2 + \sin(y)} = \frac{dy}{dx}$

57. Use implicit differentiation to find $\frac{dy}{dx}$.

$3\cos(xy) = 4x + 7y$

$\frac{dy}{dx} = $ ______

Answer: $\frac{4 + 3y\sin(xy)}{-3x\sin(xy) - 7}$

ID: 3.8.31

$-3\sin(xy) \cdot (xy)' = 4 + 7y'$

$-3\sin(xy) \cdot ((x)'(y) + (x)(y)') = 4 + 7y'$

$-3\sin(xy) \cdot ((1)y + xy') = 4 + 7y'$

$-3\sin(xy) \cdot (y + xy') = 4 + 7y'$

$-3\sin(xy) \cdot y - 3\sin(xy)xy' = 4 + 7y'$

$-3\sin(xy)xy' - 7y' = 4 + 3\sin(xy)y$

$y'(-3\sin(xy)x - 7) = 4 + 3y\sin(xy)$

$\frac{y'\cancel{(-3\sin(xy)(x) - 7)}}{\cancel{-3\sin(xy)(x) - 7}} = \frac{4 + 3y\sin(xy)}{-3\sin(xy)(x) - 7}$

$y' = \frac{4 + 3y\sin(xy)}{-3x\sin(xy) - 7}$

58. Use implicit differentiation to find $\frac{dy}{dx}$.

$e^{3xy} = 8y$

$\frac{dy}{dx} =$ ______

Answer: $\frac{3y\,e^{3xy}}{8 - 3x\,e^{3xy}}$

ID: 3.8.32

$e^{3xy}\cdot(3xy)' = 8y'$

$e^{3xy}\cdot\left((3x)'(y) + (3x)y'\right) = 8y'$

$e^{3xy}\cdot\left((3)(y + (3x)y')\right) = 8y'$

$e^{3xy}\cdot(3y + 3xy') = 8y'$

$3e^{3xy}y + 3e^{3xy}xy' = 8y'$

$3e^{3xy}y = 8y' - 3e^{3xy}xy'$

$3e^{3xy}y = y'(8 - 3e^{3xy}(x))$

$\frac{3e^{3xy}y}{8 - 3e^{3xy}(x)} = \frac{y'(8 - 3e^{3xy}(x))}{(8 - 3e^{3xy}(x))}$

$\frac{3ye^{3xy}}{8 - 3xe^{3xy}} = y'$

59. Use implicit differentiation to find $\frac{dy}{dx}$ for the following equation.

$5x^5 + 7y^5 = 12xy$

$\frac{dy}{dx} =$ ______

Answer: $\frac{25x^4 - 12y}{12x - 35y^4}$

ID: 3.8.37

$25x^4 + 35y^4\cdot y' = (12x)'(y) + (12x)(y)'$

$25x^4 + 35y^4y' = (12)(y) + (12x)y'$

$25x^4 + 35y^4y' = 12y + 12xy'$

$25x^4 - 12y = 12xy' - 35y^4y'$

$25x^4 - 12y = y'(12x - 35y^4)$

$\frac{25x^4 - 12y}{12x - 35y^4} = \frac{y'(12x - 35y^4)}{(12x - 35y^4)}$

$\frac{25x^4 - 12y}{12x - 35y^4} = y'$

60. Find $\frac{d}{dx}\left(\ln\sqrt{x^2+9}\right)$.

$\frac{d}{dx}\left(\ln\sqrt{x^2+9}\right) =$ ______

Answer: $\frac{x}{x^2+9}$

ID: 3.9.9

formula

$y = \ln f(x)$

$y' = \frac{f'(x)}{f(x)}$

$y = \ln\sqrt{x^2+9}$

$y = \ln(x^2+9)^{\frac{1}{2}}$

$y = \frac{1}{2}\ln(x^2+9)$

$y' = \frac{1}{2}\,\frac{(x^2+9)'}{(x^2+9)}$

$y' = \frac{1}{2}\,\frac{(2x+0)}{(x^2+9)}$

$y' = \frac{1}{2}\left(\frac{2x}{x^2+9}\right)$

$y' = \frac{x}{x^2+9}$

61. Express the function $f(x) = g(x)^{h(x)}$ in terms of the natural logarithmic and natural exponential functions (base $e$).

$f(x) =$ ______

Answer: $e^{h(x)\ln g(x)}$

ID: 3.9.11

$f(x) = e^{\ln(g(x))^{h(x)}}$

rewrite

$f(x) = e^{h(x)\ln(g(x))}$

62. Find the derivative.

$$\frac{d}{dx}\left(\ln\left(2x^2+3\right)\right)$$

$\frac{d}{dx}\left(\ln\left(2x^2+3\right)\right) =$ ______

Answer: $\frac{4x}{2x^2+3}$

ID: 3.9.17

$y = \ln(2x^2+3)$

$y' = \frac{(2x^2+3)'}{2x^2+3}$

$y' = \frac{4x+0}{2x^2+3}$

$y' = \frac{4x}{2x^2+3}$

$y = \ln f(x)$

$y' = \frac{f'(x)}{f(x)}$

formula

63. Evaluate the derivative.

$$y = 2x^{2\pi}$$

$y' =$ ______ (Type an exact answer.)

Answer: $4\pi x^{(2\pi-1)}$

ID: 3.9.33

$y = 2x^{2\pi}$

$y' = 2(2\pi)x^{2\pi-1}$

$y' = 4\pi x^{2\pi-1}$

64. Find $\frac{dy}{dx}$ for the function $y = 17^x$.

$\frac{dy}{dx} =$ ______

Answer: $17^x \ln 17$

ID: 3.9.37

$y = 17^x$

$y' = 17^x \cdot \ln(17)$

65. Calculate the derivative of the following function.

$$y = 8\log_4\left(x^3-3\right)$$

$\frac{d}{dx} 8\log_4\left(x^3-3\right) =$ ______

Answer: $\frac{24x^2}{\left(x^3-3\right)\ln 4}$

ID: 3.9.63

$y = 8\log_4(x^3-3)$

$y' = 8\frac{(x^3-3)'}{(x^3-3)\ln(4)}$

$y' = 8\frac{(3x^2-0)}{(x^3-3)\ln(4)}$

$y' = 8\frac{(3x^2)}{(x^3-3)\ln(4)}$

$y' = \frac{24x^2}{(x^3-3)\ln(4)}$

formula

$y = \log_b(f(x))$

$y' = \frac{f'(x)}{f(x)\cdot\ln(b)}$

66. Use logarithmic differentiation to evaluate f′(x).

$$f(x) = \frac{(x+4)^8}{(2x-4)^{12}}$$

f′(x) = ______

Answer: $\frac{(x+4)^8}{(2x-4)^{12}}\left[\frac{8}{x+4} - \frac{12}{x-2}\right]$

ID: 3.9.77

$$\ln(f(x)) = \ln \frac{(x+4)^8}{(2x-4)^{12}}$$

$$\ln f(x) = \ln(x+4)^8 - \ln(2x-4)^{12}$$

$$\ln(f(x)) = 8\ln(x+4) - 12\ln(2x-4)$$

$$\frac{f'(x)}{f(x)} = 8\frac{(x+4)'}{(x+4)} - 12\frac{(2x-4)'}{(2x-4)}$$

$$\frac{f'(x)}{f(x)} = 8\frac{(1+0)}{x+4} - 12\frac{(2-0)}{2x-4}$$

$$\frac{f'(x)}{f(x)} = 8\frac{(1)}{x+4} - 12\frac{(2)}{2x-4}$$

$$\frac{f'(x)}{f(x)} = \frac{8}{x+4} - \frac{24}{2x-4}$$

formula

$y = \ln(f(x))$

$y' = \frac{f'(x)}{f(x)}$

$$\frac{f'(x)}{f(x)} = \frac{8}{x+4} - \frac{\cancel{2}(12)}{\cancel{2}(x-2)}$$

$$\frac{f'(x)}{f(x)} = \frac{8}{x+4} - \frac{12}{x-2}$$

$$\cancel{f(x)}\left(\frac{f'(x)}{\cancel{f(x)}}\right) = f(x)\cdot\left[\frac{8}{x+4} - \frac{12}{x-2}\right]$$

$$f'(x) = \frac{(x+4)^8}{(2x-4)^{12}}\left[\frac{8}{x+4} - \frac{12}{x-2}\right]$$

67. State the derivative formulas for $\sin^{-1}x$, $\tan^{-1}x$, and $\sec^{-1}x$.

$y = \sin^{-1}(f(x))$

$y' = \dfrac{f'(x)}{\sqrt{1-(f(x))^2}}$ $-1<x<1$

What is the derivative of $\sin^{-1}x$?

- A. $\dfrac{1}{|x|\sqrt{x^2-1}}$ for $|x|>1$
- B. $-\dfrac{1}{|x|\sqrt{x^2-1}}$ for $|x|>1$
- C. $\dfrac{1}{\sqrt{1-x^2}}$ for $-1<x<1$
- D. $-\dfrac{1}{\sqrt{1-x^2}}$ for $-1<x<1$

$y' = \dfrac{1}{\sqrt{1-x^2}}$ $-1<x<1$

What is the derivative of $\tan^{-1}x$?

$y = \tan^{-1}(f(x))$

$y' = \dfrac{f'(x)}{1+(f(x))^2}$ $-\infty<x<\infty$

- A. $-\dfrac{1}{|x|\sqrt{x^2-1}}$ for $|x|>1$
- B. $-\dfrac{1}{\sqrt{1-x^2}}$ for $-1<x<1$
- C. $-\dfrac{1}{1+x^2}$ for $-\infty<x<\infty$
- D. $\dfrac{1}{1+x^2}$ for $-\infty<x<\infty$

$y' = \dfrac{1}{1+x^2}$ $-\infty<x<\infty$

What is the derivative of $\sec^{-1}x$?

$y = \sec^{-1}(f(x))$

$y' = \dfrac{f'(x)}{|f(x)|\sqrt{(f(x))^2-1}}$ $|y|>1$

- A. $-\dfrac{1}{|x|\sqrt{x^2-1}}$ for $|x|>1$
- B. $-\dfrac{1}{\sqrt{1-x^2}}$ for $-1<x<1$
- C. $\dfrac{1}{|x|\sqrt{x^2-1}}$ for $|x|>1$
- D. $\dfrac{1}{\sqrt{1-x^2}}$ for $-1<x<1$

$y' = \dfrac{1}{|x|\sqrt{x^2-1}}$ $|x|>1$

Answers C. $\frac{1}{\sqrt{1-x^2}}$ for $-1<x<1$

D. $\frac{1}{1+x^2}$ for $-\infty<x<\infty$

C. $\frac{1}{|x|\sqrt{x^2-1}}$ for $|x|>1$

ID: 3.10.1

68. Evaluate the derivative of the function.

$f(x)=\sin^{-1}(9x^4)$

$f'(x)=$ ____

Answer: $\frac{36x^3}{\sqrt{1-81x^8}}$

$y=\sin^{-1}(f(x)) \quad y'=\frac{f'(x)}{\sqrt{1-(f(x))^2}}$

$f'(x)=\frac{(9x^4)'}{\sqrt{1-(9x^4)^2}}$

$f'(x)=\frac{36x^3}{\sqrt{1-81x^8}}$

ID: 3.10.13

69. Find the derivative of the function $y=2\tan^{-1}(2x)$.

$\frac{dy}{dx}=$ ____

Answer: $\frac{4}{1+(2x)^2}$

$y=\tan^{-1}(f(x)), \quad y'=\frac{f'(x)}{1+(f(x))^2}$

$y'=2\frac{(2x)'}{1+(2x)^2}$

$y'=\frac{2(2)}{1+(2x)^2}=$

$y'=\frac{4}{1+(2x)^2}$

ID: 3.10.19

70. Evaluate the derivative of the following function.

$f(s)=\cot^{-1}(e^s)$

$\frac{d}{ds}\cot^{-1}(e^s)=$ ____

Answer: $-\frac{e^s}{1+e^{2s}}$

$y=\cot^{-1}(f(x)) \quad y'=-\frac{f'(x)}{1+(f(x))^2}$

$y'=-\frac{(e^s)'}{1+(e^s)^2}$

$y'=-\frac{e^s}{1+e^{2s}}$

ID: 3.10.39

71. The sides of a square increase in length at a rate of 3 m/sec. $\frac{ds}{dt}$

$3 = \frac{ds}{dt}$

**a.** At what rate is the area of the square changing when the sides are 15 m long?
**b.** At what rate is the area of the square changing when the sides are 28 m long?

**a.** Write an equation relating the area of a square, A, and the side length of the square, s.

$A = s^2$

Differentiate both sides of the equation with respect to t.

$\frac{dA}{dt} = (2s)\frac{ds}{dt}$

$\frac{dA}{dt} = 2(15)(3) = 90$

The area of the square is changing at a rate of ______ (1) ______ when the sides are 15 m long.

**b.** The area of the square is changing at a rate of ______ (2) ______ when the sides are 28 m long.

$\frac{dA}{dt} = 2(28)(3) = 168$

(1)
- ○ $m^3/s$
- ○ $m/s$
- ○ $m^2/s$
- ○ $m$

(2)
- ○ $m$
- ○ $m/s$
- ○ $m^2/s$
- ○ $m^3/s$

Answers $A = s^2$

$2s$

90

(1) $m^2/s$

168

(2) $m^2/s$

$A = s^2$

$\frac{dA}{dt} = 2s\frac{ds}{dt}$

ID: 3.11.11-Setup & Solve

72. The area of a circle increases at a rate of 5 $cm^2$ / s. $= \frac{dA}{dt}$

$\frac{dA}{dt} = 5$

**a.** How fast is the radius changing when the radius is 4 cm?
**b.** How fast is the radius changing when the circumference is 4 cm?

**a.** Write an equation relating the area of a circle, A, and the radius of the circle, r.

$\boxed{A = \pi r^2}$

(Type an exact answer, using $\pi$ as needed.)

Differentiate both sides of the equation with respect to t.

$\frac{dA}{dt} = (\boxed{2\pi r})\frac{dr}{dt}$

(Type an exact answer, using $\pi$ as needed.)

When the radius is 4 cm, the radius is changing at a rate of ☐ (1) ☐

(Type an exact answer, using $\pi$ as needed.)

**b.** When the circumference is 4 cm, the radius is changing at a rate of ☐ (2) ☐

(Type an exact answer, using $\pi$ as needed.)

| (1) | | (2) | |
|---|---|---|---|
| ○ | $cm^3$ / s. | ○ | cm. |
| ○ | cm. | ○ | $cm^2$ / s. |
| ○ | cm / s. | ○ | cm / s. |
| ○ | $cm^2$ / s. | ○ | $cm^3$ / s. |

Answers $A = \pi r^2$

$2\pi r$

$\frac{5}{8\pi}$

(1) cm / s.

$\frac{5}{4}$

(2) cm / s.

(a) $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$5 = 2\pi(4)\frac{dr}{dt}$

$5 = 8\pi \frac{dr}{dt}$

$\frac{5}{8\pi} = \frac{\cancel{8\pi}\frac{dr}{dt}}{\cancel{8\pi}}$

$\frac{5}{8\pi} = \frac{dr}{dt}$

ID: 3.11.15-Setup & Solve

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(b) $C = 2\pi r$

$4 = 2\pi r$

$\frac{4}{2\pi} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$

$\frac{2}{\pi} = r$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$5 = 2\pi\left(\frac{2}{\pi}\right)\frac{dr}{dt}$

$5 = 4\frac{dr}{dt}$

$\frac{5}{4} = \frac{\cancel{4}\frac{dr}{dt}}{\cancel{4}}$

$\frac{5}{4} = \frac{dr}{dt}$

73. The edges of a cube increase at a rate of 2 cm / s. How fast is the volume changing when the length of each edge is 30 cm?

$\frac{da}{dt}$

Write an equation relating the volume of a cube, V, and an edge of the cube, a.

$\frac{da}{dt}=2$

edge = a

$V = a^3$

Differentiate both sides of the equation with respect to t.

$\frac{dV}{dt} = (3a^2)\frac{da}{dt}$ (Type an expression using a as the variable.)

The rate of change of the volume is ______ (1) ______
(Simplify your answer.)

(1) ◯ cm / sec. ◯ $cm^3$.
◯ $cm^2$ / sec. ◯ cm.
◯ $cm^3$ / sec.
◯ $cm^2$.

Answers $V = a^3$

$3a^2$

5400

(1) $cm^3$ / sec.

$V = a^3$

$\frac{dV}{dt} = 3a^2 \frac{da}{dt}$

$\frac{dV}{dt} = 3(30)^2(2)$

$\frac{dV}{dt} = 3(30)(30)(2)$

$\frac{dV}{dt} = 3(900)(2)$

$\frac{dV}{dt} = 2700(2)$

$\frac{dV}{dt} = 5400$

ID: 3.11.16-Setup & Solve

74. Find an equation for the tangent to the curve at the given point.

$y = x^2 - 1$, $(-2,3)$
$x_1\ y_1$

◯ A. $y = -2x - 5$
◯ B. $y = -4x - 9$
◯ C. $y = -4x - 10$
◯ D. $y = -4x - 5$

Answer: D. $y = -4x - 5$

ID: 3.1-2

$(-2,3)$
$(x_1, y_1)$

$y = x^2 - 1$

$y' = 2x - 0$

$y' = 2x$

$y'(-2) = 2(-2)$

$y'(-2) = -4 = \text{Slope} = m$

$y - y_1 = m(x - x_1)$

$y - (3) = -4(x - (-2))$

$y - 3 = -4(x + 2)$

$y - 3 = -4x - 8$

$y - \cancel{3} + \cancel{3} = -4x - 8 + 3$

$y = -4x - 5$

$(-2, 3)$
$x_1\ y_1$

75. At time t, the position of a body moving along the s-axis is $s = t^3 - 12t^2 + 36t$ m. Find the displacement of the body from t = 0 to t = 3.

$s(t) = t^3 - 12t^2 + 36t$

$s(3) - s(0)$

$((3)^3 - 12(3)^2 + 36(3)) - ((0)^3 - 12(0) + 36(0)) =$

$(27) - (0) =$

$27 - 0 =$

$27$

- A. 27 m
- B. 63 m
- C. 37 m
- D. 32 m

Answer: A. 27 m

ID: 3.6-3

76. Use implicit differentiation to find dy/dx.

xy + x = 2

$(xy)' + 1 = 0$

$(x)'(y) + (x)(y') + 1 = 0$

$(1)(y) + (x)(y') + 1 = 0$

$y + xy' + 1 = 0$

$xy' = -1 - y$

$\frac{\cancel{x}y'}{\cancel{x}} = \frac{-1-y}{x}$

- A. $-\frac{1+y}{x}$
- B. $\frac{1+x}{y}$
- C. $-\frac{1+x}{y}$
- D. $\frac{1+y}{x}$

Answer: A. $-\frac{1+y}{x}$

ID: 3.8-4

$y' = \frac{-1-y}{x}$

$y' = -1\frac{(1+y)}{x}$

$y' = -\frac{1+y}{x}$

OR

$\frac{dy}{dx} = -\frac{1+y}{x}$

77. Find the derivative of the function.

$y = \log_2 \sqrt{9x+4}$

A. $\frac{9}{\ln 2}$

B. $\frac{9}{\ln 2\,(9x+4)}$

C. $\frac{9}{2(\ln 2)(9x+4)}$

D. $\frac{9 \ln 2}{9x+4}$

Answer: C. $\frac{9}{2(\ln 2)(9x+4)}$

ID: 3.9-11

$y = \log_2 \sqrt{9x+4}$

$y = \log_2 (9x+4)^{1/2}$

$y = \frac{1}{2}\log_2 (9x+4)$

$y' = \frac{1}{2} \frac{(9x+4)'}{(9x+4)\ln(2)}$

$y' = \frac{1}{2} \frac{(9+0)}{(9x+4)\ln(2)}$

$y' = \frac{1}{2} \frac{(9)}{(9x+4)\ln(2)}$

$y' = \frac{9}{2(9x+4)\ln(2)}$

$y' = \frac{9}{2(\ln(2))(9x+4)}$

formula

$y = \log_b f(x)$

$y' = \frac{f'(x)}{f(x)\ln(b)}$

78. Boyle's law states that if the temperature of a gas remains constant, then PV = c, where P = pressure, V = volume, and c is a constant. Given a quantity of gas at constant temperature, if V is decreasing at a rate of 10 $in^3$/sec, at what rate is P increasing when P = 50 lb/$in^2$ and V = 90 $in^3$? (Do not round your answer.)

A. $\frac{50}{9}$ lb / $in^2$ per sec

B. 450 lb / $in^2$ per sec

C. $\frac{25}{81}$ lb / $in^2$ per sec

D. 18 lb / $in^2$ per sec

Answer: A. $\frac{50}{9}$ lb / $in^2$ per sec

ID: 3.11-7

$PV = c$

$(PV)' = 0$

$(P)'(V) + (P)(V)' = 0$

$\frac{dP}{dt}V + P\frac{dV}{dt} = 0$

$\frac{dP}{dt}(90) + (50)(-10) = 0$

$90\frac{dP}{dt} - 500 = 0$

$90\frac{dP}{dt} = 500$

$\frac{90}{90}\frac{dP}{dt} = \frac{500}{90}$

$\frac{dP}{dt} = \frac{50}{9}$

$\frac{dV}{dt} = -10$

$V = 90$

$P = 50$

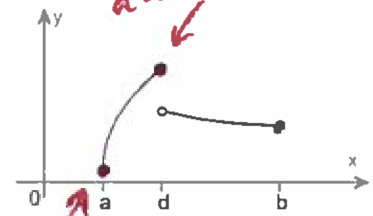
formula Product

$y = f \cdot g$

$y' = f'g + fg'$

79. Determine from the graph whether the function has any absolute extreme values on [a, b].

absolute max




absolute min

Where do the absolute extreme values of the function occur on [a, b]?

- ○ **A.** There is no absolute maximum and the absolute minimum occurs at x = a on [a, b].
- ○ **B.** There is no absolute maximum and there is no absolute minimum on [a, b].
- ○ **C.** The absolute maximum occurs at x = d and the absolute minimum occurs at x = a on [a, b].
- ○ **D.** The absolute maximum occurs at x = d and there is no absolute minimum on [a, b].

Answer: C. The absolute maximum occurs at x = d and the absolute minimum occurs at x = a on [a, b].

ID: 4.1.13

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80. Find the critical points of the following function.

$f(x) = 2x^2 - 3x + 1$

What is the derivative of $f(x) = 2x^2 - 3x + 1$?

$f'(x) =$ ______

$f(x) = 2x^2 - 3x + 1$

$f'(x) = 4x - 3 + 0$

$f'(x) = 4x - 3$

set $4x - 3 = 0$

Find the critical points, if any, of f on the domain. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$4x - 3 + 3 = 0 + 3$

- ○ **A.** The critical point(s) occur(s) at x = ______. (Use a comma to separate answers as needed.)
- ○ **B.** There are no critical points for $f(x) = 2x^2 - 3x + 1$ on the domain.

$4x = 3$

$\frac{4x}{4} = \frac{3}{4}$

$x = 3/4$

Answers $4x - 3$

A. The critical point(s) occur(s) at x = $\frac{3}{4}$ .(Use a comma to separate answers as needed.)

ID: 4.1.23-Setup & Solve

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81. Find the critical points of the following function.

$$f(x) = -\frac{x^3}{3} + 9x$$

$f(x) = -\frac{1}{3}x^3 + 9x$

$f'(x) = -\frac{1}{3}(3x^2) + 9$

$f'(x) = -x^2 + 9$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$f'(x) = 9 - x^2$

○ **A.** The critical point(s) occur(s) at x = ______
(Use a comma to separate answers as needed.)

$f'(x) = (3)^2 - (x)^2$

○ **B.** There are no critical points.

$f'(x) = (3+x)(3-x)$

Answer: A. The critical point(s) occur(s) at x = 3, −3 .(Use a comma to separate answers as needed.)

$3 + x = 0$ or $3 - x = 0$

$3 + x - 3 = 0 - 3$ or $3 - x - 3 = 0 - 3$

$x = -3$ or $-x = -3$

$-1(-x) = -1(-3)$

$x = 3$

ID: 4.1.25

82. Determine the location and value of the absolute extreme values of f on the given interval, if they exist.

$f(x) = -x^2 + 8$ on $[-3,4]$

$f(x) = -x^2 + 8$

What is/are the absolute maximum/maxima of f on the given interval? Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

$f'(x) = -2x + 0$

○ **A.** The absolute maximum/maxima is/are ______ at x = ______.
(Use a comma to separate answers as needed.)

$f'(x) = -2x$

○ **B.** There is no absolute maximum of f on the given interval.

What is/are the absolute minimum/minima of f on the given interval? Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

at $-2x = 0$

○ **A.** The absolute minimum/minima is/are ______ at x = ______.
(Use a comma to separate answers as needed.)

$\frac{-2x}{-2} = \frac{0}{-2}$

○ **B.** There is no absolute minimum of f on the given interval.

$x = 0$

critical point

Answers A. The absolute maximum/maxima is/are 8 at x = 0 .
(Use a comma to separate answers as needed.)

A. The absolute minimum/minima is/are −8 at x = 4 .
(Use a comma to separate answers as needed.)

Point

Point

ID: 4.1.43

$x = -3$ $x = 0$ $(0, 8)$ $x = 4$ $(4, -8)$

$f(x) = -x^2 + 8$

$f(-3) = -(-3)^2 + 8$

$f(-3) = -(-3)(-3) + 8$

$f(-3) = -9 + 8$

$f(-3) = -1$

$f(0) = -(0)^2 + 8$

$f(0) = -(0)(0) + 8$

$f(0) = -(0) + 8$

$f(0) = 0 + 8$

$f(0) = 8$

absolute Max

$f(4) = -(4)^2 + 8$

$f(4) = -(4)(4) + 8$

$f(4) = -(16) + 8$

$f(4) = -16 + 8$

$f(4) = -8$

absolute minimum

83. A stone is launched vertically upward from a cliff 336 ft above the ground at a speed of 64 ft / s. Its height above the ground t seconds after the launch is given by $s = -16t^2 + 64t + 336$ for $0 \le t \le 7$. When does the stone reach its maximum height?

$S = -16t^2 + 64t + 336$
$S' = -32t + 64 + 0$
$S = -32t + 64$

$t = 2$

Find the derivative of s.

s′ = ______

The stone reaches its maximum height at ______ s.
(Simplify your answer.)

set $-32t + 64 = 0$
$-32t + 64 - 64 = 0 - 64$
$-32t = -64$
$\frac{-32t}{-32} = \frac{-64}{-32}$

$S(t) = -32t + 64$
$S(1) = -32(1) + 64 = -32 + 64 = 32$ increasing
$S(3) = -32(3) + 64 = -96 + 64 = -32$ decreasing

1 2 3

Max at $X = 2$

Answers $-32t + 64$

2

ID: 4.1.73

84. Suppose a tour guide has a bus that holds a maximum of 84 people. Assume his profit (in dollars) for taking n people on a city tour is $P(n) = n(42 - 0.5n) - 84$. (Although P is defined only for positive integers, treat it as a continuous function.)

**a.** How many people should the guide take on a tour to maximize the profit?
**b.** Suppose the bus holds a maximum of 35 people. How many people should be taken on a tour to maximize the profit?

**a.** Find the derivative of the given function P(n). $P(n) = 42n - 0.5n^2 - 84$

$P'(n) = 42 - 0.5(2n) - 0$
$P'(n) = 42 - n$

P′(n) = ______

If the bus holds a maximum of 84 people, the guide should take ______ people on a tour to maximize the profit.

**b.** If the bus holds a maximum of 35 people, the guide should take ______ people on a tour to maximize the profit.

$42 - n = 0$
$42 - n - 42 = 0 - 42$
$-n = -42$
$\frac{-n}{-1} = \frac{-42}{-1}$
$n = 42$ critical point

If the bus holds 35 people the guide should take 35 to maximize profit

Answers $-n + 42$

42

35

ID: 4.1.75

85. At what points c does the conclusion of the Mean Value Theorem hold for $f(x) = x^3$ on the interval $[-17, 17]$?

$f'(x) = 3x^2$

The conclusion of the Mean Value Theorem holds for c = ______.
(Use a comma to separate answers as needed. Type an exact answer, using radicals as needed.)

Answer: $\frac{17\sqrt{3}}{3}, -\frac{17\sqrt{3}}{3}$

ID: 4.2.8

$\frac{f(b) - f(a)}{b - a} = 3x^2$

$\frac{f(17) - f(-17)}{(17) - (-17)} = 3x^2$

$\frac{(17)^3 - (-17)^3}{17 + 17} = 3x^2$

$\frac{4913 + 4913}{34} = 3x^2$

$\frac{9826}{34} = 3x^2$

$289 = 3x^2$

$\frac{289}{3} = \frac{3x^2}{3}$

$\frac{289}{3} = x^2$

$\pm\sqrt{\frac{289}{3}} = \sqrt{x^2}$

$\pm\frac{\sqrt{289}}{\sqrt{3}} = x$

$\pm\frac{\sqrt{17^2}}{\sqrt{3}} = x$

$\pm\frac{\sqrt{17^2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$

$\pm\frac{\sqrt{17^2}\sqrt{3}}{3} = x$

$\pm\frac{17\sqrt{3}}{3} = x$

$\pm\frac{17\sqrt{3}}{3} = x$

Math

86. **a.** Determine whether the Mean Value Theorem applies to the function $f(x) = -1 + x^2$ on the interval $[-2,1]$.
**b.** If so, find the point(s) that are guaranteed to exist by the Mean Value Theorem.

**a.** Choose the correct answer below.

- **A.** Yes, because the function is continuous on the interval $[-2,1]$ and differentiable on the interval $(-2,1)$.
- **B.** No, because the function is differentiable on the interval $(-2,1)$, but is not continuous on the interval $[-2,1]$.
- **C.** No, because the function is not continuous on the interval $[-2,1]$, and is not differentiable on the interval $(-2,1)$.
- **D.** No, because the function is continuous on the interval $[-2,1]$, but is not differentiable on the interval $(-2,1)$.

**b.** Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The point(s) is/are x = ______.
  (Simplify your answer. Use a comma to separate answers as needed.)
- **B.** The Mean Value Theorem does not apply in this case.

Answers A. Yes, because the function is continuous on the interval $[-2,1]$ and differentiable on the interval $(-2,1)$.

A. The point(s) is/are $x = \boxed{-\frac{1}{2}}$.

(Simplify your answer. Use a comma to separate answers as needed.)

$[-2, 1]$
$a \quad b$

ID: 4.2.21 $f(x) = -1 + x^2$

$f'(x) = 0 + 2x$

$f'(x) = 2x$

$\frac{f(b) - f(a)}{b-a} = 2x$

$\frac{f(1) - f(-2)}{(1)-(-2)} = \cancel{2x}$

$\frac{(-1+(1)^2) - (-1+(-2))^2}{1+2} = 2x$

$\frac{(-1+1) - (-1+4)}{3} = 2x$

$\frac{(0) - (3)}{3} = 2x$

$\frac{0-3}{3} = 2x$

$\frac{-3}{3} = 2x$

$-1 = 2x$

$\frac{-1}{2} = \frac{2x}{2}$

$-\frac{1}{2} = x$

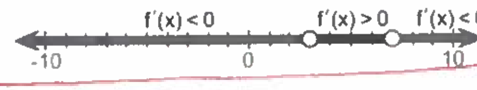
87. Sketch a function that is continuous on $(-\infty,\infty)$ and has the following properties. Use a number line to summarize information about the function.

$f'(x) < 0$ on $(-\infty,3)$; $f'(x) > 0$ on $(3,7)$; $f'(x) < 0$ on $(7,\infty)$.

Which number line summarizes the information about the function?

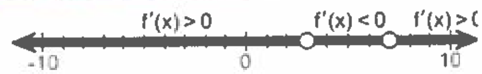
○ A.



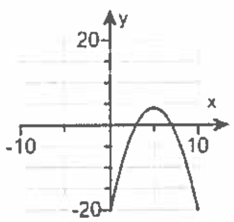
○ B.


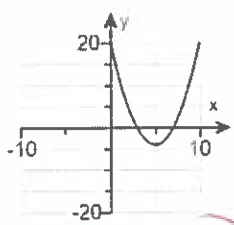

○ C.

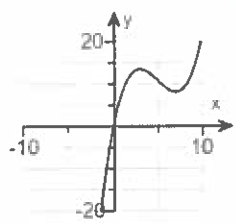


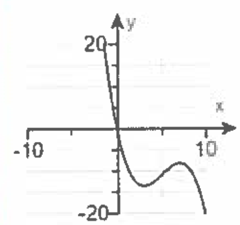
○ D.



Which of the following graphs matches the description of the given properties?

○ A.



○ B.


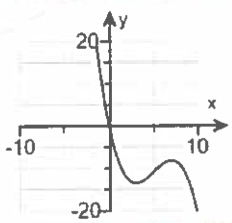

○ C.



○ D.



Answers

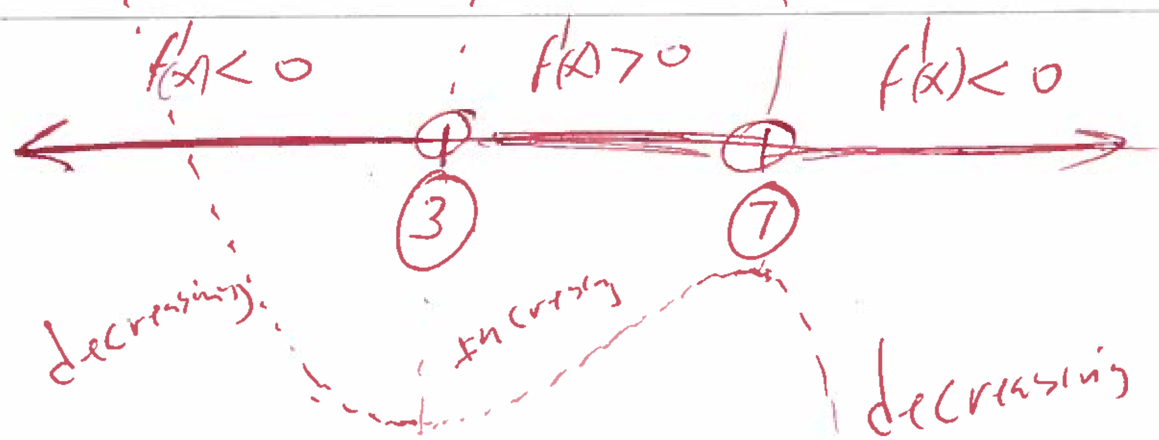
B.






D.

ID: 4.3.9



88. Find the intervals on which f is increasing and the intervals on which it is decreasing.

$f(x) = -7 + x^2$ $\longrightarrow$ $f'(x) = 0 + 2x$

$f'(x) = 2x$
set $2x = 0$
$\frac{2x}{2} = \frac{0}{2}$
$x = 0$

Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- A. The function is increasing on ______ and decreasing on ______.
  (Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)
- B. The function is increasing on ______. The function is never decreasing.
  (Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- C. The function is decreasing on ______. The function is never increasing.
  (Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- D. The function is never increasing nor decreasing.

$-1 \quad 0 \quad 1$

$f'(-1) = 2(-1) = -2 < 0$ decreasing

$f'(1) = 2(1) = 2 > 0$ increasing

decreasing $(-\infty, 0)$
increasing $(0, \infty)$

Answer: A. The function is increasing on $(0,\infty)$ and decreasing on $(-\infty,0)$.
(Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)

ID: 4.3.19

89. Find the intervals on which f is increasing and the intervals on which it is decreasing.

$f(x) = 4 - 3x + 2x^2$ $\quad f'(x) = 0 - 3 + 4x, \quad f'(x) = -3 + 4x$

Set
$-3 + 4x = 0$
$-3 + 4x + 3 = 0 + 3$
$4x = 3$
$\frac{4x}{4} = \frac{3}{4}$
$x = \frac{3}{4}$

Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- A. The function is increasing on ______ and decreasing on ______.
  (Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)
- B. The function is increasing on ______. The function is never decreasing.
  (Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- C. The function is decreasing on ______. The function is never increasing.
  (Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- D. The function is never increasing nor decreasing.

$-1 \quad \frac{3}{4} \quad 1$

$f'(-1) = -3 + 4(-1)$
$f'(-1) = -3 \cancel{-4}$
$f'(-1) = -7 < 0$
decreasing

Answer: A. The function is increasing on $\left(\frac{3}{4},\infty\right)$ and decreasing on $\left(-\infty,\frac{3}{4}\right)$.
(Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)

ID: 4.3.25

decreasing $(-\infty, \frac{3}{4})$
increasing $(\frac{3}{4}, \infty)$

$f'(1) = -3 + 4(1)$
$f'(1) = -3 + 4$
$f'(1) = 1 > 0$
increasing

90. Locate the critical points of the following function. Then use the Second Derivative Test to determine whether they correspond to local maxima, local minima, or neither.

$3x^2 - 30x = 0$

$f(x) = x^3 - 15x^2$ $\quad f'(x) = 3x^2 - 30x$ $\quad 3x(x-10) = 0$

What is(are) the critical point(s) of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$f''(x) = 6x - 30$ $\quad 3x = 0$ or $x - 10 = 0$

$\frac{3x}{3} = \frac{0}{3}$ or $x - 10 + 10 = 0 + 10$

○ **A.** The critical point(s) is(are) x = ______.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

○ **B.** There are no critical points for f.

$x = 0$ or $x = 10$

Find f''(x).

f''(x) = ______ $\quad f''(x) = 6x - 30$ $\quad f''(0) = 6(0) - 30 = 0 - 30 = -30$

concave down

What is/are the local maximum/maxima of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

max at $x = 0$

○ **A.** The local maximum/maxima of f is/are at x = ______. $\quad f''(x) = 6x - 30$
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

○ **B.** There is no local maximum of f. $\quad f''(10) = 6(10) - 30 = 60 - 30 = 30$

What is/are the local minimum/minima of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice. $> 0$

concave up

○ **A.** The local minimum/minima of f is/are at x = ______.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

○ **B.** There is no local minimum of f.

min at $x = 10$

Answers A. The critical point(s) is(are) x = **0,10**.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

6x − 30

A. The local maximum/maxima of f is/are at x = **0**.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

A. The local minimum/minima of f is/are at x = **10**.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

ID: 4.3.77-Setup & Solve

max at $x = 0$

min at $x = 10$

Can not use 2nd derivative

91. Locate the critical points of the following function. Then use the Second Derivative Test to determine whether they correspond to local maxima, local minima, or neither.

$f(x) = 2 - 2x^2$

$f'(x) = 0 - 4x$ $\quad f'(x) = -4x$ $\quad f''(x) = -4$

What is(are) the critical point(s) of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Can not use Second derivative test

$-4x = 0$

$\frac{-4x}{-4} = \frac{0}{-4}$

A. The critical point(s) is(are) x = ______.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

B. There are no critical points for f.

$x = 0$ critical point

What is/are the local maximum/maxima of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The local maximum/maxima of f is/are at x = ______.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

B. There is no local maximum of f.

$-1 \quad 0 \quad 1$

$f'(-1) = -4(-1) = 4 > 0$ inc

What is/are the local minimum/minima of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$f'(1) = -4(1) = -4 < 0$ dec

A. The local minimum/minima of f is/are at x = ______.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

B. There is no local minimum of f.

Answers A. The critical point(s) is(are) x = 0.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

A. The local maximum/maxima of f is/are at x = 0.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

B. There is no local minimum of f.

ID: 4.3.79

Critical point $x = 0$

max at $x = 0$

92. Locate the critical points of the following function. Then use the Second Derivative Test to determine whether they correspond to local maxima, local minima, or neither.

$f(x) = -3x^3 + 9x^2 + 4$

$f'(x) = -9x^2 + 18x + 0$ | $f'(x) = -9x^2 + 18x$

What is(are) the critical point(s) of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$f''(x) = -18x + 18$

$-9x^2 + 18x = 0$

- A. The critical point(s) is(are) x = ________.
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

$-9x(x-2) = 0$

- B. There are no critical points for f.

$-9x = 0$ or $x - 2 = 0$

What is/are the local minimum/minima of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$\frac{-9x}{-9} = \frac{0}{-9}$ $x - 2 + 2 = 0 + 2$

- A. The local minimum/minima of f is/are at x = ________.
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

$x = 2$

- B. There is no local minimum of f.

$x = 0$ Critical Points

What is/are the local maximum/maxima of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The local maximum/maxima of f is/are at x = ________.
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- B. There is no local maximum of f.

Answers A. The critical point(s) is(are) x = **0,2**.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

A. The local minimum/minima of f is/are at x = **0**.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

A. The local maximum/maxima of f is/are at x = **2**.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

ID: 4.3.83

$f''(x) = -18x + 18$

min

$f''(0) = -18(0) + 18 = 0 + 18 = 18 > 0$ Concave up

$f''(2) = -18(2) + 18 = -36 + 18 = -18 < 0$ Concave down

Max

minimum at $x = 0$

Maximum at $x = 2$

93 $V(x) = x(5-2x)(4-2x)$

a $V(x) = x(20 - 10x - 8x + 4x^2)$

part 1 $V(x) = x(20 - 18x + 4x^2)$

$V(x) = 20x - 18x^2 + 4x^3$

$V(x) = 4x^3 - 18x^2 + 20x$

$V'(x) = 12x^2 - 36x + 20$

$V''(x) = 24x - 36$

set $V'(x) = 12x^2 - 36x + 20 = 0$

$12x^2 - 36x + 20 = 0$

$a = 12, \quad b = -36, \quad c = 20$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-36) \pm \sqrt{(-36)^2 - 4(12)(20)}}{2(12)}$$

$$x = \frac{36 \pm \sqrt{1296 - 960}}{24}$$

$$x = \frac{36 \pm \sqrt{336}}{24}$$

$$x = \frac{36 \pm 18.33030278}{24}$$

$$x = \frac{36 + 18.33030278}{24} \quad \text{OR} \quad x = \frac{36 - 18.33030278}{24}$$

93 (a)

$x = \frac{54.33030278}{24}$ OR $x = \frac{17.66969722}{24}$

Part 2

$x = 2.263762616$ OR $x = .7362373842$

critical point critical point

$V''(x) = 24x - 36$

$V''(2.263762616) = 24(2.263762616) - 36$

$= 18.33030278 > 0$ concave up


$V''(.7362373842) = 24(.7362373842) - 36$

$= -18.33030278 < 0$ concave down max

Max at $x = .7362373842$

$V(x) = 4x^3 - 18x^2 + 20x$

$V(.7362373842) = 4(.7362373842)^3 - 18(.7362373842)^2 + 20(.7362373842)$

$V(.7362373842) = 6.564225541$

$= 6.56$ Round

MAX

93

b

part 1

$V(x) = x(m-2x)(m-2x)$

$V(x) = x(m^2 - 2mx - 2mx + 4x^2)$

$V(x) = x(m^2 - 4mx + 4x^2)$

$V(x) = m^2x^1 - 4mx^2 + 4x^3$

$V(x) = 4x^3 - 4mx^2 + m^2x$

$V'(x) = 12x^2 - 8mx + m^2$

$V''(x) = 24x - 8m$

set $V'(x) = 12x^2 - 8mx + m^2 = 0$

$a = 12, \quad b = -8m, \quad c = m^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8m) \pm \sqrt{(-8m)^2 - 4(12)(m^2)}}{2(12)}$$

$$x = \frac{8m \pm \sqrt{64m^2 - 48m^2}}{24}$$

$$x = \frac{8m \pm \sqrt{16m^2}}{24}$$

$$x = \frac{8m \pm 4m}{24}$$

$x = \frac{8m + 4m}{24}$ OR $x = \frac{8m - 4m}{24}$

$x = \frac{12m}{24}$ OR $x = \frac{4m}{24}$

Q3(b) part 2

$x = \frac{\cancel{12}(1m)}{\cancel{12}(2)}$ or $x = \frac{\cancel{4}(1m)}{\cancel{4}(6)}$

$x = \frac{m}{2}$ or $x = \frac{m}{6}$

critical point critical point

$V''(x) = 24x - 8m$

$V''\left(\frac{m}{2}\right) = 24\left(\frac{m}{2}\right) - 8m$

$= 12(m) - 8m$

$= 12m - 8m$

$= 4m > 0$ concave up


$V''\left(\frac{m}{6}\right) = 24\left(\frac{m}{6}\right) - 8m$

$= 4(m) - 8m$

$= 4m - 8m$

$= -4m < 0$ concave down


$V(x) = 4x^3 - 4mx^2 + m^2x$

$V\left(\frac{m}{6}\right) = 4\left(\frac{m}{6}\right)^3 - 4m\left(\frac{m}{6}\right)^2 + m^2\left(\frac{m}{6}\right)$

$V\left(\frac{m}{6}\right) = 4\left(\frac{m^3}{6^3}\right) - 4m\left(\frac{m^2}{6^2}\right) + m^2\left(\frac{m}{6}\right)$

$V\left(\frac{m}{6}\right) = 4\left(\frac{m^3}{216}\right) - 4m\left(\frac{m^2}{36}\right) + \frac{m^3}{6}$

$V\left(\frac{m}{6}\right) = \frac{m^3}{54} - \frac{m^3}{9} + \frac{m^3}{6}$

$V\left(\frac{m}{6}\right) = \frac{m^3}{54} - \frac{6}{6}\left(\frac{m^3}{9}\right) + \frac{9}{9}\left(\frac{m^3}{6}\right)$

93. Part 3

$$V\left(\frac{m}{6}\right) = \frac{m^3}{54} - \frac{6m^3}{54} + \frac{9m^3}{54}$$

$$V\left(\frac{m}{6}\right) = \frac{1m^3 - 6m^3 + 9m^3}{54}$$

$$V\left(\frac{m}{6}\right) = \frac{4m^3}{54}$$

$$V\left(\frac{m}{6}\right) = \frac{\cancel{2}(2)m^3}{\cancel{2}(27)}$$

$$V\left(\frac{m}{6}\right) = \frac{2m^3}{27}$$

Max at $x = \frac{m}{6}$

$$\left(\frac{m}{6}, \frac{2m^3}{27}\right)$$

(94) Use a linear approximation to estimate the following quantity. Choose a value of a to produce a small error.

$\ln(0.95)$

$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

$L(x) = f(a) + f'(a)(x - a)$

$L(0.95) = f(1) + f'(1)(0.95 - 1)$

$L(0.95) = \ln(1) + \frac{1}{(1)}(0.95 - 1)$

$L(0.95) = 0 + 1(0.95 - 1)$

$L(0.95) = 0 + 1(-.05)$

$L(0.95) = 0 - .05$

$= -0.05$

Section
ID 4.6.41

95. Consider the following function and express the relationship between a small change in x and the corresponding change in y in the form dy = f′(x)dx.

$$f(x) = 2x^3 - 5x$$

dy = [ ] dx

Answer: $6x^2 - 5$

ID: 4.6.67

$dy = f'(x)\,dx$

$f(x) = 2x^3 - 5x$

$f'(x) = 6x^2 - 5$

$\frac{dy}{dx} = 6x^2 - 5$

$dy = (6x^2 - 5)\,dx$

96. Consider the following function and express the relationship between a small change in x and the corresponding change in y in the form dy = f′(x) dx.

$$f(x) = \cot 7x$$

dy = [ ] dx

Answer: $-7\csc^2(7x)$

ID: 4.6.69

$f(x) = \cot(7x)$

$f'(x) = -\csc^2(7x)\cdot(7x)'$

$f'(x) = -\csc^2(7x)(7)$

$f'(x) = -7\csc^2(7x)$

$\frac{dy}{dx} = -7\csc^2(7x)$

$dy = -7\csc^2(7x)\,dx$

97. Evaluate the following limit. Use l'Hôpital's Rule when it is convenient and applicable.

$$\lim_{x \to 0} \frac{5 \sin 2x}{3x}$$

Use l'Hôpital's Rule to rewrite the given limit so that it is not an indeterminate form.

$$\lim_{x \to 0} \frac{5 \sin 2x}{3x} = \lim_{x \to 0} (\quad)$$

Evaluate the limit.

$$\lim_{x \to 0} \frac{5 \sin 2x}{3x} = \quad \text{(Type an exact answer.)}$$

Answers $\frac{10 \cos (2x)}{3}$

$\frac{10}{3}$

ID: 4.7.29-Setup & Solve

$\lim_{x \to 0} \frac{5\sin(2x)}{3x} =$

$\lim_{x \to 0} \frac{5\cos(2x)\cdot 2}{3} =$

$\lim_{x \to 0} \frac{5\cos(2x)\cdot 2}{3} =$

$\lim_{x \to 0} \frac{10\cos(2x)}{3} =$

$\frac{10\cos(2(0))}{3} =$

$\frac{10\cos(0)}{3} =$

$\frac{10(1)}{3} =$

$\frac{10}{3}$

formula

$y = \sin(f(x))$

$y' = \cos(f(x))\cdot f'(x)$

$y = \cos f(x)$

$y' = -\sin(f(x))\cdot f'(x)$

98. Use a calculator or program to compute the first 10 iterations of Newton's method when they are applied to the following function with the given initial approximation.

$f(x) = x^2 - 12; x_0 = 4$ $f'(x) = 2x$

| k | $x_k$ | k | $x_k$ |
|---|---|---|---|
| 0 | 4.000000 | 6 | 3.464102 |
| 1 | 3.500000 | 7 | 3.464102 |
| 2 | 3.464286 | 8 | 3.464102 |
| 3 | 3.464102 | 9 | 3.464102 |
| 4 | 3.464102 | 10 | 3.464102 |
| 5 | 3.464102 | | |

(Round to six decimal places as needed.)

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 4 - \frac{f(4)}{f'(4)}$

$x_1 = 4 - \frac{(4)^2 - 12}{2(4)}$

$x_1 = 4 - \frac{16-12}{8}$

$x_1 = 4 - \frac{4}{8}$

$x_1 = 4 - .5$

$x_1 = 3.5$

$x_2 = 3.5 - \frac{f(3.5)}{f'(3.5)}$

$x_2 = 3.5 - \frac{(3.5)^2 - 12}{2(3.5)}$

$x_2 = 3.5 - \frac{12.25-12}{7.0}$

$x_2 = 3.5 - \frac{.25}{7.0}$

$x_2 = 3.5 - .0357142857$

$x_2 = 3.4642857\,14$

Answers 4.000000

3.464102

3.500000

3.464102

3.464286

3.464102

3.464102

3.464102

3.464102

3.464102

3.464102

ID: 4.8.13-T

99. Use a calculator or program to compute the first 10 iterations of Newton's method for the given function and initial approximation.

$f(x) = 4 \sin x + x + 1$, $x_0 = 1.2$

$f'(x) = 4\cos(x) + 1$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 1.2 - \frac{f(1.2)}{f'(1.2)}$

$x_1 = 1.2 - \frac{4\sin(1.2)+(1.2)+1}{4\cos(1.2)+1}$

$x_1 = -1.22021776$

Complete the table.
(Do not round until the final answer. Then round to six decimal places as needed.)

| k | $x_k$ | k | $x_k$ |
|---|---|---|---|
| 1 | | 6 | |
| 2 | | 7 | |
| 3 | | 8 | |
| 4 | | 9 | |
| 5 | | 10 | |

Answers − 1.220218

− 0.201082

0.455144

− 0.201082

− 0.244542

− 0.201082

− 0.200905

− 0.201082

− 0.201082

− 0.201082

ID: 4.8.15-T

100. Determine the following indefinite integral. Check your work by differentiation.

$$\int (9x^{17} - 5x^9)\, dx$$

$\int (9x^{17} - 5x^9)\, dx =$ ______ (Use C as the arbitrary constant.)

Answer: $\frac{x^{18}}{2} - \frac{x^{10}}{2} + C$

ID: 4.9.23

$\int (9x^{17} - 5x^9)\, dx =$

$\frac{9x^{17+1}}{17+1} - \frac{5x^{9+1}}{9+1} + C =$

$\frac{9x^{18}}{18} - \frac{5x^{10}}{10} + C =$

$\frac{9x^{18}}{9 \cdot 2} - \frac{5x^{10}}{5 \cdot 2} + C =$

$\frac{x^{18}}{2} - \frac{x^{10}}{2} + C =$

Formula

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int a\, dx = ax + C$

101. Evaluate the following indefinite integral.

$$\int \left( \frac{3}{\sqrt{x}} + 3\sqrt{x} \right) dx$$

$\int \left( \frac{3}{\sqrt{x}} + 3\sqrt{x} \right) dx =$ ____

(Use C as the arbitrary constant.)

Answer: $6\sqrt{x} + 2x^{\frac{3}{2}} + C$

ID: 4.9.25

$\int \left( \frac{3}{x^{1/2}} + 3x^{1/2} \right) dx = \frac{2}{1} 3x^{1/2} + \frac{2}{3} 3x^{3/2} + C =$

$\int (3x^{-1/2} + 3x^{1/2}) dx =$

$\frac{3x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{3x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C =$

$\frac{3x^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}} + \frac{3x^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C =$

$\frac{3x^{1/2}}{\frac{1}{2}} + \frac{3x^{3/2}}{\frac{3}{2}} + C =$

$6x^{1/2} + 2x^{3/2} + C =$

$6\sqrt{x} + 2x^{3/2} + C =$

formula $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

102. Find $\int (6x+5)^2 dx$. =

$\int (6x+5)^2 dx =$ ____

(Use C as the arbitrary constant.)

Answer: $12x^3 + 30x^2 + 25x + C$

ID: 4.9.27

$\int (6x+5)(6x+5)\,dx =$

$\int 36x^2 + 30x + 30x + 25\,dx =$

$\int 36x^2 + 60x + 25\,dx =$

$\frac{36x^{2+1}}{2+1} + \frac{60x^{1+1}}{1+1} + 25x + C =$

$\frac{36x^3}{3} + \frac{60x^2}{2} + 25x + C$

$12x^3 + 30x^2 + 25x + C =$

formula $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

103. Determine the following indefinite integral. Check your work by differentiation.

$$\int 4m\left(12m^2 - 7m\right) dm \quad =$$

$\int 4m\left(12m^2 - 7m\right) dm =$ ____ (Use C as the arbitrary constant.)

Answer: $12m^4 - \frac{28m^3}{3} + C$

ID: 4.9.28

$\int 48m^3 - 28m^2\,dm =$

$\frac{48m^{3+1}}{3+1} - \frac{28m^{2+1}}{2+1} + C =$

$\frac{48m^4}{4} - \frac{28m^3}{3} + C =$

$12m^4 - \frac{28m^3}{3} + C =$

formula

$\int x^n dx =$

$\frac{x^{n+1}}{n+1} + C =$

104. Determine the following indefinite integral. Check your work by differentiation.

$$\int\left(3x^{\frac{1}{3}}+4x^{-\frac{2}{3}}+9\right)dx =$$

$\int\left(3x^{\frac{1}{3}}+4x^{-\frac{2}{3}}+9\right)dx =$ ______ (Use C as the arbitrary constant.)

Answer: $\frac{9}{4}x^{\frac{4}{3}}+12x^{\frac{1}{3}}+9x+C$

$$\frac{3x^{\frac{1}{3}+1}}{\frac{1}{3}+1}+\frac{4x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}+9x+C=$$

$$\frac{3x^{\frac{1}{3}+\frac{3}{3}}}{\frac{1}{3}+\frac{3}{3}}+\frac{4x^{-\frac{2}{3}+\frac{3}{3}}}{-\frac{2}{3}+\frac{3}{3}}+9x+C=$$

$$\frac{3x^{\frac{4}{3}}}{\frac{4}{3}}+\frac{4x^{\frac{1}{3}}}{\frac{1}{3}}+9x+C=$$

$$\frac{3}{4}\cdot 3x^{4/3}+\frac{3}{1}4x^{1/3}+9x+C=$$

$$\frac{9}{4}x^{4/3}+12x^{1/3}+9x+C=$$

Formula $\int x^n dx = \frac{x^{n+1}}{n+1}+C$

ID: 4.9.29

105. Determine the following indefinite integral. Check your work by differentiation.

$$\int 3\sqrt[4]{x}\,dx =$$

$\int 3\sqrt[4]{x}\,dx =$ ______ (Use C as the arbitrary constant.)

Answer: $\frac{12}{5}x^{\frac{5}{4}}+C$

Formula $\int x^n dx = \frac{x^{n+1}}{n+1}+C$

$$\int 3x^{1/4}dx =$$

$$\frac{3x^{\frac{1}{4}+1}}{\frac{1}{4}+1}+C=$$

$$\frac{3x^{\frac{1}{4}+\frac{4}{4}}}{\frac{1}{4}+\frac{4}{4}}+C=$$

$$\frac{3x^{5/4}}{\frac{5}{4}}+C=$$

$$\frac{4}{5}\cdot 3x^{5/4}+C=$$

$$\frac{12}{5}x^{5/4}+C=$$

ID: 4.9.30

106. Determine the following indefinite integral. Check your work by differentiation.

$$\int(5x+9)(1-x)\,dx = \int 5x-5x^2+9-9x\,dx =$$

$\int(5x+9)(1-x)\,dx =$ ______
(Use C as the arbitrary constant.)

Answer: $-\frac{5}{3}x^3-2x^2+9x+C$

$$\int -5x^2-4x+9\,dx =$$

$$\frac{-5x^{2+1}}{2+1}-\frac{4x^{1+1}}{1+1}+9x+C=$$

$$\frac{-5x^3}{3}-\frac{4x^2}{2}+9x+C=$$

$$-\frac{5}{3}x^3-2x^2+9x+C=$$

Formula $\int x^n dx = \frac{x^{n+1}}{n+1}+C$

ID: 4.9.31

107. Determine the following indefinite integral.

$$\int \frac{3x^5 - 6x^4}{x^3}\,dx =$$

$$\int \frac{3x^5 - 6x^4}{x^3}\,dx = \boxed{\phantom{xx}}$$

(Use C as the arbitrary constant.)

Answer: $x^3 - 3x^2 + C$

ID: 4.9.35

$$\int \frac{3x^5}{x^3} - \frac{6x^4}{x^3}\,dx = \int 3x^{5-3} - 6x^{4-3}\,dx = \int 3x^2 - 6x\,dx = \frac{3x^{2+1}}{2+1} - \frac{6x^{1+1}}{1+1} + C = \frac{3x^3}{3} - \frac{6x^2}{2} + C = x^3 - 3x^2 + C =$$

$$\boxed{\text{formula} \quad \int x^n\,dx \quad \frac{x^{n+1}}{n+1} + C}$$

108. For the following function f, find the antiderivative F that satisfies the given condition.

$f(x) = 2x^3 + 3\sin x, \; F(0) = 3$

The antiderivative that satisfies the given condition is F(x) = $\boxed{\phantom{xx}}$.

Answer: $\frac{1}{2}x^4 - 3\cos x + 6$

ID: 4.9.71

$$\int f(x)\,dx = \int 2x^3 + 3\sin(x)\,dx$$

$$F(x) = \frac{2x^{3+1}}{3+1} - 3\cos x + C$$

$$F(x) = \frac{2x^4}{4} - 3\cos(x) + C$$

$$F(0) = \frac{2(0)^4}{4} - 3\cos(0) + C = 3$$

$$0 - 3(1) + C = 3$$

$$0 - 3 + C = 3$$

$$-3 + C = 3$$

$$-3 + C + 3 = 3 + 3$$

$$C = 6$$

$$F(x) = \frac{2}{4}x^4 - 3\cos x + C$$

$$F(x) = \frac{1}{2}x^4 - 3\cos(x) + 6$$

109. For the following function f, find the antiderivative F that satisfies the given condition.

$f(u) = 2e^u + 5; \; F(0) = 8$

The antiderivative that satisfies the given condition is F(u) = $\boxed{\phantom{xx}}$.

Answer: $2e^u + 5u + 6$

ID: 4.9.74

$$\int f(u)\,du = \int 2e^u + 5\,du$$

$$F(u) = 2e^u + 5u + C$$

$$F(0) = 2e^0 + 5(0) + C = 8$$

$$F(0) = 2(1) + 0 + C = 8$$

$$2 + 0 + C = 8$$

$$2 + C = 8$$

$$2 + C - 2 = 8 - 2$$

$$C = 6$$

$$F(u) = 2e^u + 5u + C$$

$$F(u) = 2e^u + 5u + 6$$

110. Given the following velocity function of an object moving along a line, find the position function with the given initial position.

$v(t) = 6t^2 + 4t - 7; \; s(0) = 0$

The position function is s(t) = $\boxed{\phantom{xx}}$.

Answer: $2t^3 + 2t^2 - 7t$

ID: 4.9.95

$$\int v(t)\,dt = \int 6t^2 + 4t - 7\,dt$$

$$s(t) = \frac{6t^{2+1}}{2+1} + \frac{4t^{1+1}}{1+1} - 7t + C$$

$$s(t) = \frac{6t^3}{3} + \frac{4t^2}{2} - 7t + C$$

$$s(t) = 2t^3 + 2t^2 - 7t + C$$

$$s(0) = 2(0)^3 + 2(0)^2 - 7(0) + C = 0$$

$$0 + 0 - 0 + C = 0$$

$$C = 0$$

$$s(t) = 2t^3 + 2t^2 - 7t$$

111. Find the absolute extreme values of the function on the interval.

$F(x) = \sqrt[3]{x}, \ -8 \le x \le 27$

- ○ A. absolute maximum is 3 at x = −27; absolute minimum is −2 at x = 27
- ○ B. absolute maximum is 3 at x = 27; absolute minimum is −2 at x = −8
- ○ C. absolute maximum is 0 at x = 0; absolute minimum is −2 at x = −8
- ○ D. absolute maximum is 3 at x = 27; absolute minimum is 0 at x = 0

$F(x) = x^{1/3}$

$F'(x) = \frac{1}{3}x^{1/3 - 1}$ $\quad \frac{1}{3} - \frac{3}{3}$

$F'(x) = \frac{1}{3}x^{-2/3}$

$F'(x) = \frac{1}{3x^{2/3}}$ undefined $x=0$

$x=-8$ $\quad x=0$ $\quad x=27$

Answer: B. absolute maximum is 3 at x = 27; absolute minimum is −2 at x = −8

$f(-8) = \sqrt[3]{-8} = -2$ absolute min | $F(0) = \sqrt[3]{0} = 0$ | $F(27) = \sqrt[3]{27} = 3$ absolute max

ID: 4.1-13

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112. Find the value or values of c that satisfy the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ in the conclusion of the Mean Value Theorem for the function and interval.

$f(x) = x^2 + 3x + 3, \ [-2,3]$

$[-2, 3]$
$\ \ a \quad b$

- ○ A. $-\frac{1}{2}, \frac{1}{2}$
- ○ B. $0, \frac{1}{2}$
- ○ C. $\frac{1}{2}$
- ○ D. $-2, 3$

$f'(x) = 2x + 3 + 0$

$f'(x) = 2x + 3$

$\frac{f(b) - f(a)}{b - a} = 2x + 3$

$\frac{f(3) - f(-2)}{(3) - (-2)} = 2x + 3$

$\frac{((3)^2 + 3(3) + 3) - ((-2)^2 + 3(-2) + 3)}{3 + 2} = 2x + 3$

Answer: C. $\frac{1}{2}$

$\frac{(9 + 9 + 3) - (4 - 6 + 3)}{5} = 2x + 3$

ID: 4.2-1

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$\frac{(21) - (1)}{5} = 2x + 3$

$\frac{20}{5} = 2x + 3$

$4 = 2x + 3$

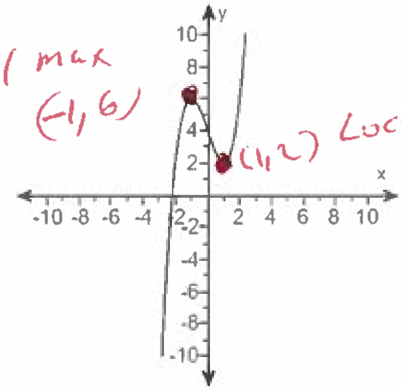
$4 - 3 = 2x + 3 - 3$

$1 = 2x$

$\frac{1}{2} = \frac{2x}{2}$

$\frac{1}{2} = x$

113. Use the graph of the function f(x) to locate the local extrema and identify the intervals where the function is concave up and concave down.




A. Local minimum at $x = 1$; local maximum at $x = -1$; concave down on $(-\infty, \infty)$

B. Local minimum at $x = 1$; local maximum at $x = -1$; concave down on $(0, \infty)$; concave up on $(-\infty, 0)$

C. Local minimum at $x = 1$; local maximum at $x = -1$; concave up on $(0, \infty)$; concave down on $(-\infty, 0)$

D. Local minimum at $x = 1$; local maximum at $x = -1$; concave up on $(-\infty, \infty)$

Answer: C. Local minimum at $x = 1$; local maximum at $x = -1$; concave up on $(0, \infty)$; concave down on $(-\infty, 0)$

ID: 4.3-9

114. From a thin piece of cardboard 20 in by 20 in, square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

A. $13.3 \text{ in} \times 13.3 \text{ in} \times 6.7 \text{ in}$; $1{,}185.2 \text{ in}^3$

B. $13.3 \text{ in} \times 13.3 \text{ in} \times 3.3 \text{ in}$; $592.6 \text{ in}^3$

C. $6.7 \text{ in} \times 6.7 \text{ in} \times 6.7 \text{ in}$; $296.3 \text{ in}^3$

D. $10.0 \text{ in} \times 10.0 \text{ in} \times 5.0 \text{ in}$; $500.0 \text{ in}^3$

Answer: B. $13.3 \text{ in} \times 13.3 \text{ in} \times 3.3 \text{ in}$; $592.6 \text{ in}^3$

$V(x) = x(20-2x)(20-2x)$
$V(x) = x(400 - 40x - 40x + 4x^2)$
$V(x) = x(400 - 80x + 4x^2)$
$V(x) = 400x - 80x^2 + 4x^3$
$V(x) = 4x^3 - 80x^2 + 400x$
$V'(x) = 12x^2 - 160x + 400$
$V''(x) = 24x - 160 + 0$
$V''(x) = 24x - 160$

$V'(x) = 12x^2 - 160x + 400 = 0$
$4(3x^2 - 40x + 100) = 0$
$4(3x - 10)(x - 10) = 0$
$3x - 10 = 0$ or $x - 10 = 0$
$3x = 10$ or $x = 10$
$\frac{3x}{3} = \frac{10}{3}$
$x = \frac{10}{3}$ or $x = 10$ concave up
max

Max at $x = \frac{10}{3} = 3\frac{1}{3}$ $(= 3.33333)$

$V''\left(\frac{10}{3}\right) = 24\left(\frac{10}{3}\right) - 160 = 8(10) - 160 = 80 - 160 = -80 < 0$

ID: 4.5-1

115. Solve the initial value problem.

$$\frac{ds}{dt} = \cos t - \sin t, \quad s\left(\frac{\pi}{2}\right) = 7$$

A. $s = \sin t + \cos t + 8$

B. $s = 2\sin t + 5$

C. $s = \sin t - \cos t + 6$

D. $s = \sin t + \cos t + 6$

Answer: D. $s = \sin t + \cos t + 6$

$\int ds = \int \cos(t) - \sin(t)\, dt$
$s = \sin(t) + \cos(t) + C$
$s(t) = \sin(t) + \cos(t) + C$
$s\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) + C = 7$
$1 + 0 + C = 7$
$1 + C = 7$
$\cancel{1} + C \cancel{- 1} = 7 - 1$
$C = 6$
$s(t) = \sin(t) + \cos(t) + C$
$s(t) = \sin(t) + \cos(t) + 6$

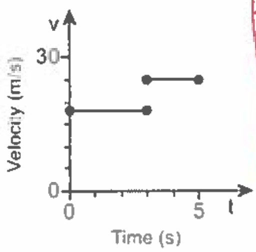
Formula
$\boxed{\int \cos(f(x)) f'(x)\, dx = \sin(f(x)) + C}$
$\boxed{\int -\sin(f(x)) f'(x)\, dx = \cos(f(x)) + C}$

ID: 4.9-16

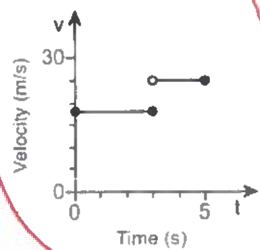
116. Suppose an object moves along a line at 18 m/s for $0 \le t \le 3$ s and at 25 m/s for $3 < t \le 5$ s. Sketch the graph of the velocity function and find the displacement of the object for $0 \le t \le 5$.

Sketch the graph of the velocity function. Choose the correct graph below.

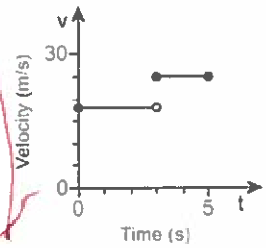
A.



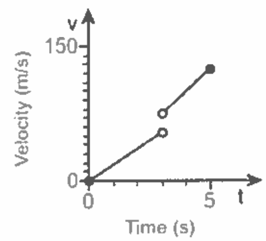

B.




C.

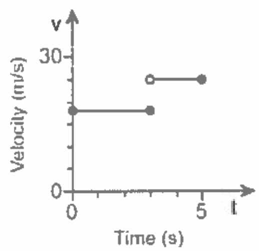



D.




The displacement of the object for $0 \le t \le 5$ is ______ m. (Simplify your answer.)

Answers

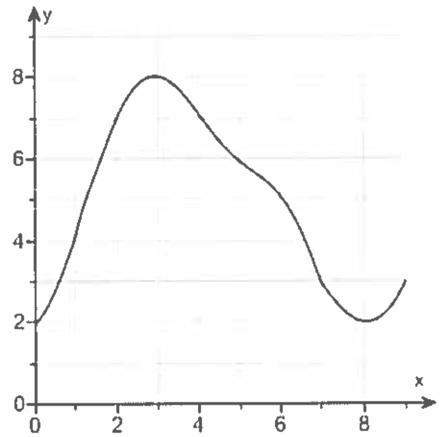



B.

104

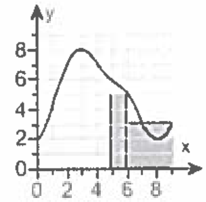
ID: 5.1.1

117. Approximate the area of the region bounded by the graph of f(x) (shown below) and the x-axis by dividing the interval [5,9] into n = 4 subintervals. Use a left and right Riemann sum to obtain two different approximations. Draw the approximating rectangles.

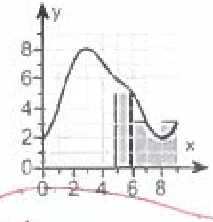



In which graph below are the selected points the left endpoints of the 4 approximating rectangles?

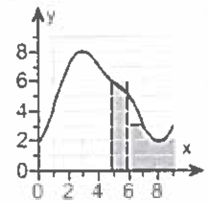
A.



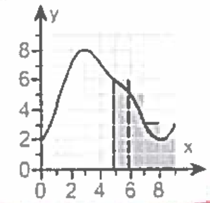
B.



C.



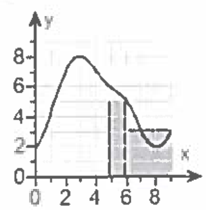
D.



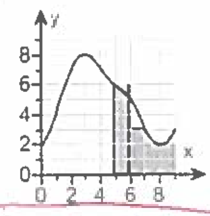
Using the specified rectangles, approximate the area.

In which graph below are the selected points the right endpoints of the 4 approximating rectangles?

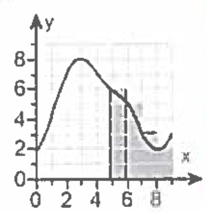
A.



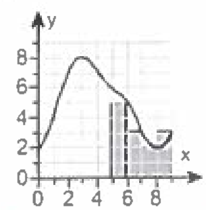
B.



C.

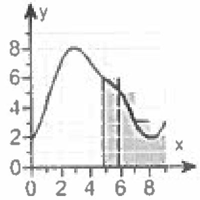


D.



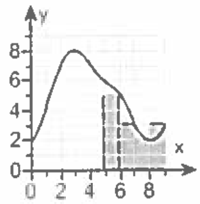
Using the specified rectangles, approximate the area.

Answers




D.

16




D.

13

ID: 5.1.9

118. Evaluate the following expressions.

a. $\sum_{k=1}^{16} k$ b. $\sum_{k=1}^{5} (3k+2)$ c. $\sum_{k=1}^{8} k^2$ d. $\sum_{n=1}^{5} \left(3+n^2\right)$

e. $\sum_{m=1}^{4} \frac{4m+4}{9}$ f. $\sum_{j=1}^{3} (5j-6)$ g. $\sum_{k=1}^{9} k(6k+5)$ h. $\sum_{n=0}^{7} \sin\frac{n\pi}{2}$

a. $\sum_{k=1}^{16} k =$ 136 (Type an integer or a simplified fraction.)

b. $\sum_{k=1}^{5} (3k+2) =$ 55 (Type an integer or a simplified fraction.)

c. $\sum_{k=1}^{8} k^2 =$ 204 (Type an integer or a simplified fraction.)

d. $\sum_{n=1}^{5} \left(3+n^2\right) =$ 70 (Type an integer or a simplified fraction.)

e. $\sum_{m=1}^{4} \frac{4m+4}{9} =$ $\frac{56}{9}$ (Type an integer or a simplified fraction.)

f. $\sum_{j=1}^{3} (5j-6) =$ 12 (Type an integer or a simplified fraction.)

g. $\sum_{k=1}^{9} k(6k+5) =$ 1935 (Type an integer or a simplified fraction.)

h. $\sum_{n=0}^{7} \sin\frac{n\pi}{2} =$ 0 (Type an integer or a simplified fraction.)

use graphing calculator
math, summation $\Sigma$, enter
$\sum_{k=1}^{16} (k) = 136$

Answers 136

55

204

70

$\frac{56}{9}$

12

1935

0

ID: 5.1.49

119.

The functions f and g are integrable and $\int_3^7 f(x)dx = 6$, $\int_3^7 g(x)dx = 3$, and $\int_4^7 f(x)dx = 4$. Evaluate the integral below or state that there is not enough information.

$$-\int_7^3 3f(x)dx$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$-\int_7^3 3f(x)dx =$

$\int_3^7 3f(x)dx =$

$3\int_3^7 f(x)dx =$

$3(6) =$

$18 =$

○ A. $-\int_7^3 3f(x)dx =$ ________ (Simplify your answer.)

○ B. There is not enough information to evaluate $-\int_7^3 3f(x)dx$.

Answer: A. $-\int_7^3 3f(x)dx = \boxed{18}$ (Simplify your answer.)

ID: EXTRA 5.18

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120.

Evaluate $\frac{d}{dx}\int_a^x f(t)\,dt$ and $\frac{d}{dx}\int_a^b f(t)\,dt$, where a and b are constants.

$\frac{d}{dx}\int_a^x f(t)\,dt = \boxed{f(x)}$ (Simplify your answer.)

$\frac{d}{dx}\int_a^b f(t)\,dt = \boxed{0}$ (Simplify your answer.)

Answers f(x)

0

ID: 5.3.9

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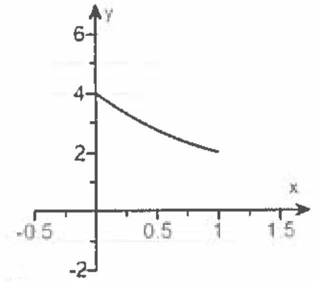
121. Evaluate the following integral using the Fundamental Theorem of Calculus. Discuss whether your result is consistent with the figure.




$$\int_0^1 (x^2 - 3x + 4)\,dx$$

$\left(\frac{x^{2+1}}{2+1} - \frac{3x^2}{2} + 4x\right)\Big|_0^1$

$\left(\frac{x^3}{3} - \frac{3x^2}{2} + 4x\right)\Big|_0^1 =$

$\left(\frac{(1)^3}{3} - \frac{3(1)^2}{2} + 4(1)\right) - \left(\frac{(0)^3}{3} - \frac{3(0)^2}{2} + 4(0)\right) =$

$$\int_0^1 (x^2 - 3x + 4)\,dx = \underline{\qquad}$$

$\left(\frac{1}{3} - \frac{3}{2} + 4\right) - (0) =$

$\left(\frac{1}{3}\left(\frac{2}{2}\right) - \frac{3}{2}\left(\frac{3}{3}\right) + \frac{4}{1}\left(\frac{6}{6}\right) - (0)\right) =$

$\left(\frac{2}{6} - \frac{9}{6} + \frac{24}{6}\right) - (0) =$

$\left(\frac{2-9+24}{6}\right) - (0) =$

$\left(\frac{17}{6}\right) - (0) =$

$\frac{17}{6} =$

Is your result consistent with the figure?

- A. No, because the definite integral is positive and the graph of f lies below the x-axis.
- B. Yes, because the definite integral is positive and the graph of f lies above the x-axis.
- C. Yes, because the definite integral is negative and the graph of f lies below the x-axis.
- D. No, because the definite integral is negative and the graph of f lies above the x-axis.

formula

$\int x^n dx$

$\frac{x^{n+1}}{n+1} + C$

Answers $\frac{17}{6}$

B. Yes, because the definite integral is positive and the graph of f lies above the x-axis.

ID: 5.3.23

---

$\int_0^1 (x^2 - 3x + 4)\,dx =$

$\left(\frac{x^{2+1}}{2+1} - \frac{3x^{1+1}}{1+1} + 4x\right)\Big|_0^1 =$

$\left(\frac{x^3}{3} - \frac{3x^2}{2} + 4x\right)\Big|_0^1 =$

$\left(\frac{(1)^3}{3} - \frac{3(1)^2}{2} + 4(1)\right) - \left(\frac{(0)^3}{3} - \frac{3(0)^2}{2} + 4(0)\right) =$

$\left(\frac{1}{3} - \frac{3(1)}{2} + 4\right) - (0 - 0 + 0) =$

$\left(\frac{1}{3} - \frac{3}{2} + \frac{4}{1}\right) - (0) =$

$\frac{1}{3}\left(\frac{2}{2}\right) - \frac{3}{2}\left(\frac{3}{3}\right) + \frac{4}{1}\left(\frac{6}{6}\right) =$

$\frac{2}{6} - \frac{9}{6} + \frac{24}{6} =$

$\frac{2-9+24}{6} =$

$\frac{17}{6} =$

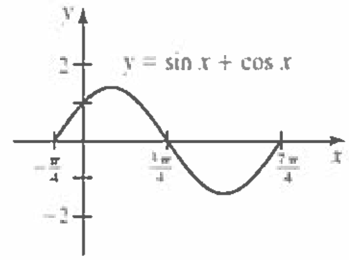
formula

$\int x^n dx =$

$\frac{x^{n+1}}{n+1} + C =$

122.

Evaluate the integral $\int_{-\pi/4}^{7\pi/4} (\sin x + \cos x)\,dx$ using the fundamental theorem of calculus. Discuss whether your result is consistent with the figure shown to the right.




$\int_{-\pi/4}^{7\pi/4} (\sin x + \cos x)\,dx =$ ______

Is this value consistent with the given figure?

- ○ **A.** The value is consistent with the figure because the total area can be approximated using a rectangle of base $\pi$ and height $\sin(\pi/4) + \cos(\pi/4) = \sqrt{2}$.
- ○ **B.** The value is not consistent with the figure because the total area could be approximated using a rectangle of base $\pi$ and height $\sin(\pi/4) + \cos(\pi/4) = \sqrt{2}$.
- ○ **C.** The value is not consistent with the figure because the figure is a graph of the base function, f(x), instead of a graph of the area function, A(x).
- ○ **D.** The value is consistent with the figure because the area below the x-axis appears to be equal to the area above the x-axis.

$\int_{-\pi/4}^{7\pi/4} (\sin(x) + \cos(x))\,dx$

Answers 0

D.

The value is consistent with the figure because the area below the x-axis appears to be equal to the area above the x-axis.

$\left(-\cos x + \sin(x)\right)\Big|_{-\pi/4}^{7\pi/4}$

ID: 5.3.24

$\left(-\cos\left(\frac{7\pi}{4}\right) + \sin\left(\frac{7\pi}{4}\right)\right) - \left(-\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)\right) =$

$(-.7071 - .7071) - (-.7071 - .7071) =$

$(-1.4142) - (-1.4142) =$

$-1.4142 + 1.4142 =$

$0 =$

123. Evaluate the following integral using the fundamental theorem of calculus. Sketch the graph of the integrand and shade the region whose net area you have found.

$$\int_{-3}^{1} \left(x^2+2x-3\right)\,dx = \left(\frac{x^3}{3}+\frac{2x^2}{2}-3x\right)\Big|_{-3}^{1} = \left(\frac{x^3}{3}+x^2-3x\right)\Big|_{-3}^{1} =$$

$$\left(\frac{(1)^3}{3}+(1)^2-3(1)\right)-\left(\frac{(-3)^3}{3}+(-3)^2-3(-3)\right)=$$

$$\left(\frac{1}{3}+1-3\right)-\left(-\frac{27}{3}+9+9\right)=$$

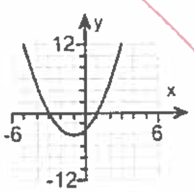
$$\int_{-3}^{1} \left(x^2+2x-3\right)\,dx = \square$$

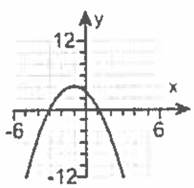
$$\left(\frac{1}{3}-\frac{2}{1}\right)-\left(\cancel{-9}\cancel{+9}+9\right)=$$

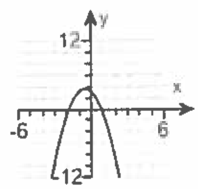
$$\left(\frac{1}{3}-\frac{2}{1}\left(\frac{3}{3}\right)\right)-(9)=$$

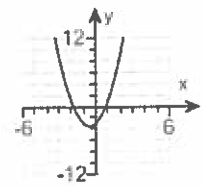
Choose the correct sketch below.

$$\frac{1}{3}-\frac{6}{3}-\frac{9}{1}\left(\frac{3}{3}\right)$$

A. 

B. 

C. 

D. 

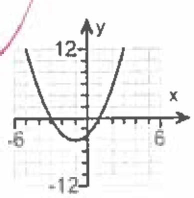
$$\frac{1}{3}-\frac{6}{3}-\frac{27}{3}=$$

$$\frac{1-6-27}{3}=$$

$$-\frac{32}{3}$$

Answers $\frac{32}{3}$

formula
$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C=$$



A.

ID: 5.3.25

124. Use the fundamental theorem of calculus to evaluate the following definite integral.

$$\int_{0}^{3} \left(5x^2+4\right)\,dx = \left(\frac{5x^{2+1}}{2+1}+4x\right)\Big|_0^3 =$$

$$\left(\frac{5x^3}{3}+4x\right)\Big|_0^3 =$$

formula
$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C=$$

$$\int_{0}^{3} \left(5x^2+4\right)\,dx = \square$$

$$\left(\frac{5(3)^3}{3}+4(3)\right)-\left(\frac{5(0)^3}{3}+4(0)\right)=$$

(Type an exact answer.)

$$\left(\frac{5\cdot 3\cdot 3\cdot \cancel{3}}{\cancel{3}}+4(3)\right)-(0+0)=$$

Answer: 57

$$(5\cdot 3\cdot 3+4(3))-(0)=$$

ID: 5.3.29

$$(45+12)-(0)=$$

$$57-0=$$

$$57=$$

125. Evaluate the following integral using the Fundamental Theorem of Calculus.

$$\int_4^9 (4-x)(x-9)\,dx$$

$= \int_4^9 (4x - 36 - x^2 + 9x)\,dx = \int_4^9 -x^2 + 13x - 36\,dx$

$= \frac{x^3}{3} + \frac{13x^2}{2} - 36x \Big|_4^9$

$$\int_4^9 (4-x)(x-9)\,dx = \underline{\qquad}$$

$= \left(-\frac{x^3}{3} + \frac{13x^2}{2} - 36x\right)\Big|_4^9 =$

(Type an exact answer.)

$\left(-\frac{(9)^3}{3} + \frac{13(9)^2}{2} - 36(9)\right) - \left(-\frac{(4)^3}{3} + \frac{13(4)^2}{2} - 36(4)\right) =$

Answer: $\frac{125}{6}$

$\left(-\frac{729}{3} + \frac{1053}{2} - \frac{324}{1}\right) - \left(-\frac{64}{3} + \frac{208}{2} - \frac{144}{1}\right) =$

$\frac{125}{6}$

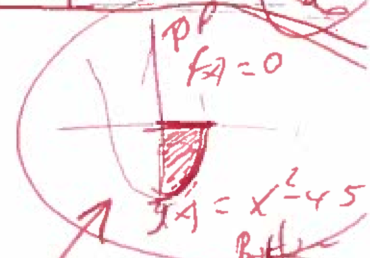
ID: 5.3.41

$\left(-\frac{729}{3}\left(\frac{2}{2}\right) + \frac{1053}{2}\left(\frac{3}{3}\right) - \frac{324}{1}\left(\frac{6}{6}\right)\right) - \left(-\frac{64}{3}\left(\frac{2}{2}\right) + \frac{207}{2}\left(\frac{3}{3}\right) - \frac{144}{1}\left(\frac{6}{6}\right)\right) = \frac{125}{6}$

126. Find the area of the region bounded by the graph of f and the x-axis on the given interval.

$f(x) = x^2 - 45;\ [3,4]$

$\int_3^4 x^2 - 45 = \frac{x^3}{3} - 45x \Big|_3^4 =$




The area is $\underline{\qquad}$. (Type an integer or a simplified fraction.)

$\left(\frac{(4)^3}{3} - 45(4)\right) - \left(\frac{(3)^3}{3} - 45(3)\right) =$

Answer: $\frac{98}{3}$

$\left(\frac{64}{3} - \frac{180}{1}\right) - \left(\frac{27}{3} - \frac{135}{1}\right) =$

positive area only

ID: 5.3.67

$\frac{64}{3} - \frac{180}{1}\left(\frac{3}{3}\right) - \frac{27}{3} + \frac{135}{1}\left(\frac{3}{3}\right) =$

$\frac{64}{3} - \frac{540}{3} - \frac{27}{3} + \frac{405}{3} = -\frac{98}{3} \rightarrow \frac{98}{3}$

127. Simplify the following expression.

$$\frac{d}{dx}\int_x^3 \sqrt{t^2+1}\,dt$$

$$\frac{d}{dx}\int_x^3 \sqrt{t^2+1}\,dt = \underline{\qquad}$$

$\frac{d}{dx}\left(-\int_3^x \sqrt{t^2+1}\,dt\right) =$

Answer: $-\sqrt{x^2+1}$

$-1\sqrt{x^2+1} =$

ID: 5.3.75

$-\sqrt{x^2+1} =$

128. Simplify the following expression.

$$\frac{d}{dx}\int_{10}^{x^3}\frac{dp}{p^2}$$

$$\frac{d}{dx}\int_{10}^{x^3}\frac{dp}{p^2} = \square$$

Answer: $\frac{3}{x^4}$

ID: 5.3.77

$\frac{d}{dx}\int_{10}^{x^3}\frac{1}{p^2}\,dp =$

$\frac{1}{(x^3)^2}\cdot(x^3)' =$

$\frac{1}{x^6}\cdot(3x^2) =$

$\frac{3x^2}{x^6} =$

$\frac{3}{x^{6-2}} =$

$\frac{3}{x^4} =$

129. Evaluate the following integral.

$$\int_0^4 (x-5)^2\,dx =$$

$$\int_0^4 (x-5)^2\,dx = \square$$

(Type an exact answer. Use C as the arbitrary constant as needed.)

Answer: $\frac{124}{3}$

ID: EXTRA 5.57

$\int_0^4 (x-5)^2\cdot(1)\,dx =$

$\frac{(x-5)^{2+1}}{2+1}\Big|_0^4 =$

$\frac{(x-5)^3}{3}\Big|_0^4 =$

Formula

$\int (f(x))^n\cdot f'(x)\,dx =$

$\frac{(f(x))^{n+1}}{n+1}+C =$

$\frac{(4-5)^3}{3} - \frac{(0-5)^3}{3} =$

$\frac{(-1)^3}{3} - \frac{(-5)^3}{3} =$

$\frac{(-1)(-1)(-1)}{3} - \frac{(-5)(-5)(-5)}{3} =$

$\frac{-1}{3} - \frac{-125}{3} =$

$-\frac{1}{3} + \frac{125}{3} =$

$\frac{-1+125}{3} =$

$\frac{124}{3} =$

130. Find the average value of the following function over the given interval. Draw a graph of the function and indicate the average value.

$f(x) = x(x-1)$; $[5,7]$

$a$ $b$

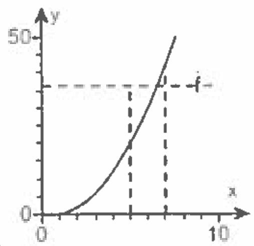
The average value of the function is $\bar{f} =$ ______.

Choose the correct graph of f(x) and $\bar{f}$ below.

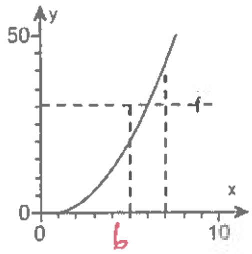
formula

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C =$

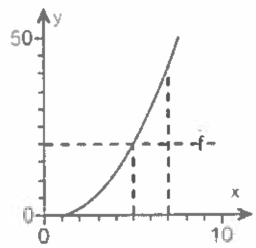
A.



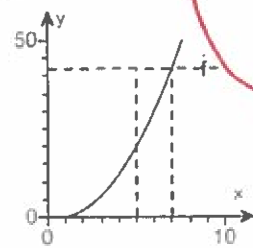

B.




C.




D.

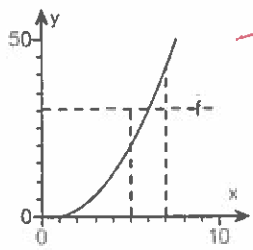



$b$

$\frac{1}{b-a}\int_a^b f(x)\,dx =$

Answers $\frac{91}{3}$

$\frac{1}{7-5}\int_5^7 x(x-1)\,dx =$




B.

$\frac{1}{2}\int_5^7 (x^2 - x)\,dx =$

$\frac{1}{2}\left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_5^7 =$

ID: 5.4.30

$\frac{1}{2}\left(\frac{(7)^3}{3} - \frac{(7)^2}{2}\right) - \frac{1}{2}\left(\frac{(5)^3}{3} - \frac{(5)^2}{2}\right) =$

$\frac{1}{2}\left(\frac{343}{3} - \frac{49}{2}\right) - \frac{1}{2}\left(\frac{125}{3} - \frac{25}{2}\right) =$

$\frac{1}{2}\left(\frac{343}{3}\left(\frac{2}{2}\right) - \frac{49}{2}\left(\frac{3}{3}\right)\right) - \frac{1}{2}\left(\frac{125}{3}\left(\frac{2}{2}\right) - \frac{25}{2}\left(\frac{3}{3}\right)\right)$

$\frac{1}{2}\left(\frac{686}{6} - \frac{147}{6}\right) - \frac{1}{2}\left(\frac{250}{6} - \frac{75}{6}\right)$

$\frac{1}{2}\left(\frac{686-147}{6}\right) - \frac{1}{2}\left(\frac{250-75}{6}\right)$

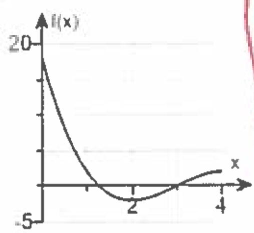
$\frac{1}{2}\left(\frac{539}{6}\right) - \frac{1}{2}\left(\frac{175}{6}\right)$

$\frac{539}{12} - \frac{175}{12} = \frac{364}{12} = \frac{\cancel{4}(91)}{\cancel{4}(3)} = \frac{91}{3}$

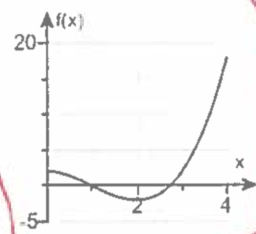
131. The elevation of a path is given by $f(x) = x^3 - 3x^2 + 2$, where x measures horizontal distances. Draw a graph of the elevation function and find its average value for $0 \le x \le 4$.

Choose the correct graph below.

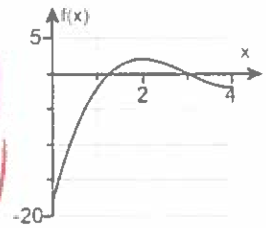
○ A.



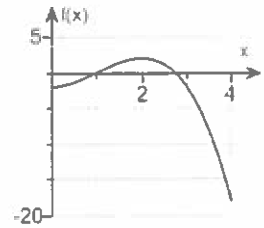

○ B.




○ C.

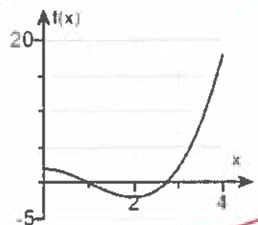



○ D.




The average value is 2 . (Type an integer or a simplified fraction.)

Answers




B.

2

ID: 5.4.34

132. Find the point(s) at which the function $f(x) = 5 - 6x$ equals its average value on the interval [0,4].

$\frac{1}{b-a}\int_a^b f(x)\,dx$

The function equals its average value at x = ____.
(Use a comma to separate answers as needed.)

Answer: 2

$\frac{1}{4-0}\int_0^4 (5-6x)\,dx$

$\frac{1}{4}\left(5x-\frac{6x^2}{2}\right)\Big|_0^4$

$\frac{1}{4}(5x-3x^2)\Big|_0^4 =$

$\frac{1}{4}(5(4)-3(4)^2)-\frac{1}{4}(5(0)-3(0)^2)=$

$\frac{1}{4}(20-48)-\frac{1}{4}(0)=\frac{1}{4}(-28)-0=-7$

$5-6x=-7$ Set

$5-6x-5=-7-5$

$-6x=-12$

$\frac{-6x}{-6}=\frac{-12}{-6}$

$x=2$

ID: 5.4.39

133. Use the substitution $u = x^2 + 15$ to find the following indefinite integral. Check your answer by differentiation.

$$\int 2x\left(x^2+15\right)^7 dx = \int (x^2+15)^7(2x)\,dx =$$

$\int 2x\left(x^2+15\right)^7 dx =$ ____

(Use C as the arbitrary constant.)

$\frac{(x^2+15)^{7+1}}{7+1}+C=$

$\frac{(x^2+15)^{8}}{8}+C=$

formula

$\int (f(x))^N \cdot f'(x)\,dx = \frac{(f(x))^{N+1}}{N+1}+C=$

Answer: $\frac{1}{8}\left(x^2+15\right)^8 + C$

$\frac{1}{8}(x^2+15)^8+C=$

ID: 5.5.7

134. Use the substitution $u = 7x^2 - 1$ to find the following indefinite integral. Check your answer by differentiation.

$$\int 14x \cos\left(7x^2 - 1\right)\, dx = \int \cos(7x^2-1)(14x)\,dx = \sin(7x^2-1) + C =$$

formula

$$\int \cos(f(x))\, f'(x)\, dx = \sin(f(x)) + C =$$

$\int 14x \cos\left(7x^2 - 1\right)\, dx =$ ☐

(Use C as the arbitrary constant.)

Answer: $\sin\left(7x^2 - 1\right) + C$

ID: 5.5.8

---

135. Use the substitution $u = 7x^2 + 4x$ to evaluate the indefinite integral below.

$$\int (14x+4)\sqrt{7x^2+4x}\, dx = \int \sqrt{7x^2+4x}\,(14x+4)\,dx = \int (7x^2+4x)^{1/2}(14x+4)\,dx =$$

$$\frac{(7x^2+4x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{(7x^2+4x)^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C = \frac{(7x^2+4x)^{3/2}}{3/2} + C = \frac{2}{3}(7x^2+4x)^{3/2} + C =$$

Write the integrand in terms of u.

$\int (14x+4)\sqrt{7x^2+4x}\, dx = \int$ (☐) du

Evaluate the integral.

$\int (14x+4)\sqrt{7x^2+4x}\, dx =$ ☐

(Use C as the arbitrary constant.)

Answers $\sqrt{u}$

$$\frac{2}{3}\left(7x^2+4x\right)^{\frac{3}{2}} + C$$

ID: 5.5.10-Setup & Solve

formula

$$\int (f(x))^N \cdot f'(x)\, dx = \frac{(f(x))^{N+1}}{N+1} + C =$$

136. Use a change of variables or the accompanying table to evaluate the following indefinite integral.

$$\int \frac{e^{2x}}{e^{2x}+7}\,dx \quad = \qquad \frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+7}\,dx =$$

[2] Click the icon to view the table of general integration formulas.

Determine a change of variables from x to u. Choose the correct answer below. $\frac{1}{2}\int \frac{e^{2x}\cdot 2}{e^{2x}+7}\,dx =$

- A. $u = 2x$
- B. $u = e^{2x}$
- C. $u = e^{2x} + 7$
- D. $u = \frac{1}{e^{2x}+7}$

$\frac{1}{2}\ln\left|e^{2x}+7\right| + C =$

formula

$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + C$

Write the integral in terms of u.

$$\int \frac{e^{2x}}{e^{2x}+7}\,dx = \int (\square)\,du$$

Evaluate the integral.

$$\int \frac{e^{2x}}{e^{2x}+7}\,dx = \square$$

(Use C as the arbitrary constant.)

2: General Integration Formulas

$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$

$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$

$\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$

$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax + C$

$\int \csc ax \cot ax\,dx = -\frac{1}{a}\csc ax + C$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$

$\int b^x\,dx = \frac{1}{\ln b}b^x + C,\ b > 0,\ b \neq 1$

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C,\ a > 0$

$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C,\ a > 0$

Answers C. $u = e^{2x} + 7$

$\frac{1}{2u}$

$\frac{1}{2}\ln\left|e^{2x}+7\right| + C$

ID: 5.5.44-Setup & Solve

137. Use a change of variables or the table to evaluate the following definite integral.

$$\int_0^{\pi/12} \cos 2x\, dx =$$

$$\frac{1}{2}\int_0^{\pi/12} \cos(2x)(2)\, dx =$$

$$\frac{1}{2}\sin(2x)\Big|_0^{\pi/12} =$$

[3] Click to view the table of general integration formulas.

$$\int_0^{\pi/12} \cos 2x\, dx = \square \text{ (Type an exact answer.)}$$

$$\frac{1}{2}\sin\left(2\left(\frac{\pi}{12}\right)\right) - \frac{1}{2}\sin(2(0)) =$$

$$\frac{1}{2}\sin\left(\frac{\pi}{6}\right) - \frac{1}{2}\sin(0) =$$

$$\frac{1}{2}\left(\frac{1}{2}\right) - \frac{1}{2}(0) =$$

$$\frac{1}{4} - 0 =$$

$$\frac{1}{4} =$$

3: General Integration Formulas

$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$

$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$

$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax + C$

$\int \csc^2 ax\, dx = -\frac{1}{a}\cot ax + C$

$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax + C$

$\int \csc ax \cot ax\, dx = -\frac{1}{a}\csc ax + C$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$

$\int b^x dx = \frac{1}{\ln b}b^x + C,\ b > 0,\ b \neq 1$

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C,\ a > 0$

$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C,\ a > 0$

Answer: $\frac{1}{4}$

formula

$$\int \cos(f(x)) \cdot f'(x)\, dx =$$

$$\sin(f(x)) + C =$$

ID: 5.5.45

138. Use a change of variables or the table to evaluate the following definite integral.

$$\int_0^1 6e^{3x}\,dx$$

[4] Click to view the table of general integration formulas.

$\int_0^1 6e^{3x}\,dx =$ ______ (Type an exact answer.)

4: General Integration Formulas

$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$

$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$

$\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$

$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax + C$

$\int \csc ax \cot ax\,dx = -\frac{1}{a}\csc ax + C$

$\int e^{ax}dx = \frac{1}{a}e^{ax} + C$

$\int b^x dx = \frac{1}{\ln b}b^x + C,\ b > 0,\ b \neq 1$

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C,\ a > 0$

$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C,\ a > 0$

Answer: $2e^3 - 2$

ID: 5.5.46

$$\int_0^1 6e^{3x}\,dx =$$

$$2\int_0^1 3e^{3x}\,dx =$$

$$2\int_0^1 e^{3x}\cdot 3\,dx =$$

$$2e^{3x}\Big|_0^1 =$$

$$2e^{3(1)} - 2e^{3(0)} =$$

$$2e^3 - 2e^0 =$$

$$2e^3 - 2(1) =$$

$$2e^3 - 2 =$$

Formula

$$\int e^{f(x)}\cdot f'(x)\,dx =$$

$$e^{f(x)} + C =$$

139. Use a change of variables or the table to evaluate the following definite integral.

$$\int_0^1 4x^3\left(9-x^4\right)dx = -1\int_0^1 (9-x^4)^1(-4x^3)\,dx =$$

$$\int (f(x))^N \cdot f'(x)\,dx = \frac{(f(x))^{N+1}}{N+1} + C =$$

$$-1\,\frac{(9-x^4)^{1+1}}{1+1}\Big|_0^1 =$$

5 Click to view the table of general integration formulas.

$$-1\,\frac{(9-x^4)^2}{2}\Big|_0^1 =$$

$\int_0^1 4x^3\left(9-x^4\right)dx =$ ______ (Type an exact answer.)

$$-1\,\frac{(9-(1)^4)^2}{2} - -1\,\frac{(9-(0)^4)^2}{2} =$$

5: General Integration Formulas

$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$

$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$

$\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$

$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax + C$

$\int \csc ax \cot ax\,dx = -\frac{1}{a}\csc ax + C$

$\int e^{ax}dx = \frac{1}{a}e^{ax} + C$

$\int b^x dx = \frac{1}{\ln b}b^x + C,\ b > 0,\ b \neq 1$

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C,\ a > 0$

$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C,\ a > 0$

$$-1\,\frac{(9-1)^2}{2} - -1\,\frac{(9-0)^2}{2} =$$

$$-1\,\frac{(8)^2}{2} - -1\,\frac{(9)^2}{2} =$$

$$\frac{-1(64)}{2} - \frac{-1(81)}{2} =$$

$$\frac{-64}{2} + \frac{81}{2} =$$

$$\frac{-64+81}{2} =$$

$$\frac{17}{2} =$$

Answer: $\frac{17}{2}$

ID: 5.5.47

140. Use a finite approximation to estimate the area under the graph of the given function on the stated interval as instructed.

$f(x) = x^2$ between $x = 0$ and $x = 2$, using a right sum with two rectangles of equal width

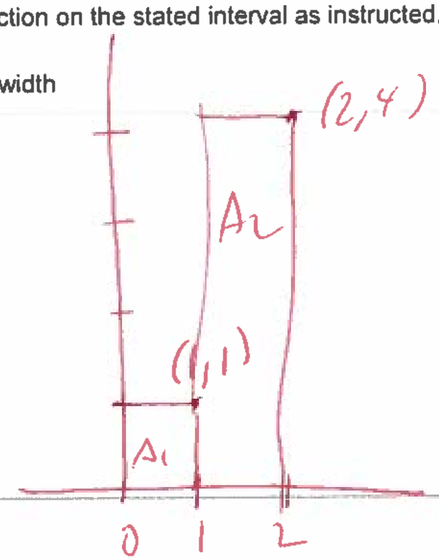
- A. 4.5
- B. 1
- C. 5
- D. 2.5

$A_1 + A_2 = LW$

$(1)(1) + (1)(4) =$

$1 + 4 =$

$5 =$




Answer: C. 5

ID: 5.1-1

141. Suppose that $\int_1^2 f(x)dx = -2$. Find $\int_1^2 3f(u)du$ and $\int_1^2 -f(u)du$.

$\int_1^2 3\,f(u)\,du =$

$3\int_1^2 f(u)\,du =$

$3(-2) =$

$-6 =$

$\int_1^2 -f(u)\,du =$

$-1\int_1^2 f(u)\,du =$

$-1(-2) =$

$2 =$

- A. $-6; -\frac{1}{2}$
- B. 3; 2
- C. −6; 2
- D. 1; −2

Answer: C. −6; 2

ID: 5.2-12

142. Find the derivative.

$$\frac{d}{dx}\int_1^{\sqrt{x}} 16t^9\,dt$$

- A. $\frac{16}{5}x^{3.5} - \frac{16}{5}$
- B. $16x^{9/2}$
- C. $\frac{32}{3}x^{3.5}$
- D. $8x^{1.5}4$

Answer: D. $8x^{1.5}4$

ID: 5.3-18

143. Evaluate the integral using the given substitution.

$$\int x\cos\left(10x^2\right)dx,\ u = 10x^2$$

- A. $\frac{1}{u}\sin(u) + C$
- B. $\frac{x^2}{2}\sin\left(10x^2\right) + C$
- C. $\sin\left(10x^2\right) + C$
- D. $\frac{1}{20}\sin\left(10x^2\right) + C$

Answer: D. $\frac{1}{20}\sin\left(10x^2\right) + C$

$\int \cos(10x^2)(x)\,dx =$

$\frac{1}{20}\int \cos(10x^2)(20x)\,dx =$

$\frac{1}{20}\sin(10x^2) + C =$

Formula

$\int \cos(f(x))\,f'(x)\,dx =$

$\sin(f(x)) + C =$

ID: 5.5-1

144. Evaluate the following integral.

$$\int \frac{dx}{x^2 - 2x + 5}$$

$\int \frac{dx}{x^2 - 2x + 5} =$ ______

(Use C as the arbitrary constant as needed.)

Answer: $\frac{1}{2}\tan^{-1}\frac{x-1}{2} + C$

Complete Square

$\int \frac{dx}{x^2 - 2x + \left(\frac{1}{2}(-2)\right)^2 + 5 - \left(\frac{1}{2}(-2)\right)^2}$

$\int \frac{dx}{x^2 - 2x + (-1)^2 + 5 - (-1)^2} =$

$\int \frac{dx}{x^2 - 2x + 1 + 5 - 1} =$

$\int \frac{dx}{(x-1)(x-1) + 4} =$

$\int \frac{dx}{(x-1)^2 + 4} =$

$\int \frac{dx}{(x-1)^2 + (2)^2} =$

$\frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}\right) + C =$

ID: 8.1.31

145. If the general solution of a differential equation is $y(t) = Ce^{-4t} + 10$, what is the solution that satisfies the initial condition $y(0) = 6$?

$y(t) =$ ______

Answer: $-4e^{-4t} + 10$

$y(t) = Ce^{-4t} + 10$

$y(0) = Ce^{-4(0)} + 10 = 6$

$Ce^{0} + 10 = 6$

$C(1) + 10 = 6$

$C + 10 = 6$

$C + \cancel{10} - 10 = 6 - 10$

$C = -4$

$y(t) = Ce^{-4t} + 10$

$y(t) = -4e^{-4t} + 10$

ID: 9.1.2

146. Find the general solution of the following equation. Express the solution explicitly as a function of the independent variable.

$t^{-4}y'(t) = 4$

y = 

Answer: $\frac{4t^5}{5} + C$

ID: 9.3.5

$t^{-4} y'(t) = 4$

$\frac{y'(t)}{t^4} = 4$

$\frac{y'(t)}{\cancel{t^4}} \cdot \cancel{t^4} = 4 \cdot t^4$

$y'(t) = 4t^4$

$\frac{dy}{dt} = 4t^4$

$\int dy = \int 4t^4\,dt$

$y = \frac{4t^{4+1}}{4+1} + C$

$y = \frac{4t^5}{5} + C$

formula

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

147. Find the general solution of the differential equation $\frac{dy}{dt} = \frac{3t^2}{4y}$.

Choose the correct answer below.

- A. $y = 2t^3 + C$
- B. $y = \pm\sqrt{2t^3 + C}$
- C. $y = \pm\sqrt{\frac{t^3}{2} + C}$
- D. $y = \frac{t^3}{2} + C$

Answer: C. $y = \pm\sqrt{\frac{t^3}{2} + C}$

ID: 9.3.7

$4y\,dy = 3t^2\,dt$

$\int 4y\,dy = \int 3t^2\,dt$

$\frac{4y^{1+1}}{1+1} = \frac{3t^{2+1}}{2+1} + C_1$

$\frac{4y^2}{2} = \frac{\cancel{3}t^3}{\cancel{3}} + C_1$

$2y^2 = t^3 + C_1$

$\frac{\cancel{2}y^2}{\cancel{2}} = \frac{t^3 + C_1}{2}$

$y^2 = \frac{t^3 + C_1}{2}$

$\sqrt{y^2} = \pm\sqrt{\frac{t^3 + C_1}{2}}$

$y = \pm\sqrt{\frac{t^3}{2} + \frac{C_1}{2}}$

$y = \pm\sqrt{\frac{t^3}{2} + C}$

formula

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

148. The general solution of a first-order linear differential equation is $y(t) = Ce^{-10t} - 19$. What solution satisfies the initial condition $y(0) = 5$?

The solution is y(t) = .

Answer: $24e^{-10t} - 19$

ID: 9.4.1

$y(t) = Ce^{-10t} - 19$

$y(0) = Ce^{-10(0)} - 19 = 5$

$Ce^0 - 19 = 5$

$C(1) - 19 = 5$

$C - 19 = 5$

$C - \cancel{19} + \cancel{19} = 5 + 19$

$C = 24$

$y(t) = Ce^{-10t} - 19$

$y(t) = 24e^{-10t} - 19$

149. Find the general solution of the following equation.

$y'(t) = 4y - 5$

$y(t) =$ ☐

(Use C as the arbitrary constant.)

Answer: $Ce^{4t} + \frac{5}{4}$

ID: 9.4.5

$k=4$

$b=-5$

formula

$y'(t) = ky + b$

$y(t) = Ce^{kt} - \frac{b}{k}$

$y'(t) = 4y - 5$

$y(t) = Ce^{kt} - \left(\frac{-5}{4}\right)$

$y(t) = Ce^{kt} + \frac{5}{4}$ OR

$y(t) = Ce^{4t} + \frac{5}{4}$

First order linear Differential Equation

always

150. Find the general solution of the following equation.

$y'(x) = -y + 4$

$y(x) =$ ☐

(Use C as the arbitrary constant.)

Answer: $Ce^{-x} + 4$

ID: 9.4.6

formula

$y'(t) = ky + b$

$y(t) = Ce^{kt} - \frac{b}{k}$

first order linear Differential Equation

always

$y'(x) = -y + 4$

$y'(x) = -1y + 4$

$k=-1$

$b=4$

$y(x) = Ce^{kx} - \left(\frac{4}{-1}\right)$

$y(x) = Ce^{kx} + \frac{4}{1}$

$y(x) = Ce^{kx} + 4$

OR

$y(x) = Ce^{-1x} + 4$