10-22-18
10-24-18
10-26-18
10-25-18
10-28-18

Student: ____________	Instructor: Alfredo Alvarez	Assignment:
Date: ____________	Course: 2413 Cal I	calmath2413alvarez139nextfinaal

1. The function s(t) represents the position of an object at time t moving along a line. Suppose s(2) = 135 and s(4) = 183. Find the average velocity of the object over the interval of time [2,4].

The average velocity over the interval [2,4] is v_{av} = ______. (Simplify your answer.)

Answer: 24

$$\frac{s(4)-s(2)}{4-2} = \frac{183-135}{2} = \frac{48}{2} = 24$$

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2. The position of an object moving along a line is given by the function $s(t) = -15t^2 + 120t$. Find the average velocity of the object over the following intervals.

(a) [1, 10]
(b) [1, 9]
(c) [1, 8]
(d) [1, 1 + h] where h > 0 is any real number.

(a) The average velocity of the object over the interval [1, 10] is −45.

(b) The average velocity of the object over the interval [1, 9] is −30.

(c) The average velocity of the object over the interval [1, 8] is −15.

(d) The average velocity of the object over the interval [1, 1 + h] is −15h + 90.

Answers −45

−30

−15

−15h + 90

$s(t) = -15t^2 + 120t$

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2 a [1, 10] $\frac{s(10)-s(1)}{(10)-(1)} = -45$

2 b [1, 9] $\frac{s(9)-s(1)}{(9)-(1)} = -30$

2 c [1, 8] $\frac{s(8)-s(1)}{(8)-(1)} = -15$

2d

$$\frac{s'(1+h) - s'(1)}{(1+h)-(1)} =$$

$$\frac{\left(-15(1+h)^2+120(1+h)\right) - \left(-15(1)^2+120(1)\right)}{(1+h)-(1)} =$$

$$\frac{\left(-15(1+h)(1+h)+120(1+h)\right) - \left(-15(1)(1)+120(1)\right)}{h} =$$

$$\frac{\left(-15(1+1h+1h+h^2)+120+120h\right) - \left(-15+120\right)}{h} =$$

$$\frac{\left(-15(1+2h+h^2)+120+120h\right) - (105)}{h} =$$

$$\frac{\cancel{-15} - 30h - 15h^2 \cancel{+120} + 120h \cancel{-105}}{h} =$$

$$\frac{-15h^2+90h}{h} =$$

$$\frac{\cancel{h}(-15h+90)}{\cancel{h}} =$$

$$-15h+90 =$$

3. For the position function $s(t) = -16t^2 + 111t$, complete the following table with the appropriate average velocities. Then make a conjecture about the value of the instantaneous velocity at $t = 1$.

Time Interval	[1, 2]	[1, 1.5]	[1, 1.1]	[1, 1.01]	[1, 1.001]
Average Velocity	___	___	___	___	___

Complete the following table.

Time Interval	[1, 2]	[1, 1.5]	[1, 1.1]	[1, 1.01]	[1, 1.001]
Average Velocity	63	71	77.4	78.84	78.984

(Type exact answers. Type integers or decimals.)

The value of the instantaneous velocity at t = 1 is 79.
(Round to the nearest integer as needed.)

Answers 63

71

77.4

78.84

78.984

79

$s(t) = -16t^2 + 111t$

$[1, 2]$ $\quad \dfrac{s(2) - s(1)}{(2) - (1)} = 63$

$[1, 1.5]$ $\quad \dfrac{s(1.5) - s(1)}{(1.5) - (1)} = 71$

$[1, 1.1]$ $\quad \dfrac{s(1.1) - s(1)}{(1.1) - (1)} = 77.4$

$[1, 1.01]$ $\quad \dfrac{s(1.01) - s(1)}{(1.01) - (1)} = 78.84$

$[1, 1.001]$ $\quad \dfrac{s(1.001) - s(1)}{(1.001) - (1)} = 78.984$

4. For the function $f(x) = 20x^3 - x$, make a table of slopes of secant lines and make a conjecture about the slope of the tangent line at $x = 1$.

Complete the table.

(Round the final answer to three decimal places as needed. Round all intermediate values to four decimal places as needed.)

Interval	Slope of secant line
[1, 2]	139.000
[1, 1.5]	94.000
[1, 1.1]	65.200
[1, 1.01]	59.600
[1, 1.001]	59.100

An accurate conjecture for the slope of the tangent line at $x = 1$ is 59.
(Round to the nearest integer as needed.)

Answers 139.000

94.000

65.200

59.600

59.100

59

$f(x) = 20x^3 - x$

$[1, 2]$ $\quad \dfrac{f(2) - f(1)}{(2) - (1)} = 139.000$

$[1, 1.5]$ $\quad \dfrac{f(1.5) - f(1)}{(1.5) - (1)} = 94.000$

$[1, 1.1]$ $\quad \dfrac{f(1.1) - f(1)}{(1.1) - (1)} = 65.200$

$[1, 1.01]$ $\quad \dfrac{f(1.01) - f(1)}{(1.01) - (1)} = 59.600$

$[1, 1.001]$ $\quad \dfrac{f(1.001) - f(1)}{(1.001) - (1)} = 59.100$

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$f(x)=20x^3-x$ find the slope of the tangent line at $x=1$

another method

$f'(x)=20(3x^2)-(1)$

$f'(x)=60x^2-1$

$f'(1)=60(1)^2-1$

$f'(1)=60(1)(1)-1$

$f'(1)=60-1$

$f'(1)=59$

Formula

$y=x^n$

$y'=nx^{n-1}$

$y=ax$

$y'=a$

$y=a$

$y'=0$

Slope of the tangent line at $X=1$

5. Let $f(x) = \frac{x^2-16}{x+4}$. **(a)** Calculate f(x) for each value of x in the following table. **(b)** Make a conjecture about the value of $\lim_{x \to -4} \frac{x^2-16}{x+4}$.

(a) Calculate f(x) for each value of x in the following table.

x	−3.9	−3.99	−3.999	−3.9999
$f(x) = \frac{x^2-16}{x+4}$	−7.9	−7.99	−7.999	−7.9999
x	−4.1	−4.01	−4.001	−4.0001
$f(x) = \frac{x^2-16}{x+4}$	−8.1	−8.01	−8.001	−8.0001

(Type an integer or decimal rounded to four decimal places as needed.)

(b) Make a conjecture about the value of $\lim_{x \to -4} \frac{x^2-16}{x+4}$.

a

$\lim_{x \to -4} \frac{x^2-16}{x+4} =$ −8 (Type an integer or a decimal.)

$f(x) = \frac{x^2-16}{x+4}$

Answers −7.9

−7.99

−7.999

−7.9999

−8.1

−8.01

−8.001

−8.0001

−8

$f(-3.9) = \frac{(-3.9)^2-16}{(-3.9)+4} = -7.9$

$f(-3.99) = \frac{(-3.99)^2-16}{(-3.99)+4} = -7.99$

$f(-3.999) = \frac{(-3.999)^2-16}{(-3.999)+4} = -7.999$

$f(-3.9999) = \frac{(-3.9999)^2-16}{(-3.9999)+4} = -7.9999$

$f(-4.1) = \frac{(-4.1)^2-16}{(-4.1)+4} = -8.1$

$f(-4.01) = \frac{(-4.01)^2-16}{(-4.01)+4} = -8.01$

$f(-4.001) = \frac{(-4.001)^2-16}{(-4.001)+4} = -8.001$

$f(-4.0001) = \frac{(-4.0001)^2-16}{(-4.1001)+4} = -8.0001$

5 b

$$\lim_{x \to -4} \frac{x^2-16}{x+4} =$$

Formula

$a^2-b^2=$

$(a+b)(a-b)$

$$\lim_{x \to -4} \frac{(x)^2-(4)^2}{x+4} =$$

$$\lim_{x \to -4} \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}} =$$

$$\lim_{x \to -4} (x-4) =$$

$$-4-4 =$$

$$-8 =$$

6. Let $g(t) = \frac{t-4}{\sqrt{t}-2}$.

a. Make two tables, one showing the values of g for t = 3.9, 3.99, and 3.999 and one showing values of g for t = 4.1, 4.01, and 4.001.

b. Make a conjecture about the value of $\lim_{t \to 4} \frac{t-4}{\sqrt{t}-2}$.

a. Make a table showing the values of g for t = 3.9, 3.99, and 3.999.

t	3.9	3.99	3.999
g(t)	3.9748	3.9975	3.9997

(Round to four decimal places.)

Make a table showing the values of g for t = 4.1, 4.01, and 4.001.

t	4.1	4.01	4.001
g(t)	4.0248	4.0025	4.0003

(Round to four decimal places.)

b. Make a conjecture about the value of $\lim_{t \to 4} \frac{t-4}{\sqrt{t}-2}$. Select the correct choice below and fill in any answer boxes in your choice.

a

$$g(t) = \frac{t-4}{\sqrt{t}-2}$$

○ **A.** $\lim_{t \to 4} \frac{t-4}{\sqrt{t}-2} =$ 4 (Simplify your answer.)

○ **B.** The limit does not exist.

$$g(3.9) = \frac{(3.9)-4}{\sqrt{3.9}-2} = 3.9748$$

$$g(3.99) = \frac{(3.99)-4}{\sqrt{3.99}-2} = 3.9975$$

$$g(3.999) = \frac{(3.999)-4}{\sqrt{3.999}-2} = 3.9997$$

Answers 3.9748

3.9975

3.9997

4.0248

4.0025

4.0003

A. $\lim_{t \to 4} \frac{t-4}{\sqrt{t}-2} =$ 4 (Simplify your answer.)

$$g(4.1) = \frac{(4.1)-4}{\sqrt{4.1}-2} = 4.0248$$

$$g(4.01) = \frac{(4.01)-4}{\sqrt{4.01}-2} = 4.0025$$

$$g(4.001) = \frac{(4.001)-4}{\sqrt{4.001}-2} = 4.0003$$

6 b

$$\lim_{t \to 4} \frac{t-4}{\sqrt{t}-2} =$$

$$\lim_{t \to 4} \left(\frac{t-4}{\sqrt{t}-2}\right)\left(\frac{\sqrt{t}+2}{\sqrt{t}+2}\right) =$$

$$\lim_{t \to 4} \frac{(t-4)(\sqrt{t}+2)}{(\sqrt{t})^2 + \cancel{2\sqrt{t}} - \cancel{2\sqrt{t}} - 4} =$$

$$\lim_{t \to 4} \frac{(t-4)(\sqrt{t}+2)}{\sqrt{t^2} - 4} =$$

$$\lim_{t \to 4} \frac{\cancel{(t-4)}(\sqrt{t}+2)}{\cancel{(t-4)}} =$$

$$\lim_{t \to 4} (\sqrt{t}+2) =$$

$$\sqrt{4} + 2 =$$

$$2 + 2 =$$

$$4 =$$

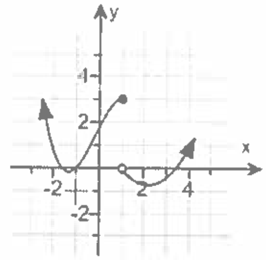
7. Use the graph to find the following limits and function value.

a. $\lim_{x \to 1^-} f(x)$

b. $\lim_{x \to 1^+} f(x)$

c. $\lim_{x \to 1} f(x)$

d. f(1)



a. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 1^-} f(x) =$ 3 (Type an integer.)

B. The limit does not exist.

b. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 1^+} f(x) =$ 0 (Type an integer.)

B. The limit does not exist.

c. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 1} f(x) =$ ______ (Type an integer.)

B. The limit does not exist.

d. Find the function value. Select the correct choice below and fill in any answer boxes in your choice.

A. f(1) = 3 (Type an integer.)

B. The answer is undefined.

Answers A. $\lim_{x \to 1^-} f(x) =$ 3 (Type an integer.)

A. $\lim_{x \to 1^+} f(x) =$ 0 (Type an integer.)

B. The limit does not exist.

A. f(1) = 3 (Type an integer.)

8.

Explain why $\lim_{x \to -5} \frac{x^2+8x+15}{x+5} = \lim_{x \to -5} (x+3)$, and then evaluate $\lim_{x \to -5} \frac{x^2+8x+15}{x+5}$.

Choose the correct answer below.

- A. Since each limit approaches -5, it follows that the limits are equal.
- B. Since $\frac{x^2+8x+15}{x+5} = x+3$ whenever $x \neq -5$, it follows that the two expressions evaluate to the same number as x approaches -5.
- C. The numerator of the expression $\frac{x^2+8x+15}{x+5}$ simplifies to $x+3$ for all x, so the limits are equal.
- D. The limits $\lim_{x \to -5} \frac{x^2+8x+15}{x+5}$ and $\lim_{x \to -5} (x+3)$ equal the same number when evaluated using direct substitution.

Now evaluate the limit.

$\lim_{x \to -5} \frac{x^2+8x+15}{x+5} =$ -2 (Simplify your answer.)

Answers B.

Since $\frac{x^2+8x+15}{x+5} = x+3$ whenever $x \neq -5$, it follows that the two expressions evaluate to the same number as x approaches -5.

-2

$$\lim_{x \to -5} \frac{x^2+8x+15}{x+5} =$$

$$\lim_{x \to -5} \frac{(x+3)\cancel{(x+5)}}{\cancel{(x+5)}} =$$

$$\lim_{x \to -5} (x+3) =$$

$$-5+3 =$$

$$-2 =$$

9

$$\lim_{x \to 9} f(x) = 21$$

$$\lim_{x \to 9} h(x) = 3$$

Given

find

$$\lim_{x \to 9} \frac{f(x)}{h(x)} =$$

$$\frac{\lim_{x \to 9} f(x)}{\lim_{x \to 9} h(x)} =$$

$$\frac{21}{3} =$$

$$7 =$$

(10)

$$\lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5} =$$

$$\lim_{x \to 5} \frac{(x-1)(x-5)}{(x-5)} =$$

$$\lim_{x \to 5} \frac{(x-1)\cancel{(x-5)}}{\cancel{(x-5)}} =$$

$$\lim_{x \to 5} (x-1) =$$

$$5 - 1 =$$

$$4 =$$

(11.) $$\lim_{x \to 100} \frac{\sqrt{x}-10}{x-100} =$$

$$\lim_{x \to 100} \left(\frac{\sqrt{x}-10}{x-100}\right)\left(\frac{\sqrt{x}+10}{\sqrt{x}+10}\right) = \text{mult}$$

$$\lim_{x \to 100} \left(\frac{(\sqrt{x})^2 + \cancel{10\sqrt{x}} - \cancel{10\sqrt{x}} - 100}{(x-100)(\sqrt{x}+10)}\right) =$$

$$\lim_{x \to 100} \frac{(\sqrt{x})^2 - 100}{(x-100)(\sqrt{x}+10)} =$$

$$\lim_{x \to 100} \frac{(x-100)}{(x-100)(\sqrt{x}+10)} =$$

$$\lim_{x \to 100} \frac{1\cancel{(x-100)}}{\cancel{(x-100)}(\sqrt{x}+10)} =$$

$$\lim_{x \to 100} \frac{1}{\sqrt{x}+10} =$$

$$\frac{1}{\sqrt{100}+10} =$$

$$\frac{1}{10+10} = \frac{1}{20}$$

(12) If $f(x) \to 200{,}000$ and $g(x) \to \infty$

as $x \to \infty$

find $\lim_{x \to \infty} \frac{f(x)}{g(x)} =$

$\lim_{x \to \infty} f(x) = 200{,}000$

$\lim_{x \to \infty} g(x) = \infty$

$\lim_{x \to \infty} \frac{f(x)}{g(x)} =$

$\frac{\lim_{x \to \infty} f(x)}{\lim_{x \to \infty} g(x)} =$

$\frac{200{,}000}{\infty} =$

$0 =$

(13) $\lim_{x \to \infty} \frac{5+3x+4x^2}{x^2}$

$\lim_{x \to \infty} \left(\frac{5+3x+4x^2}{x^2}\right)\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right) =$ Mult

$\lim_{x \to \infty} \left(\frac{\frac{5}{x^2} + \frac{3x}{x^2} + \frac{4\cancel{x^2}}{\cancel{x^2}}}{\frac{x^2}{x^2}}\right) =$

$\lim_{x \to \infty} \left(\frac{\frac{5}{x^2} + \frac{3}{x} + 4}{1}\right) =$

formula

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to \infty} \frac{1}{x^n} = 0$

$\frac{0+0+4}{1} =$

$\frac{4}{1} =$

$4 =$

(14) $\lim_{w\to\infty} \frac{8w^2+3w+9}{\sqrt{4w^4+w^3}}$

formula
$\lim_{x\to\infty} \frac{1}{x}=0$

$\lim_{x\to\infty} \frac{1}{x^n}=0$

$\lim_{w\to\infty} \left(\frac{8w^2+3w+9}{\sqrt{4w^4+w^3}}\right)\left(\frac{\sqrt{\frac{1}{w^4}}}{\sqrt{\frac{1}{w^4}}}\right) =$ Mult

$\lim_{w\to\infty} \frac{(8w^2+3w+9)\frac{1}{w^2}}{\sqrt{4w^4(\frac{1}{w^4})+w^3(\frac{1}{w^4})}} =$

$\lim_{w\to\infty} \frac{\frac{8\cancel{w^2}}{\cancel{w^2}}+\frac{3w}{w^2}+\frac{9}{w^2}}{\sqrt{\frac{4\cancel{w^4}}{\cancel{w^4}}+\frac{w^3}{w^4}}} =$

$\lim_{w\to\infty} \frac{8+\frac{3}{w}+\frac{9}{w^2}}{\sqrt{4+\frac{1}{w}}} =$

$\frac{8+0+0}{\sqrt{4+0}} =$

$\frac{8}{\sqrt{4}} =$

$\frac{8}{2} =$

$4 =$

(15) $\lim_{x \to -\infty} \frac{\sqrt{121x^2+x}}{x}$

$$\lim_{x \to -\infty} \left(\frac{\sqrt{121x^2+x}}{x}\right)\left(\frac{\sqrt{\frac{1}{x^2}}}{\sqrt{\frac{1}{x^2}}}\right) =$$

Mult

Formula

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to \infty} \frac{1}{x^n} = 0$

$$\lim_{x \to -\infty} \left(\frac{\sqrt{121x^2\left(\frac{1}{x^2}\right) + x\left(\frac{1}{x^2}\right)}}{x \cdot \left(\frac{1}{x}\right)}\right) =$$

$$\lim_{x \to -\infty} \frac{\sqrt{\frac{121\cancel{x^2}}{\cancel{x^2}} + \frac{x}{x^2}}}{\frac{x}{x}} =$$

$$\lim_{x \to -\infty} \frac{\sqrt{121 + \frac{1}{x}}}{1} =$$

$$-\frac{\sqrt{121+0}}{1} =$$

$$-\sqrt{121} =$$

$$-11 =$$

(16)

$$\lim_{x\to\infty} \frac{3x}{21x+3} =$$

mult

$$\lim_{x\to\infty} \left(\frac{3x}{21x+3}\right)\left(\frac{\frac{1}{x}}{\frac{1}{x}}\right) =$$

formula

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} \left(\frac{\frac{3x}{x}}{\frac{21x}{x} + \frac{3}{x}}\right) =$$

$$\lim_{x\to\infty} \left(\frac{3}{21 + \frac{3}{x}}\right) =$$

$$\frac{3}{21+0} =$$

$$\frac{3}{21} =$$

$$\frac{\cancel{3}(1)}{\cancel{3}(7)} =$$

$$\frac{1}{7}$$

17 $\lim_{x\to\infty} \dfrac{16x^2-9x+7}{4x^2+1} =$

formula
$\lim_{x\to\infty} \dfrac{1}{x} = 0$
$\lim_{x\to\infty} \dfrac{1}{x^n} = 0$

$\lim_{x\to\infty} \left(\dfrac{16x^2-9x+7}{4x^2+1}\right) \dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} =$ Mult

$\lim_{x\to\infty} \dfrac{\frac{16\cancel{x^2}}{\cancel{x^2}} - \frac{9x}{x^2} + \frac{7}{x^2}}{\frac{4\cancel{x^2}}{\cancel{x^2}} + \frac{1}{x^2}} =$

$\lim_{x\to\infty} \dfrac{16 - \frac{9}{x} + \frac{7}{x^2}}{4 + \frac{1}{x^2}} =$

$\dfrac{16-0+0}{4+0} =$

$\dfrac{16}{4} =$

$4 =$

(18) $\lim_{x\to\infty} \frac{24x^8-2}{3x^8-4x^7}$

Formula
$\lim_{x\to\infty} \frac{1}{x} = 0$
$\lim_{x\to\infty} \frac{1}{x^n} = 0$

$\lim_{x\to\infty} \left(\frac{24x^8-2}{3x^8-4x^7}\right)\left(\frac{\frac{1}{x^8}}{\frac{1}{x^8}}\right) =$ mult

$\lim_{x\to\infty} \left(\frac{\frac{24\cancel{x^8}}{\cancel{x^8}} - \frac{2}{x^8}}{\frac{3\cancel{x^8}}{\cancel{x^8}} - \frac{4x^7}{x^8}}\right) =$

$\lim_{x\to\infty} \left(\frac{24 - \frac{2}{x^8}}{3 - \frac{4}{x}}\right) =$

$\frac{24-0}{3-0} =$

$\frac{24}{3} =$

$8 =$

(19) $\lim_{x\to\infty} \dfrac{5x^4-8}{x^5+2x^3} =$

formula

$\lim_{x\to\infty} \dfrac{1}{x} = 0$

$\lim_{x\to\infty} \dfrac{1}{x^n} = 0$

$\lim_{x\to\infty} \left(\dfrac{5x^4-8}{x^5+2x^3}\right)\left(\dfrac{\frac{1}{x^5}}{\frac{1}{x^5}}\right)$ mult

$\lim_{x\to\infty} \dfrac{\frac{5x^4}{x^5} - \frac{8}{x^5}}{\frac{\cancel{x^5}}{\cancel{x^5}} + \frac{2x^3}{x^5}} =$

$\lim_{x\to\infty} \left(\dfrac{\frac{5}{x} - \frac{8}{x^5}}{1 + \frac{2}{x^2}}\right) =$

$\dfrac{0-0}{1+0} =$

$\dfrac{0}{1} =$

$0 =$

(20) Find all asymptotes

$$\lim_{x\to\infty} \frac{2x^2+4}{2x^2-x-36} =$$

$$\lim_{x\to\infty} \left(\frac{2x^2+4}{2x^2-x-36}\right)\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right) = \text{Mult}$$

$$\lim_{x\to\infty} \left(\frac{\frac{2x^2}{x^2}+\frac{4}{x^2}}{\frac{2x^2}{x^2}-\frac{x}{x^2}-\frac{36}{x^2}}\right) =$$

formulas

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} \frac{2+\frac{4}{x^2}}{2-\frac{1}{x}-\frac{36}{x^2}} =$$

$$\frac{2+0}{2-0-0} =$$

$$\frac{2}{2} =$$

$$1 =$$

One horizontal asymptote

$y = 1$

Vertical asymptote

Let $2x^2-x-36=0$

$(2x-9)(x+4)=0$

$2x-9=0$ OR $x+4=0$

$2x-9+9=0+9$ OR $x+4-4=0-4$

$2x=9$ OR $x=-4$

$\frac{2x}{2}=\frac{9}{2}$

$x=9/2$

Two vertical asymptotes

$x=-4$ or $x=\frac{9}{2}$

$\boxed{\text{NO slant asymptotes}}$

since power are equal $\frac{x^2}{x^2}$

21. Determine whether the following function is continuous at a. Use the continuity checklist to justify your answer.

$$f(x) = \frac{2x^2 + 7x + 3}{x^2 - 7x}, \; a = 7$$

Select all that apply.

- [ ] **A.** The function is continuous at $a = 7$.
- [x] **B.** The function is not continuous at $a = 7$ because $f(7)$ is undefined.
- [x] **C.** The function is not continuous at $a = 7$ because $\lim_{x \to 7} f(x)$ does not exist.
- [x] **D.** The function is not continuous at $a = 7$ because $\lim_{x \to 7} f(x) \neq f(7)$.

Answer: B. The function is not continuous at $a = 7$ because $f(7)$ is undefined. , C. The function is not continuous at $a = 7$ because $\lim_{x \to 7} f(x)$ does not exist. , D. The function is not continuous at $a = 7$ because $\lim_{x \to 7} f(x) \neq f(7)$.

22. Determine whether the following function is continuous at a. Use the continuity checklist to justify your answer.

$$f(x) = \begin{cases} \dfrac{x^2 - 169}{x - 13} & \text{if } x \neq 13 \\ 10 & \text{if } x = 13 \end{cases} \; ; a = 13$$

Select all that apply.

- [ ] **A.** The function is continuous at $a = 13$.
- [ ] **B.** The function is not continuous at $a = 13$ because $f(13)$ is undefined.
- [ ] **C.** The function is not continuous at $a = 13$ because $\lim_{x \to 13} f(x)$ does not exist.
- [x] **D.** The function is not continuous at $a = 13$ because $\lim_{x \to 13} f(x) \neq f(13)$.

Answer: D. The function is not continuous at $a = 13$ because $\lim_{x \to 13} f(x) \neq f(13)$.

23. Determine the intervals on which the following function is continuous.

$$f(x) = \frac{x^2 - 4x + 3}{x^2 - 9}$$

Pc.+1

On what interval(s) is f continuous?

$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

(Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)

Answer: $(-\infty, -3), (-3, 3), (3, \infty)$

23 Part 2 Determine the intervals on which the function is continuous

$$f(x) = \frac{x^2 - 4x + 3}{x^2 - 9}$$

formula
$a^2 - b^2$
$(a+b)(a-b)$

Let $x^2 - 9 = 0$

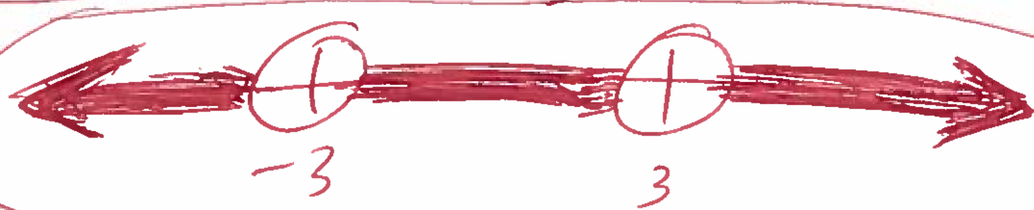
$(x)^2 - (3)^2 = 0$

$(x+3)(x-3) = 0$

$x+3=0$ or $x-3=0$

$x+3-3=0-3$ OR $x-3+3=0+3$

$x \neq -3$ or $x \neq 3$



$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

Continuous

24

$$\lim_{x \to 5} \sqrt{x^2+24} =$$

$$\sqrt{(5)^2+24} =$$

$$\sqrt{25+24} =$$

$$\sqrt{49} =$$

$$7 =$$

because x^2+24 is continuous for all x and the square root function is continuous for all $x \geq 0$

(25) Suppose x lives in the interval $(4,6)$ with $x \neq 5$. Find the smallest positive value of δ such that the inequality $0 < |x-5| < \delta$ is true for all possible values of x.

$$0 < |x-5| < \delta$$

$$|x-5| < \delta$$

$$-\delta < x-5 < \delta \qquad (4, 6)$$

$$-\delta + 5 < x - \cancel{5} + \cancel{5} < \delta + 5$$

$$-\delta + 5 < x < \delta + 5$$

Set $-\delta + 5 = \underline{4}$ OR $\delta + 5 = \underline{6}$

$-\delta + \cancel{5} - \cancel{5} = 4 - 5$ OR $\delta + \cancel{5} - \cancel{5} = 6 - 5$

$-\delta = -1$ OR $\delta = 1$

$-1(-\delta) = -1(-1)$ OR

$\delta = 1$

$\delta = 1$

(28) Use the precise definition of a limit to prove the following limit. Specify a relationship between ϵ and δ that guarantees the limit exists.

$$\lim_{x \to 0} (8x-5) = -5$$

$$|(8x-5)-(-5)| < \epsilon$$

$$|8x-5+5| < \epsilon$$

$$|8x| < \epsilon$$

$$|8| \cdot |x| < \epsilon$$

$$8|x| < \epsilon$$

$$\frac{\cancel{8}|x|}{\cancel{8}} < \frac{\epsilon}{8}$$

$$|x| < \frac{\epsilon}{8}$$

$$|x-0| < \frac{\epsilon}{8}$$

Choose $\delta = \frac{\epsilon}{8}$ then $|(8x-5)-(-5)| < \epsilon$ whenever $0 < |x-0| < \delta = \frac{\epsilon}{8}$

(29) Find the derivative of the function at the point

$f(x) = 3x^2 - 5x$ $\quad (-1, 8)$ point

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x \quad y$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \to 0} \left(\frac{3(x+h)^2 - 5(x+h)) - (3x^2 - 5x)}{h} \right) =$$

$$\lim_{h \to 0} \left(\frac{(3(x+h)(x+h) - 5x - 5h) - (3x^2 - 5x)}{h} \right) =$$

$$\lim_{h \to 0} \left(\frac{3(x^2 + xh + xh + h^2) - 5x - 5h) - (3x^2 - 5x)}{h} \right) =$$

$$\lim_{h \to 0} \frac{(3(x^2 + 2xh + h^2) - 5x - 5h) - (3x^2 - 5x)}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{- 5x} - 5h \cancel{- 3x^2} \cancel{+ 5x}}{h} =$$

$$\lim_{h \to 0} \frac{6xh + 3h^2 - 5h}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{h}(6x + 3h - 5)}{\cancel{h}} =$$

$$\lim_{h \to 0} (6x + 3h - 5) =$$

$$6x + 3(0) - 5$$

$$6x - 5$$

$$f'(x) = 6x - 5$$

Subst

$$f'(-1) = 6(-1) - 5$$

$$f'(-1) = -6 - 5$$

$$f'(-1) = -11$$

(30) use definition $m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$f(x) = x^2 - 3$ $(-3, 6)$ point

(a)

$$\lim_{h \to 0} \frac{((a+h)^2 - 3) - ((a)^2 - 3)}{h} =$$

$$\lim_{h \to 0} \frac{(a+h)(a+h) - 3 - a^2 + 3}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{a^2} + ah + ah + h^2 - \cancel{3} - \cancel{a^2} + \cancel{3}}{h} =$$

$$\lim_{h \to 0} \frac{2ah + h^2}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{h}(2a + h)}{\cancel{h}} =$$

$$\lim_{h \to 0} 2a + h =$$

$$2a + (0)$$

$$2a$$

(b)

$f'(a) = 2a$

$(-3, 6)$
$x_1\ y_1$

determine an equation of the tangent line at P

point $(-3, 6)$

$f'(-3) = 2(-3)$

$f'(-3) = -6 = \text{slope} = m$

$y - y_1 = m(x - x_1)$

$y - (6) = -6(x - (-3))$

$y - 6 = -6(x + 3)$

$y - 6 = -6x - 18$

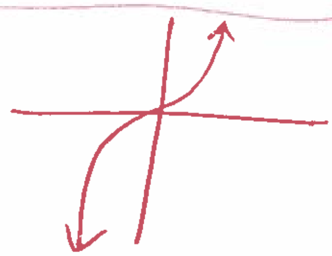
$y - \cancel{6} + \cancel{6} = -6x - 18 + 6$

$y = -6x - 12$

31 Match the graph with its derivative

Example

$y = x^3$



$y' = 3x^2$

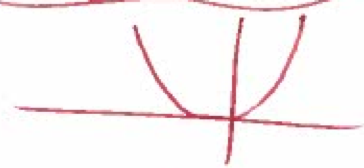
Example

$y = x^4$

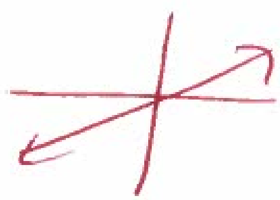
$y' = 4x^3$

Example

$y = x^2$

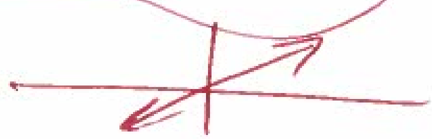


$y' = 2x$



Example

$y = x$



$y' = 1$

(32) a line perpendicular to another line or to a tangent line is called a normal line.

Find an equation of the line perpendicular to the line that is tangent to the following curve at the given point.

$y = 11x + 23$ $\quad P(-2, 1)$ Point
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x_1 \; y_1$

$m = 11$

$m_{per} = \frac{-1}{11}$ = Slope = m_{per}

$y - y_1 = m(x - x_1)$

$y - (1) = -\frac{1}{11}(x - (-2))$

$y - 1 = -\frac{1}{11}(x + 2)$

$y - 1 = -\frac{1}{11}x - \frac{2}{11}$

$y - \cancel{1} + \cancel{1} = -\frac{1}{11}x - \frac{2}{11} + 1$

$y = -\frac{1}{11}x - \frac{2}{11} + \frac{11}{11}$

$y = -\frac{1}{11}x + \frac{-2+11}{11}$

$y = \frac{-1}{11}x + \frac{9}{11}$

(33) Find derivative (use limit definition)

$f(x) = x^2 - 8x + 5$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \to 0} \frac{((x+h)^2 - 8(x+h) + 5) - (x^2 - 8x + 5)}{h} =$$

$$\lim_{h \to 0} \frac{((x+h)(x+h) - 8(x+h) + 5) - (x^2 - 8x + 5)}{h} =$$

$$\lim_{h \to 0} \frac{x^2 + xh + xh + h^2 - 8x - 8h + 5 - x^2 + 8x + 5}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{- 8x} - 8h \cancel{+ 5} \cancel{- x^2} \cancel{+ 8x} \cancel{+ 5}}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2 - 8h}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + h - 8)}{\cancel{h}} =$$

$$\lim_{h \to 0} (2x + h - 8) =$$

$2x + (0) - 8 =$

$2x - 8$

$f'(x) = 2x - 8$

34

$$y = \frac{x-6}{4x-3}$$

$$y' = \frac{(x-6)'(4x-3)-(x-6)(4x-3)'}{(4x-3)^2}$$

$$y' = \frac{(1-0)(4x-3)-(x-6)(4-0)}{(4x-3)^2}$$

$$y' = \frac{(1)(4x-3)-(x-6)(4)}{(4x-3)^2}$$

$$y' = \frac{\cancel{4x}-3-\cancel{4x}+24}{(4x-3)^2}$$

$$y' = \frac{-3+24}{(4x-3)^2}$$

$$y' = \frac{21}{(4x-3)^2}$$

[illegible]

$$y = \frac{f}{g}$$

$$y' = \frac{f'g-fg'}{(g)^2}$$

$$y = x^n$$

$$y' = nx^{n-1}$$

$$y = a$$

$$y' = 0$$

(35) $g(x) = \frac{2x^2}{4x+5}$ find $g'(1) =$

$y = \frac{f}{g}$

$y' = \frac{f'g - fg'}{(g)^2}$

$g'(x) = \frac{(2x^2)'(4x+5) - (2x^2)(4x+5)'}{(4x+5)^2}$

$g'(x) = \frac{(4x)(4x+5) - (2x^2)(4+0)}{(4x+5)^2}$

$y = x^n$

$y' = nx^{n-1}$

$g'(x) = \frac{(4x)(4x+5) - (2x^2)(4)}{(4x+5)^2}$

$y = a$

$y' = 0$

$g'(x) = \frac{16x^2 + 20x - 8x^2}{(4x+5)^2}$

$g'(x) = \frac{8x^2 + 20x}{(4x+5)^2}$

$g'(1) = \frac{8(1)^2 + 20(1)}{(4(1)+5)^2}$

$g'(1) = \frac{28}{81}$

$g'(1) = \frac{8(1)(1) + 20(1)}{(4+5)^2}$

$g'(1) = \frac{8+20}{(9)^2}$

(36) $f(x) = (x-3)(2x+4)$

$f'(x) = (x-3)'(2x+4) + (x-3)(2x+4)'$

$f'(x) = (1-0)(2x+4) + (x-3)(2+0)$

$f'(x) = (1)(2x+4) + (x-3)(2)$

formula

$y = f \cdot g$

$y' = f'g + fg'$

$f'(x) = 2x+4+2x-6$

$f'(x) = 4x-2$

OR

$f(x) = (x-3)(2x+4)$

Expand

$f(x) = 2x^2+4x-6x-12$

$f(x) = 2x^2-2x-12$

formula

$y = x^n$

$y' = nx^{n-1}$

$f'(x) = 4x-2-0$

$y = ax$

$y' = a$

$f'(x) = 4x-2$

(37) $h(w) = \dfrac{5w^6 - w}{w}$

$h'(w) = \dfrac{(5w^6 - w)'(w) - (5w^6 - w)(w)'}{(w)^2}$

$h'(w) = \dfrac{(30w^5 - 1)(w) - (5w^6 - w)(1)}{(w)^2}$

$h'(w) = \dfrac{30w^6 - \cancel{w} - 5w^6 + \cancel{w}}{(w)^2}$

$h'(w) = \dfrac{25w^6}{w^2}$

$h'(w) = 25w^4$ reduce (simp)

Formula

$y = \dfrac{f}{s}$

$y' = \dfrac{f's - fs'}{(s)^2}$

OR

$h(w) = \dfrac{5w^6 - w}{w}$

$h(w) = \dfrac{5w^6}{w^1} - \dfrac{w^1}{w^1}$

$h(w) = 5w^{6-1} - 1$

$h(w) = 5w^5 - 1$

$h'(w) = 25w^4$

$y = x^n$

$y' = nx^{n-1}$

$y = a$

$y' = 0$

(38) $\lim_{x\to 0} \frac{\sin(5x)}{\sin(2x)}$

$\lim_{x\to 0} \left(\frac{\sin(5x)}{\sin(2x)}\right) \frac{\frac{1}{x}}{\frac{1}{x}} =$ mult

formula

$\lim_{x\to 0} \frac{\sin(x)}{(x)} = 1$

$\lim_{x\to 0} \frac{\sin(mx)}{(mx)} = 1$

$\lim_{x\to 0} \frac{\frac{\sin(5x)}{x}}{\frac{\sin(2x)}{x}} =$

$\lim_{x\to 0} \frac{\frac{\sin(5x)}{x}\left(\frac{5}{5}\right)}{\frac{\sin(2x)}{x}\cdot\left(\frac{2}{2}\right)} =$ Mult again

$\lim_{x\to 0} \frac{\frac{\sin(5x)}{5x}\cdot(5)}{\frac{\sin(2x)}{2x}\cdot(2)} =$

$\frac{5}{2} \lim_{x\to 0} \frac{\frac{\sin(5x)}{(5x)}}{\frac{\sin(2x)}{(2x)}} =$

$\frac{5}{2} \frac{\lim_{x\to 0} \frac{\sin(5x)}{(5x)}}{\lim_{x\to 0} \frac{\sin(2x)}{(2x)}} =$

$\frac{5}{2} \cdot \left(\frac{1}{1}\right) =$

$\frac{5}{2}$

(39) $\lim_{\theta \to 0} \frac{\cos^2(\theta) - 1}{\theta} =$

Formula

$\lim_{x \to 0} \frac{\sin(x)}{(x)} = 1$

$\lim_{x \to 0} \frac{\cos(x) - 1}{(x)} = 0$

$\lim_{\theta \to 0} \frac{\cos^2\theta - (1)^2}{\theta} =$

$\lim_{\theta \to 0} \frac{(\cos(\theta) + 1)(\cos(\theta) - 1)}{\theta} =$

$\lim_{\theta \to 0} \frac{\cos(\theta) + 1}{1} \cdot \lim_{\theta \to 0} \frac{\cos(\theta) - 1}{\theta} =$

$\frac{\cos(0) + 1}{1} \cdot 0 =$

$\frac{1 + 1}{1} \cdot 0 =$

$\frac{2}{1} \cdot 0 =$

$2 \cdot 0$

$0 =$

40

$y = 9\sin(x) + 8\cos(x)$

$y' = 9(\cos(x)(x)') + 8(-\sin(x)(x)')$

$y' = 9(\cos(x)\cdot(1)) + 8(-\sin(x)(1))$

$y' = 9\cos(x) - 8\sin(x)$

OR

$\frac{dy}{dx} = 9\cos(x) - 8\sin(x)$

formula

$y = \sin(f(x))$

$y' = \cos(f(x)) \cdot f'(x)$

$y = \cos(f(x))$

$y' = -\sin(f(x)) \cdot f'(x)$

(41) $y = e^{-x} \sin(x)$

$y' = (e^{-x})'(\sin(x)) + (e^{-x})(\sin(x))'$

$y' = (e^{-x}(-x)')(\sin(x)) + (e^{-x})(\cos(x) \cdot (x)')$

$y' = (e^{-x}(-1))(\sin(x)) + (e^{-x})(\cos(x) \cdot 1)$

$y' = -e^{-x} \sin(x) + e^{-x} \cos(x)$

$y' = e^{-x} \cos(x) - e^{-x} \sin(x)$ rewrite

$y' = e^{-x}(\cos(x) - \sin(x))$

or

$\frac{dy}{dx} = e^{-x}(\cos(x) - \sin(x))$

Formula

$y = f \cdot g$
$y' = f'g + fg'$

$y = e^{f(x)}$
$y' = e^{f(x)} \cdot f'(x)$

$y = \sin(f(x))$
$y' = \cos(f(x)) f'(x)$

(42) find the equation of the line tangent to the following curve at point

$$y = -18x^2 + 5\sin x$$

$P(0,0)$
$x_1 \quad y_1$

$$y' = -36x + 5\cos(x)(x)'$$

$$y' = -36x + 5\cos(x)(1)$$

$$y' = -36x + 5\cos(x)$$

$$y'(0) = -36(0) + 5\cos(0)$$

$$y'(0) = 0 + 5(1)$$

$$y'(0) = 5 = m = \text{slope}$$

$$y - y_1 = m(x - x_1)$$

$$y - (0) = 5(x - (0))$$

$$y - 0 = 5(x - 0)$$

$$y = 5(x)$$

$$y = 5x$$

formula

$$y = x^n$$
$$y' = nx^{n-1}$$

$$y = \sin(f(x))$$
$$y' = \cos(f(x)) \cdot f'(x)$$

43 Let $h(x) = f(g(x))$ and $p(x) = g(f(x))$

find $h'(2)$

$p'(4)$

x	1	2	3	4
$f(x)$	1	4	3	2
$f'(x)$	-7	-8	-9	-2
$g(x)$	3	2	1	4
$g'(x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{7}{9}$	$\frac{4}{9}$

$$\boxed{h'(x) = f'(g(x)) \cdot g'(x)}$$

$$\boxed{p'(x) = g'(f(x)) \cdot f'(x)}$$

composite function

$$h'(x) = f'(g(x)) \cdot g'(x)$$
$$h'(2) = f'(g(2)) \cdot g'(2)$$
$$h'(2) = f'(2) \cdot \left(\frac{2}{9}\right)$$
$$h'(2) = (-8) \cdot \left(\frac{2}{9}\right)$$
$$h'(2) = -\frac{16}{9}$$

$$p'(x) = g'(f(x)) \cdot f'(x)$$
$$p'(4) = g'(f(4)) \cdot f'(4)$$
$$p'(4) = g'(2) \cdot (-2)$$
$$p'(4) = \left(\frac{2}{9}\right) \cdot (-2)$$
$$p'(4) = \frac{-4}{9}$$

4c

$$y = (5x+6)^7$$

$$y' = 7(5x+6)^{7-1} \cdot (5x+6)'$$

$$y' = 7(5x+6)^6 (5+0)$$

$$y' = 7(5x+6)^6 (5)$$

$$y' = 35(5x+6)^6$$

formula

$$y = (f(x))^N$$

$$y' = N(f(x))^{N-1} \cdot f'(x)$$

45

$$y = 5(2x^4+9)^{-8}$$

$$y' = 5(-8)(2x^4+9)^{-8-1} \cdot (2x^4+9)'$$

$$y' = -40(2x^4+9)^{-9}(8x^3+0)$$

$$y' = -40(2x^4+9)^{-9}(8x^3)$$

$$y' = -320x^3(2x^4+9)^{-9}$$

$$y' = \frac{-320x^3}{(2x^4+9)^9}$$ Hw. L

OR

$$\frac{dy}{dx} = \frac{-320x^3}{(2x^4+9)^9}$$

Formula

$$y = (f(x))^n$$

$$y' = n(f(x))^{n-1} \cdot f(x)'$$

(46) $y = \sin(13t+14)$

$y' = \cos(13t+14) \cdot (13t+14)'$

$y' = \cos(13t+14) \cdot (13+0)$

$y' = \cos(13t+14) \cdot (13)$

$y' = 13\cos(13t+14)$

OR

$\frac{dy}{dt} = 13\cos(13t+14)$

formula

$y = \sin(f(x))$

$y' = \cos(f(x)) \cdot f'(x)$

47)

$y = \tan(e^x)$

$y' = \sec^2(e^x) \cdot (e^x)'$

$y' = \sec^2(e^x)\,(e^x \cdot (x)')$

$y' = \sec^2(e^x)\,(e^x(1))$

$y' = \sec^2(e^x)\,(e^x)$

$y' = e^x \sec^2(e^x)$

OR

$\frac{dy}{dx} = e^x \sec^2(e^x)$

Formula

$y = \tan(f(x))$

$y' = \sec^2(f(x)) \cdot f'(x)$

$y = e^{f(x)}$

$y' = e^{f(x)} \cdot f'(x)$

48

$$y = \cos(3\sin(x))$$

$$y' = -\sin(3\sin(x)) \cdot (3\sin(x))'$$

$$y' = -\sin(3\sin(x)) \cdot (3\cos(x)(x)')$$

$$y' = -\sin(3\sin(x)) \cdot (3\cos(x)(1))$$

$$y' = -\sin(3\sin(x)) \cdot (3\cos(x))$$

$$y' = -3\sin(3\sin(x)) \cdot \cos(x)$$

OR

$$\frac{dy}{dx} = -3\sin(3\sin(x)) \cdot \cos(x)$$

formula

$$y = \cos(f(x))$$

$$y' = -\sin(f(x)) \cdot f'(x)$$

$$y = (f(x))^N$$

$$y' = N(f(x))^{N-1} \cdot f'(x)$$

49 use implicit differentiation

$$75x = y^2$$

$$75 = 2yy'$$

$$\frac{75}{2y} = \frac{\cancel{2y}y'}{\cancel{2y}}$$

$$\frac{75}{2y} = y'$$

$$y' = \frac{75}{2y}$$

or

$$\frac{dy}{dx} = \frac{75}{2y}$$

(50) use implicit differentiation

$\cos(y) + 4 = x$

$-\sin(y)\,y' + 0 = 1$

$-\sin(y)\,y' = 1$

$$\frac{\cancel{-\sin(y)}\,y'}{\cancel{-\sin(y)}} = \frac{1}{-\sin(y)}$$

$$y' = -\frac{1}{\sin(y)}$$

$y' = -\csc(y)$

OR

$$\frac{dy}{dx} = -\csc(y)$$

Formula

$$\frac{1}{\sin(x)} = \csc(x)$$

$y = \cos(f(x))$

$y' = -\sin(f(x)) \cdot f'(x)$

(57.) $x = y^5$ Use implicit differentiation

$$1 = 5y^4 y'$$

$$\frac{1}{5y^4} = \frac{\cancel{5y^4} y'}{\cancel{5y^4}}$$

$$\frac{1}{5y^4} = y'$$

$$y' = \frac{1}{5y^4}$$

$$y' = \frac{1}{5} y^{-4}$$

$$y'' = \frac{1}{5}(-4)\, y^{-4-1} \cdot y'$$

$$y'' = \frac{-4}{5} y^{-5} \cdot y'$$

Subst

$$y'' = -\frac{4}{5} y^{-5} \cdot \left(\frac{1}{5} y^{-4}\right)$$

$$y'' = \frac{-4}{25} y^{-5-4}$$

$$y'' = -\frac{4}{25} \cdot y^{-9}$$

$$y'' = \frac{-4}{25y^9}$$ rewrite

OR $$\frac{d^2y}{dx^2} = \frac{-4}{25y^9}$$

52. use implicit differentiation

find the slope of the curve at the given point

$$x^5 + y^5 = -275$$

$(-3, -2)$
$x \quad y$

point

$$5x^4 + 5y^4 y' = 0$$

$$5y^4 y' = -5x^4$$

$$\frac{\cancel{5y^4} y'}{\cancel{5y^4}} = \frac{-5x^4}{5y^4}$$

$$y' = -\frac{x^4}{y^4}$$

$$y'(\underset{x}{-3}, \underset{y}{-2}) = \frac{-(-3)^4}{(-2)^4}$$

$$y'(-3, -2) = \frac{-(-3)(-3)(-3)(-3)}{(-2)(-2)(-2)(-2)}$$

$$y'(-3, -2) = \frac{-81}{16}$$

(53) $\cos(y) + \sin(x) = 3y$ Implicit differentiation

$$-\sin(y)\,y' + \cos(x)(x)' = 3y'$$

$$-\sin(y)\,y' + \cos(x)(1) = 3y'$$

$$-\sin(y)\,y' + \cos(x) = 3y'$$

$$\cos(x) = 3y' + \sin(y)\,y'$$

$$\cos(x) = y'(3 + \sin(y))$$

$$\frac{\cos(x)}{(3+\sin(y))} = \frac{y'\cancel{(3+\sin(y))}}{\cancel{(3+\sin(y))}}$$

$$\frac{\cos(x)}{3+\sin(y)} = y'$$

OR

$$\frac{dy}{dx} = \frac{\cos(x)}{3+\sin(y)}$$

54 $2\cos(xy) = 5x + 9y$

$$-2\sin(xy)\cdot(xy)' = 5 + 9y'$$

$$-2\sin(xy)\cdot\left((x)'(y) + (x)(y')\right) = 5 + 9y'$$

$$-2\sin(xy)\left((1)(y) + (x)(y')\right) = 5 + 9y'$$

$$-2\sin(xy)\left(y + xy'\right) = 5 + 9y'$$

$$-2\sin(xy)\cdot y - 2\sin(xy)\cdot xy' = 5 + 9y'$$

$$-2\sin(xy)\,xy' - 9y' = 5 + 2\sin(xy)\cdot y$$

$$y'\left(-2\sin(xy)\,x - 9\right) = 5 + 2\sin(xy)\cdot y$$

$$y'\left(-2x\sin(xy) - 9\right) = 5 + 2\sin(xy)\cdot y$$

$$y'\left(-2x\sin(xy) - 9\right) = 5 + 2y\sin(xy)$$

$$\frac{y'\cancel{(-2x\sin(xy) - 9)}}{\cancel{(-2x\sin(xy) - 9)}} = \frac{5 + 2y\sin(xy)}{(-2x\sin(xy) - 9)}$$

$$y' = \frac{5 + 2y\sin(xy)}{-2x\sin(xy) - 9} = \frac{dy}{dx}$$

53-

$$e^{xy} = 6y$$

$$e^{xy} \cdot (xy)' = 6y'$$

$$e^{xy} \cdot ((x)'(y) + (x)(y)') = 6y'$$

$$e^{xy} \cdot ((1)(y) + (x)(y')) = 6y'$$

$$e^{xy} \cdot (y + xy') = 6y'$$

$$e^{xy} \cdot y + e^{xy} \cdot xy' = 6y'$$

$$e^{xy} \cdot y = 6y' - e^{xy} xy'$$

$$e^{xy} \cdot y = y'(6 - e^{xy} x)$$

$$e^{xy} y = y'(6 - xe^{xy})$$

$$\frac{e^{xy} y}{(6 - xe^{xy})} = \frac{y'(6 - xe^{xy})}{(6 - xe^{xy})}$$

$$\frac{e^{xy} \cdot y}{6 - xe^{xy}} = y'$$

OR

$$\frac{ye^{xy}}{6 - xe^{xy}} = \frac{dy}{dx}$$

(56)

$$8x^4 + 3y^4 = 11xy$$

$$32x^3 + 12y^3y' = (11x)'(y) + (11x)(y)'$$

$$32x^3 + 12y^3y' = (11)(y) + (11x)(y')$$

$$32x^3 + 12y^3y' = 11y + 11xy'$$

$$12y^3y' - 11xy' = 11y - 32x^3$$

$$y'(12y^3 - 11x) = 11y - 32x^3$$

$$\frac{y'\cancel{(12y^3 - 11x)}}{\cancel{(12y^3 - 11x)}} = \frac{11y - 32x^3}{(12y^3 - 11x)}$$

$$y' = \frac{-32x^3 + 11y}{-11x + 12y^3}$$

$$y' = \frac{-1(-32x^3 + 11y)}{-1(-11x + 12y^3)}$$

$$y' = \frac{32x^3 - 11y}{11x - 12y^3} = \frac{dy}{dx}$$

(57) $y = \ln\sqrt{x^2+17}$

$y = \ln(x^2+17)^{1/2}$

$y = \frac{1}{2}\ln(x^2+17)$

$y' = \frac{1}{2}\,\frac{(x^2+17)'}{(x^2+17)}$

$y' = \frac{1}{2}\,\frac{(2x+0)}{(x^2+17)}$

$y' = \frac{1}{2}\,\frac{2x}{(x^2+17)}$

$y' = \frac{1}{\cancel{2}}\,\frac{\cancel{2}x}{(x^2+17)}$

$y' = \frac{x}{x^2+17}$

formula

$y = \ln\sqrt{f(x)}$

$y = \ln(f(x))^{1/2}$

$y = \frac{1}{2}\ln(f(x))$

$y = \ln(f(x))$

$y' = \frac{f'(x)}{f(x)}$

(58) express $f(x) = g(x)^{h(x)}$ in terms of the natural logarithmic and natural exponential function

formal

$$e^{\ln(f(x))} = f(x)$$

$$e^{\ln(f(x))^N} = (f(x))^N$$

$$e^{\ln(g(x))^{h(x)}} = (g(x))^{h(x)}$$

$$e^{\ln (g(x))^{h(x)}} =$$

$$e^{h(x)\ln(g(x))} =$$

59 $y = \ln(8x^2+1)$

formula

$y = \ln(f(x))$

$y' = \dfrac{f'(x)}{f(x)}$

$$y' = \frac{(8x^2+1)'}{8x^2+1}$$

$$y' = \frac{16x+0}{8x^2+1}$$

$$y' = \frac{16x}{8x^2+1}$$

60)

$y = 2x^{2\pi}$

$y' = 2(2\pi)\, x^{2\pi - 1}$

$y' = 4\pi\, x^{2\pi - 1}$

Format

$y = x^n$

$y' = n x^{n-1}$

61

$$y = 23^x$$

$$y' = 23^x \ln(23)$$

formula

$$y = a^x$$

$$y' = a^x \cdot \ln(a)$$

62

$$y = 4\log_4(x^2-3)$$

formula

$$y = \log_b f(x)$$

$$y' = \frac{f'(x)}{f(x)\ln(b)}$$

$$y' = 4\,\frac{(x^2-3)'}{(x^2-3)\ln(4)}$$

$$y' = 4\,\frac{(2x-0)}{(x^2-3)\ln(4)}$$

$$y' = 4\,\frac{2x}{(x^2-3)\ln(4)}$$

$$y' = \frac{8x}{(x^2-3)\ln(4)}$$

63 use logarithmic differentiation

$$f(x) = \frac{(x+2)^9}{(2x-2)^{11}}$$

formula

$$y = \frac{f}{g}$$

$$y' = \frac{f'g - fg'}{(g)^2}$$

$$y = \ln(f(x))$$

$$y' = \frac{f'(x)}{f(x)}$$

$$\ln(f(x)) = \ln\left(\frac{(x+2)^9}{(2x-2)^{11}}\right)$$

$$\ln(f(x)) = \ln(x+2)^9 - \ln(2x-2)^{11}$$

$$\ln(f(x)) = 9\ln(x+2) - 11\ln(2x-2)$$

$$\frac{f'(x)}{f(x)} = 9\,\frac{(x+2)'}{(x+2)} - 11\,\frac{(2x-2)'}{(2x-2)}$$

$$\frac{f'(x)}{f(x)} = 9\,\frac{(1+0)}{(x+2)} - 11\,\frac{(2-0)}{(2x-2)}$$

$$\frac{f'(x)}{f(x)} = 9\left(\frac{1}{x+2}\right) - 11\left(\frac{2}{2x-2}\right)$$

$$\frac{f'(x)}{f(x)} = \frac{9}{x+2} - 11\left(\frac{\cancel{2}\,1}{\cancel{2}(x-1)}\right)$$

substitute

$$f'(x) = f(x)\left[\frac{9}{x+2} - \frac{11}{x-1}\right] = \frac{(x+2)^9}{(2x-2)^{11}}\left[\frac{9}{x+2} - \frac{11}{x-1}\right]$$

64 $y = \sin^{-1}(f(x))$

$$y' = \frac{f'(x)}{\sqrt{1-(f(x))^2}}$$

OR

$y = \sin^{-1}(x)$

$$y' = \frac{1}{\sqrt{1-x^2}} \quad -1 < x < 1$$

$y = \tan^{-1}(f(x))$

$$y' = \frac{f'(x)}{1+(f(x))^2}$$

OR

$y = \tan^{-1}(x)$

$$y' = \frac{1}{1+x^2} \quad -\infty < x < \infty$$

$y = \sec^{-1}(f(x))$

$$y' = \frac{f'(x)}{|f(x)|\sqrt{(f(x))^2-1}} \quad |f(x)| > 1$$

OR

$y = \sec^{-1}(x)$

$$y' = \frac{1}{|x|\sqrt{x^2-1}} \quad |x| > 1$$

65

$$y = \sin^{-1}(2x^5)$$

$$y' = \frac{(2x^5)'}{\sqrt{1-(2x^5)^2}}$$

$$y' = \frac{10x^4}{\sqrt{1-(2x^5)(2x^5)}}$$

$$y' = \frac{10x^4}{\sqrt{1-4x^{5+5}}}$$

$$y' = \frac{10x^4}{\sqrt{1-4x^{10}}}$$

Formula

$$y = \sin^{-1}(f(x))$$

$$y' = \frac{f'(x)}{\sqrt{1-(f(x))^2}}$$

66

$$y = 6\tan^{-1}(9x)$$

$$y' = 6\,\frac{(9x)'}{1+(9x)^2}$$

$$y' = 6\,\frac{(9)}{1+(9x)(9x)}$$

$$y' = 6\,\frac{(9)}{1+81x^2}$$

$$y' = \frac{54}{1+81x^2}$$

OR

$$y' = \frac{54}{1+(9x)^2}$$

formula

$$y = \tan^{-1}(f(x))$$

$$y' = \frac{f'(x)}{1+(f(x))^2}$$

67.

$$y = \cot^{-1}(e^{s})$$

$$y' = -\frac{(e^{s})'}{1+(e^{s})^2}$$

$$y' = -\frac{e^{s}(s)'}{1+e^{2s}}$$

$$y' = -\frac{e^{s}(1)}{1+e^{2s}}$$

$$y' = -\frac{e^{s}}{1+e^{2s}}$$

OR

$$\frac{dy}{ds} = -\frac{e^{s}}{1+e^{2s}}$$

Formula

$$y = \cot^{-1}(f(x))$$

$$y' = -\frac{f'(x)}{1+(f(x))^2}$$

68. The sides of a square increase in length at a rate of 4 m/sec. $\longleftrightarrow$ $\frac{ds}{dt}$

a. At what rate is the area of the square changing when the sides are 16 m long?
b. At what rate is the area of the square changing when the sides are 27 m long?

a. Write an equation relating the area of a square, A, and the side length of the square, s.

$\boxed{A = s^2}$

Differentiate both sides of the equation with respect to t.

$\frac{dA}{dt} = (\boxed{2s})\frac{ds}{dt}$

$\frac{dA}{dt} = 2(16)(4) = 128 \ m^2/s$

The area of the square is changing at a rate of $\boxed{128}$ (1) $\boxed{m^2/s}$ when the sides are 16 m long.

b. The area of the square is changing at a rate of $\boxed{216}$ (2) $\boxed{m^2/s}$ when the sides are 27 m long.

$\frac{dA}{dt} = 2(27)(4) = 216 \ m^2/s$

(1) ○ m
○ m / s
○ m^3 / s
◉ m^2 / s

(2) ○ m / s
◉ m^2 / s
○ m
○ m^3 / s

Answers $A = s^2$

2s

128

(1) m^2 / s

216

(2) m^2 / s

$A = s^2$

$\frac{dA}{dt} = 2s\frac{ds}{dt}$

69. The area of a circle increases at a rate of 1 cm^2/s. ⟷ $\frac{dA}{dt}$

a. How fast is the radius changing when the radius is 5 cm?
b. How fast is the radius changing when the circumference is 5 cm?

a. Write an equation relating the area of a circle, A, and the radius of the circle, r.

$A = \pi r^2$

(Type an exact answer, using π as needed.)

Differentiate both sides of the equation with respect to t.

$$\frac{dA}{dt} = (2\pi r)\frac{dr}{dt}$$

(Type an exact answer, using π as needed.)

When the radius is 5 cm, the radius is changing at a rate of $\frac{1}{10\pi}$ (1) cm/s
(Type an exact answer, using π as needed.)

b. When the circumference is 5 cm, the radius is changing at a rate of $\frac{1}{5}$ (2) cm/s
(Type an exact answer, using π as needed.)

(1) ◉ cm/s. ○ cm. ○ cm^2/s. ○ cm^3/s.

(2) ○ cm^3/s. ○ cm. ○ cm^2/s. ◉ cm/s.

Answers $A = \pi r^2$

$2\pi r$

$\frac{1}{10\pi}$

(1) cm/s.

$\frac{1}{5}$

(2) cm/s.

area of circle

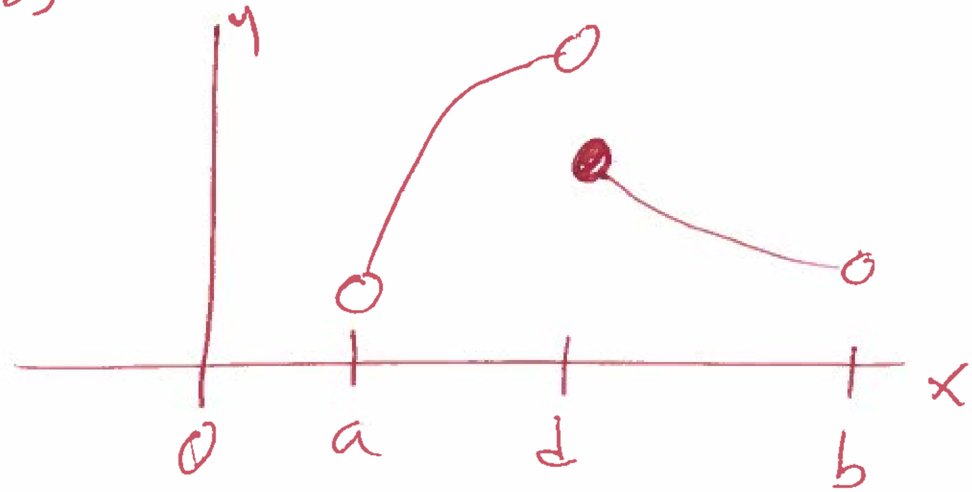
Part 1

$$A = \pi r^2$$
$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$
$$1 = 2\pi(5)\frac{dr}{dt}$$
$$1 = 10\pi\frac{dr}{dt}$$
$$\frac{1}{10\pi} = \frac{\cancel{10\pi}\frac{dr}{dt}}{\cancel{10\pi}}$$
$$\frac{1}{10\pi} = \frac{dr}{dt}$$

Part 2 circumference

$$C = 2\pi r$$
$$5 = 2\pi r$$
$$\frac{5}{2\pi} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$$
$$\frac{5}{2\pi} = r$$
$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$
$$1 = 2\pi\left(\frac{5}{2\pi}\right)\frac{dr}{dt}$$
$$1 = 5\frac{dr}{dt}$$
$$\frac{1}{5} = \frac{5}{5}\frac{dr}{dt}$$
$$\frac{1}{5} = \frac{dr}{dt}$$

(70) determine from the graph whether the function has any absolute extreme value on [a, b]



there is no absolute maximum and there is no absolute minimum. on [a, b]

71. find critical points

$f(x) = 4x^2 - 5x + 1$

$f'(x) = 8x - 5 + 0$

$f'(x) = 8x - 5$

set $8x - 5 = 0$

$8x \cancel{-5} \cancel{+5} = 0 + 5$

$8x = 5$

$\frac{8x}{8} = \frac{5}{8}$

$x = 5/8$ critical point

$y = x^n$

$y' = nx^{n-1}$

$y = ax$

$y' = a$

72 find Critical Points

$$f(x) = \frac{-x^3}{3} + 100x$$

$$f'(x) = \frac{-3x^2}{3} + 100$$

$$f'(x) = -x^2 + 100$$

set $-x^2 + 100 = 0$

$$-x^2 = -100$$

$$\frac{-x^2}{-1} = \frac{-100}{-1}$$

$$x^2 = 100$$

$$\sqrt{x^2} = \pm\sqrt{100}$$

$$x = \pm 10$$

$x = 10$ or $x = -10$

Critical Points

$y = x^n$

$y' = nx^{n-1}$

$y = ax$

$y' = a$

(73) determine the location and value of the absolute extreme value of f on the interval given if they exist.

$f(x) = x^2 - 5$ on $[\underset{a}{-2}, \underset{b}{4}]$

$f'(x) = 2x - 0$

$f'(x) = 2x$

set $2x = 0$

$\frac{2x}{2} = \frac{0}{2}$

$x = 0$

Critical point

$f(-2) = (-2)^2 - 5$

$f(-2) = (-2)(-2) - 5$

$f(-2) = 4 - 5$

$f(-2) = -1$

$f(4) = (4)^2 - 5$

$f(4) = (4)(4) - 5$

$f(4) = 16 - 5$

$f(4) = 11$

$f(0) = (0)^2 - 5$

$f(0) = (0)(0) - 5$

$f(0) = 0 - 5$

$f(0) = -5$

absolute maximum $f(4) = 11$

absolute minimum $f(0) = -5$

(74) find max

$s' = -16t^2 + 96t + 112$ $\quad 0 \le t \le 7$

$s'(t) = -32t + 96 + 0$

$s'(t) = -32t + 96$ derivative

set $-32t + 96 = 0$

$-32t + \cancel{96} - \cancel{96} = 0 - 96$

$-32t = -96$

$$\frac{-32t}{-32} = \frac{-96}{-32}$$

$t = 3$ critical point

$s(t) = -16t^2 + 96t + 112$

$s'(3) = -16(3)^2 + 96(3) + 112$

$s'(3) = -16(3)(3) + 96(3) + 112$

$s'(3) = -144 + 288 + 112$

$s'(3) = 256$

critical point

Maximum

Maximum at $t = 3$

$s(3) = 256$

$(3, 256)$

(7) Suppose a tour guide has a bus that holds a maximum of 88 people. Assume his profit (in dollars) for taking n people on a city tour is

$$P(n) = n(44 - 0.5n) - 88.$$

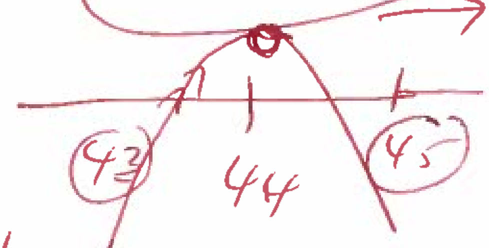
(a) How many people should the guide take on tour to maximize the profit?

(b) Suppose the bus holds a maximum of 38 people. How many people should be taken on a tour to maximize the profit.

(a)

$$P(n) = n(44 - 0.5n) - 88$$
$$P(n) = 44n - 0.5n^2 - 88$$
$$P'(n) = 44 - 0.5(2n) - 0$$
$$P'(n) = 44 - 1n$$
$$P'(n) = -n + 44$$
$$-n + 44 = 0$$
$$-n + 44 - 44 = 0 - 44$$
$$-n = -44 \rightarrow n = 44$$

Critical point



$$P'(n) = -n + 44$$
$$P'(43) = -43 + 44 = 1 > 0 \text{ increas}$$
$$P'(45) = -(45) + 44 = -45 + 44 = -1 < 0 \text{ decreas}$$

Max at $n = 44$

(b)

If the bus holds a max of 38 people the guide should take 38 people

(76) at what point c does the conclusion of the Mean Value Theorem hold for $f(x)=x^3$ on the interval $[-20, 20]$, $a=-20$, $b=20$

$f'(x)=3x^2$

$$\frac{f(b)-f(a)}{b-a}=f'(c)$$ MEAN VALUE Theorem

$$\frac{f(20)-f(-20)}{(20)-(-20)}=3x^2$$

$$\frac{(20)^3-(-20)^3}{20+20}=3x^2$$

$$\frac{8000-(-8000)}{40}=3x^2$$

$$\frac{8000+8000}{40}=3x^2$$

$$\frac{16000}{40}=3x^2$$

$$400=3x^2$$

$$\frac{400}{3}=\frac{\cancel{3}x^2}{\cancel{3}}$$

$$\frac{400}{3}=x^2$$

$$\pm\sqrt{\frac{400}{3}}=\sqrt{x^2}$$

$$\pm\frac{\sqrt{400}}{\sqrt{3}}=x$$

$$\pm\frac{20}{\sqrt{3}}=x$$

$$\pm\frac{20}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=x$$

$$\pm\frac{20\sqrt{3}}{\sqrt{9}}=x$$

$$\pm\frac{20\sqrt{3}}{3}=x$$

$x=\frac{20\sqrt{3}}{3}$ OR $x=-\frac{20\sqrt{3}}{3}$

(77) determine whether the mean value theorem applies to the function $f(x) = 3 - x^2$ on the interval $[\underset{a}{-2}, \underset{b}{1}]$.

$f'(x) = 0 - 2x$

$f'(x) = -2x$

$$\frac{f(b) - f(a)}{(b) - (a)} = f'(c)$$

Mean value theorem

$$\frac{f(1) - f(-2)}{(1) - (-2)} = -2x$$

$$\frac{(3 - (1)^2) - (3 - (-2)^2)}{(1) - (-2)} = -2x$$

$$\frac{(3 - 1) - (3 - (4))}{1 + 2} = -2x$$

$$\frac{(2) - (-1)}{3} = -2x$$

$$\frac{2 + 1}{3} = -2x$$

$$\frac{3}{3} = -2x$$

$$1 = -2x$$

set $-2x = 1$

$$\frac{-2x}{-2} = \frac{1}{-2}$$

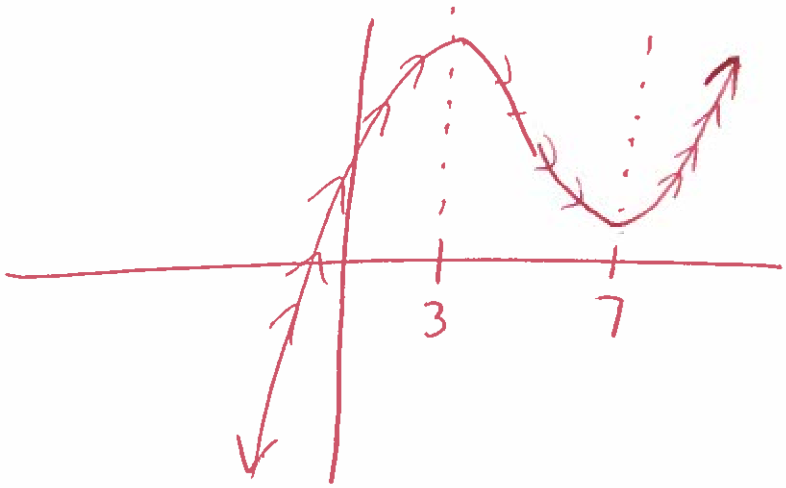
$$x = -1/2$$

78. Sketch a function that is continuous on $(-\infty, \infty)$ and has the following properties.

$f'(x) > 0$ on $(-\infty, 3)$ function increasing

$f'(x) < 0$ on $(3, 7)$ function decreasing

$f'(x) > 0$ on $(7, \infty)$ function increasing



(79) Find the intervals on which f is increasing and the intervals on which it is decreasing.

$$f(x) = -2 + x^2$$

$$f'(x) = 0 + 2x$$

$$f'(x) = 2x$$

set $2x = 0$

$$\frac{2x}{2} = \frac{0}{2}$$

$x = 0$ Critical point

Check $\boxed{-1}$ 0 $\boxed{1}$ Check

$$f'(x) = 2x$$

$$f'(-1) = 2(-1)$$

$f'(-1) = -2 < 0$ decreasing $(-\infty, 0)$

$$f'(x) = 2x$$

$$f'(1) = 2(1)$$

$f'(1) = 2 > 0$ increasing $(0, \infty)$

(80) Find the intervals on which f is increasing and the intervals on which it is decreasing.

$$f(x) = -10 - x + 2x^2$$

$$f'(x) = 0 - 1 + 4x$$

$$f'(x) = -1 + 4x$$

set $-1 + 4x = 0$

$$\cancel{-1} + 4x \cancel{+1} = 0 + 1$$

$$4x = 1$$

$$\frac{4x}{4} = \frac{1}{4}$$

$x = \frac{1}{4}$ Critical point

$\boxed{0}$ $\quad \frac{1}{4} \quad$ $\boxed{1}$

$$f'(x) = -1 + 4x$$

$$f'(0) = -1 + 4(0)$$

$$f'(0) = -1 + 0$$

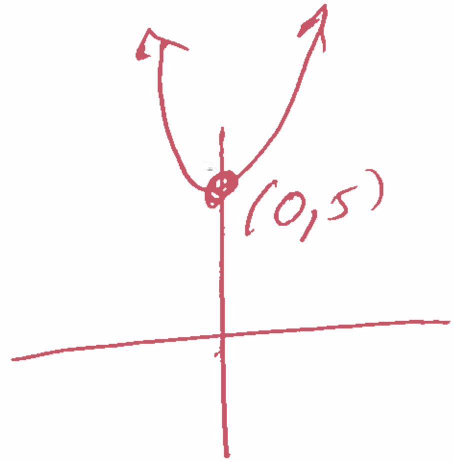
$f'(0) = -1 < 0$ decreasing $(-\infty, \frac{1}{4})$

$f'(1) = -1 + 4(1)$

$f'(1) = -1 + 4$

$f'(1) = 3 > 0$ increasing $(\frac{1}{4}, \infty)$

(81.) find max or min



$f(x) = 5 + x^2$

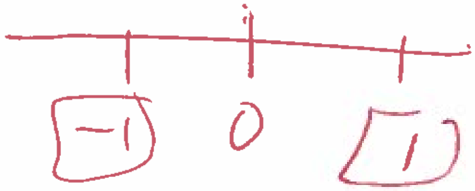
$f'(x) = 0 + 2x$

$f'(x) = 2x$

set $2x = 0$

$\frac{2x}{2} = \frac{0}{2}$

$x = 0$ critical points



$f'(x) = 2x$

$f'(-1) = 2(-1)$

$f'(-1) = -2 < 0$ decreasing on $(-\infty, 0)$

$f'(x) = 2x$

$f'(1) = 2(1)$

$f'(1) = 2 > 0$ increasing on $(0, \infty)$

Minimum at $x = 0$ → $f(0) = 5 + 0$

$f(x) = 5 + x^2$

$f(0) = 5 + (0)^2$

$f(0) = 5$

$(0, 5)$ Minimum at $x = 0$

8C find max or min

$f(x) = x^3 + 12x^2 + 5$

$f'(x) = 3x^2 + 24x + 0$

$f'(x) = 3x^2 + 24x$

set $3x^2 + 24x = 0$

$3x(x+8) = 0$

$3x = 0$ or $x + 8 = 0$

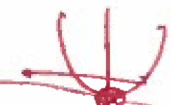
$\frac{3x}{3} = \frac{0}{3}$ or $x + \cancel{8} - \cancel{8} = 0 - 8$

$x = 0$ or $x = -8$ critical points

$f'(x) = 3x^2 + 24x$

$f''(x) = 6x + 24$

$f''(0) = 6(0) + 24 = 0 + 24 = 24 > 0$ concave up



minimum at $x = 0$

$f''(-8) = 6(-8) + 24 = -48 + 24 = -24 < 0$ concave down

Maximum at $x = -8$

$f(x) = x^3 + 12x^2 + 5$ (0, 5) minimum point

$f(0) = (0)^3 + 12(0)^2 + 5 = 0 + 0 + 5 = 5$

$f(x) = x^3 + 12x^2 + 5$ $f(-8) = 261$ Maximum point

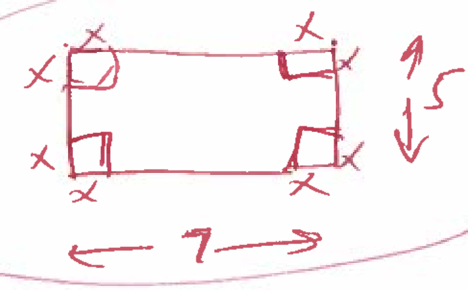
$f(-8) = (-8)^3 + 12(-8)^2 + 5$

$f(-8) = -512 + 768 + 5$

83

$V(x) = x(7-2x)(5-2x)$

find max rectangle



$V(x) = x(35 - 14x - 10x + 4x^2)$

$V(x) = x(35 - 24x + 4x^2)$

$V(x) = 35x - 24x^2 + 4x^3$

$V(x) = 4x^3 - 24x^2 + 35x$

$V'(x) = 12x^2 - 48x + 35$ and $V''(x) = 24x - 48$

$a = 12, \quad b = -48, \quad c = 35$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-48) \pm \sqrt{(-48)^2 - 4(12)(35)}}{2(12)}$$

$$x = \frac{48 \pm \sqrt{2304 - 1680}}{24}$$

$$x = \frac{48 \pm \sqrt{624}}{24}$$

$V(x) = x(7-2x)(5-2x)$

$V(.96) = .96(7-2(.96))(5-2(.96))$

$V(.96) \cong 15.02$

Max Volume

$x = (48 + \sqrt{624}) \div 24$ OR $x = (48 - \sqrt{625}) \div 24$

$x = 3.040833$ OR $x \cong .959167003$

$V''(x) = 24x - 48$

$V''(3.040833) = 24(3.040833) - 48 = 24.979992 > 0$ concave up

$x = 3.040833 \to$ min

$V''(.959167003) = 24(.959167003) - 48 = -24.97999199 < 0$

maximum $x = .959167003$ concave down

83 Part 2 find max

let side = s = m

$V(x) = x(m-2x)(m-2x)$

$V(x) = x(m^2 - 2mx - 2mx + 4x^2)$

$V(x) = x(m^2 - 4mx + 4x^2)$

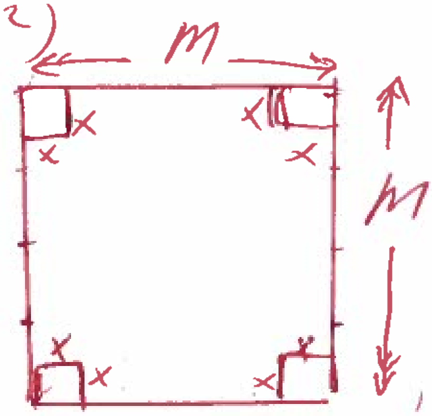
$V(x) = m^2x - 4mx^2 + 4x^3$

$V'(x) = m^2(1) - 4m(2x) + 4(3x^2)$

$V'(x) = m^2 - 8mx + 12x^2$

$V'(x) = 12x^2 - 8mx + m^2$

$V'(x) = (2x - m)(6x - m)$



set $2x - m = 0$ or $6x - m = 0$

$2x - m + m = 0 + m$ or $6x - m + m = 0 + m$

$2x = m$ or $6x = m$

$\frac{2x}{2} = \frac{m}{2}$ or $\frac{6x}{6} = \frac{m}{6}$

$x = \frac{m}{2}$ or $x = \frac{m}{6}$

critical points

83 Part 3

$$x = \frac{m}{6}$$

$$V(x) = x(m-2x)(m-2x)$$

$$V\left(\frac{m}{6}\right) = \left(\frac{m}{6}\right)\left(m - 2\left(\frac{m}{6}\right)\right)\left(m - 2\left(\frac{m}{6}\right)\right)$$

$$V\left(\frac{m}{6}\right) = \left(\frac{m}{6}\right)\left(m - \frac{\cancel{2}m}{\cancel{2}(3)}\right)\left(m - \frac{\cancel{2}m}{\cancel{2}(3)}\right)$$

$$V\left(\frac{m}{6}\right) = \left(\frac{m}{6}\right)\left(m - \frac{m}{3}\right)\left(m - \frac{m}{3}\right)$$

$$V\left(\frac{m}{6}\right) = \left(\frac{m}{6}\right)\left(\frac{3m}{3} - \frac{m}{3}\right)\left(\frac{3m}{3} - \frac{m}{3}\right)$$

$$V\left(\frac{m}{6}\right) = \left(\frac{m}{6}\right)\left(\frac{3m-1m}{3}\right)\left(\frac{3m-1m}{3}\right)$$

$$V\left(\frac{m}{6}\right) = \left(\frac{m}{6}\right)\left(\frac{2m}{3}\right)\left(\frac{2m}{3}\right)$$

$$V\left(\frac{m}{6}\right) = \frac{4m^3}{54}$$

$$V\left(\frac{m}{6}\right) = \frac{\cancel{2}(2m^3)}{\cancel{2}(27)}$$

$$V\left(\frac{m}{6}\right) = \frac{2m^3}{27}$$

Max volume

83 Part 4 $x = \frac{m}{2}$

$V(x) = x(m-2x)(m-2x)$

$V(\frac{m}{2}) = (\frac{m}{2})(m - 2(\frac{m}{2}))(m - 2(\frac{m}{2}))$

$V(\frac{m}{2}) = (\frac{m}{2})(m-m)(m-m)$

$V(\frac{m}{2}) = (\frac{m}{2})(0)(0)$

$V(\frac{m}{2}) = 0$ min volume

(84) Use a linear approximation to estimate the quantity. Choose a value to produce a small error

$\ln(1.05)$

$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

$f'(1) = \frac{1}{1} = 1$

$f'(1) = 1$

$L(x) = f(a) + f'(a)(x - a)$

$L(1.05) = f(1) + f'(1)(1.05 - 1)$

$L(1.05) = \ln(1) + \left(\frac{1}{1}\right)(1.05 - 1)$

$L(1.05) = 0 + (1)(0.05)$

$L(1.05) = 0 + 0.05$

$L(1.05) = 0.05$

85 express the relationship between a small change in x and the corresponding change in y in the form $dy = f'(x)\,dx$

$f(x) = e^{20x}$

$f'(x) = e^{20x}(20x)'$

$f'(x) = e^{20x}(20)$

$f'(x) = 20e^{20x}$

$\frac{dy}{dx} = 20e^{20x}$

$\frac{dy}{\cancel{dx}}(\cancel{dx}) = 20e^{20x}\,dx$

$dy = 20e^{20x} \cdot dx$

formula

$y = e^{f(x)}$

$y' = e^{f(x)} \cdot f'(x)$

(8c) express the relationship between a small change in x and the following corresponding change in y in the form $dy = f'(x)\,dx$

$$f(x) = 2x^3 - 4x$$

$$f'(x) = 6x^2 - 4$$

$$\frac{dy}{dx} = 6x^2 - 4$$

$$\frac{dy}{\cancel{dx}}(\cancel{dx}) = (6x^2-4)\,dx$$

$$dy = (6x^2-4)\,dx$$

$$y = x^n$$
$$y' = nx^{n-1}$$

$$y = ax$$
$$y' = a$$

87. express the relationship between a small change in x and the corresponding change in y in the form $dy = f'(x)dx$

$f(x) = \cot(4x)$

$f'(x) = -\csc^2(4x) \cdot (4x)'$

$f'(x) = -\csc^2(4x)(4)$

$f'(x) = -4\csc^2(4x)$

$\frac{dy}{dx} = -4\csc^2(4x)$

$\frac{dy}{\cancel{dx}}(\cancel{dx}) = -4\csc^2(4x)(dx)$

$dy = (-4\csc^2(4x))\,dx$

formula

$y = \cot f(x)$

$y' = -\csc^2(f(x)) \cdot f'(x)$

(88) use l Hopitals Rule

$$\lim_{x \to 0} \frac{7\sin(9x)}{5x} =$$

$$\lim_{x \to 0} \frac{7\cos(9x)\cdot(9x)'}{(5x)'} =$$

$$\lim_{x \to 0} \frac{7\cos(9x)(9)}{5} =$$

$$\lim_{x \to 0} \frac{63\cos(9x)}{5} =$$

$$\frac{63\cos(9(0))}{5} =$$

$$\frac{63\cos(0)}{5} =$$

$$\frac{63(1)}{5} =$$

$$\frac{63}{5} =$$

formula

$$\lim_{x \to a} \frac{f(x)}{g(x)} =$$

$$\lim_{x \to a} \frac{f'(x)}{g'(x)} =$$

formula

$$y = \sin(f(x))$$

$$y' = \cos(f(x))\, f'(x)$$

(89) Use l Hopital Rule

$$\lim_{u \to \frac{\pi}{4}} \frac{\tan(u) - \cot(u)}{u - \frac{\pi}{4}} =$$

Formula

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} =$$

$$\lim_{u \to \frac{\pi}{4}} \frac{\sec^2(u) \cdot (u)' - (-\csc^2(u)\, u')}{1 - 0} =$$

$$\lim_{u \to \frac{\pi}{4}} \frac{\sec^2(u) \cdot (1) + \csc^2(u)(1)}{1} =$$ ~~Formula~~

$$\lim_{u \to \frac{\pi}{4}} \sec^2(u) + \csc^2(u) =$$

$$\sec^2\left(\frac{\pi}{4}\right) + \csc^2\left(\frac{\pi}{4}\right) =$$

$$\frac{1}{\cos^2\left(\frac{\pi}{4}\right)} + \frac{1}{\sin^2\left(\frac{\pi}{4}\right)} =$$

$$\frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} + \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} =$$

$$\frac{1}{\frac{(\sqrt{2})^2}{2^2}} + \frac{1}{\frac{(\sqrt{2})^2}{2^2}} =$$

$$\frac{1}{\frac{2}{4}} + \frac{1}{\frac{2}{4}} =$$

$$\frac{1}{\frac{1}{2}} + \frac{1}{\frac{1}{2}} =$$

$$\frac{1}{1} \cdot \frac{2}{1} + \frac{1}{1} \cdot \frac{2}{1} =$$

$$\frac{2}{1} + \frac{2}{1} =$$

$$2 + 2 =$$

$$4 =$$

90. Use a calculator or program to compute the first 10 iterations of Newton's method when they are applied to the following function with the given initial approximation.

$f(x) = x^2 - 24; x_0 = 5$

$f'(x) = 2x$ $x_0 = 5$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 5 - \frac{f(5)}{f'(5)}$

$x_1 = 5 - \frac{(5)^2 - 24}{2(5)}$

$x_1 = 4.9000000$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

k	x_k	k	x_k
0	5.000000	6	4.898979
1	4.900000	7	4.898979
2	4.898980	8	4.898979
3	4.898979	9	4.898979
4	4.898979	10	4.898979
5	4.898979		

(Round to six decimal places as needed.)

Answers 5.000000

4.898979

4.900000

4.898979

4.898980

4.898979

4.898979

4.898979

4.898979

4.898979

4.898979

91. Use a calculator or program to compute the first 10 iterations of Newton's method for the given function and initial approximation.

$f(x) = 5\sin x - 3x + 1,\ x_0 = 1.4$

$f'(x) = 5\cos(x) - 3$

$x_0 = 1.4$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

Complete the table.
(Do not round until the final answer. Then round to six decimal places as needed.)

k	x_k	k	x_k
1	2.203310	6	1.906800
2	1.938494	7	1.906800
3	1.907290	8	1.906800
4	1.906800	9	1.906800
5	1.906800	10	1.906800

$x_1 = 1.4 - \frac{f(1.4)}{f'(1.4)}$

$x_1 = 1.4 - \frac{5\sin(1.4) - 3(1.4) + 1}{5\cos(1.4) - 3}$

$x_1 = 2.203310$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

Answers 2.203310

1.906800

1.938494

1.906800

1.907290

1.906800

1.906800

1.906800

1.906800

1.906800

---

92. Determine the following indefinite integral. Check your work by differentiation.

$$\int \left(5x^9 - 3x^5\right) dx$$

$\int \left(5x^9 - 3x^5\right) dx =$ ☐ (Use C as the arbitrary constant.)

Answer: $\frac{x^{10}}{2} - \frac{x^6}{2} + C$

$\int (5x^9 - 3x^5) dx =$

$\frac{5x^{9+1}}{9+1} - \frac{3x^{5+1}}{5+1} + C =$

$\frac{5x^{10}}{10} - \frac{3x^6}{6} + C =$

$\frac{\cancel{5}x^{10}}{(\cancel{5})(2)} - \frac{\cancel{3}x^6}{(\cancel{3})(2)} + C =$

$\frac{x^{10}}{2} - \frac{x^6}{2} + C =$

---

formula

$\int x^n dx = \frac{x^{n+1}}{n+1} + C =$

(93) $\int \left(\frac{12}{\sqrt{x}} + 12\sqrt{x}\right) dx$

Formula
$\int x^n dx =$
$\frac{x^{n+1}}{n+1} + C =$

$\int \left(\frac{12}{x^{1/2}} + 12x^{1/2}\right) dx =$

$\int \left(12x^{-1/2} + 12x^{1/2}\right) dx =$

$\frac{12x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{12x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C =$

$\frac{12x^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}} + \frac{12x^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C =$

$\frac{12x^{\frac{-1+2}{2}}}{\frac{-1+2}{2}} + \frac{12x^{\frac{1+2}{2}}}{\frac{1+2}{2}} + C =$

$\frac{12x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{12x^{\frac{3}{2}}}{\frac{3}{2}} + C =$

$\frac{2}{1}(12x^{1/2}) + \frac{2}{3}(12x^{3/2}) + C =$

$24x^{1/2} + \frac{24}{3}x^{3/2} + C =$

$24x^{1/2} + \frac{\cancel{3}(8)}{\cancel{3}(1)}x^{3/2} + C =$

$24x^{1/2} + 8x^{3/2} + C =$

$24\sqrt{x} + 8x^{3/2} + C =$

(94) $\int \left(\frac{2}{s^2} - 9s^4 \right) ds =$

$\int (2s^{-2} - 9s^4) ds =$

$\frac{2s^{-2+1}}{-2+1} - \frac{9s^{4+1}}{4+1} + C =$

$\frac{2s^{-1}}{-1} - \frac{9s^5}{5} + C =$

$-2s^{-1} - \frac{9}{5}s^5 + C =$

formula

$\int x^n dx =$

$\frac{x^{n+1}}{n+1} + C =$

95.

$$\int (6x+5)^2\,dx =$$

$$\int (6x+5)(6x+5)\,dx =$$

$$\int (36x^2+30x+30x+25)\,dx =$$

$$\int (36x^2+60x+25)\,dx =$$

$$\frac{36x^{2+1}}{2+1}+\frac{60x^{1+1}}{1+1}+25x+C=$$

$$\frac{36x^3}{3}+\frac{60x^2}{2}+25x+C=$$

$$12x^3+30x^2+25x+C=$$

formula

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C$$

$$\int a\,dx = ax+C$$

96

$$\int 4m(8m^2 - 5m)\,dm =$$

$$\int (32m^3 - 20m^2)\,dm =$$

$$\frac{32m^{3+1}}{3+1} - \frac{20m^{2+1}}{2+1} + C =$$

$$\frac{32m^4}{4} - \frac{20m^3}{3} + C =$$

$$8m^4 - \frac{20m^3}{3} + C =$$

formula

$$\int x^n\,dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

(97) $\int (2x^{2/3} + 4x^{-1/3} + 7)\,dx =$

$\frac{2x^{2/3+1}}{\frac{2}{3}+1} + \frac{4x^{-1/3+1}}{-\frac{1}{3}+1} + 7x + C =$

$\frac{2x^{\frac{2}{3}+\frac{3}{3}}}{\frac{2}{3}+\frac{3}{3}} + \frac{4x^{-\frac{1}{3}+\frac{3}{3}}}{-\frac{1}{3}+\frac{3}{3}} + 7x + C =$

$\frac{2x^{\frac{2+3}{3}}}{\frac{2+3}{3}} + \frac{4x^{\frac{-1+3}{3}}}{\frac{-1+3}{3}} + 7x + C =$

$\frac{2x^{5/3}}{\frac{5}{3}} + \frac{4x^{2/3}}{\frac{2}{3}} + 7x + C =$

$\left(\frac{3}{5}\right)(2x^{5/3}) + \frac{3}{2}(4x^{2/3}) + 7x + C =$

$\frac{6}{5}x^{5/3} + \frac{12}{2}x^{2/3} + 7x + C =$

$\frac{6}{5}x^{5/3} + 6x^{2/3} + 7x + C =$

formula

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C =$

$\int a\,dx = ax + C$

(98) $\int 4\sqrt[9]{x}\,dx =$

$\int 4x^{1/9}\,dx =$

$\dfrac{4x^{1/9+1}}{\frac{1}{9}+1} + C =$

$\dfrac{4x^{1/9+9/9}}{\frac{1}{9}+\frac{9}{9}} + C =$

$\dfrac{4x^{\frac{1+9}{9}}}{\frac{1+9}{9}} + C =$

$\dfrac{4x^{10/9}}{\frac{10}{9}} + C =$

$\frac{9}{10}\left(4x^{10/9}\right) + C =$

$\frac{36}{10}x^{10/9} + C =$

$\frac{\cancel{2}(18)}{\cancel{2}(5)}x^{10/9} + C =$

$\frac{18}{5}x^{10/9} + C =$

formula

$\int x^n\,dx =$

$\dfrac{x^{n+1}}{n+1} + C =$

99.

$$\int (9x+1)(7-x)\,dx =$$

$$\int (63x - 9x^2 + 7 - 1x)\,dx =$$

$$\int (-9x^2 + 62x + 7)\,dx =$$

$$\frac{-9x^{2+1}}{2+1} + \frac{62x^{1+1}}{1+1} + 7x + C =$$

$$\frac{-9x^3}{3} + \frac{62x^2}{2} + 7x + C =$$

$$-3x^3 + 31x^2 + 7x + C =$$

Formula

$$\int x^n\,dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

$$\int a\,dx =$$

$$ax + C =$$

100

$$\int \left(\frac{8}{x^4} + 2 - \frac{8}{x^2}\right) dx =$$

$$\int 8x^{-4} + 2 - 8x^{-2}) dx =$$

$$\frac{8x^{-4+1}}{-4+1} + 2x - \frac{8x^{-2+1}}{-2+1} + C =$$

$$\frac{8x^{-3}}{-3} + 2x - \frac{8x^{-1}}{-1} + C =$$

$$-\frac{8x^{-3}}{3} + 2x + 8x^{-1} + C =$$

$$\frac{-8}{3x^3} + 2x + \frac{8}{x} + C =$$

Formula

$$\int x^n dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

$$\int a\, dx =$$

$$ax + C =$$

101

$$\int \frac{4x^4+15x^3}{x}\,dx =$$

$$\int\left(\frac{4x^4}{x^1}+\frac{15x^3}{x^1}\right)dx =$$

$$\int\left(4x^{4-1}+15x^{3-1}\right)dx =$$

$$\int\left(4x^3+15x^2\right)dx =$$

$$\frac{4x^{3+1}}{3+1}+\frac{15x^{2+1}}{2+1}+C =$$

$$\frac{4x^4}{4}+\frac{15x^3}{3}+C =$$

$$x^4+5x^3+C =$$

formula

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C =$$

102 for the function f, find the antiderivative F that satisfies the given condition:

$f(x) = 6x^3 + 9\sin(x)$ $\quad F(0) = 4$

formula

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int \sin x \, dx = -\cos(x) + C$

$\int f(x)\, dx = \int (6x^3 + 9\sin(x))\, dx$

$F(x) = \frac{6x^{3+1}}{3+1} + 9(-\cos(x)) + C$

$F(x) = \frac{6x^4}{4} - 9\cos(x) + C$

$F(x) = \frac{\cancel{2}(3)x^4}{\cancel{2}(2)} - 9\cos(x) + C$

$F(x) = \frac{3}{2}x^4 - 9\cos(x) + C$

$F(0) = \frac{3}{2}(0)^4 - 9\cos(0) + C = 4$

$\frac{3}{2}(0)(0)(0)(0) - 9(1) + C = 4$

$0 - 9 + C = 4$

$-9 + C = 4$

$\cancel{-9} + C \cancel{+9} = 4 + 9$

$C = 13$

$\boxed{F(x) = \frac{3}{2}x^4 - 9\cos(x) + 13}$

103. For the function f, find the antiderivative F that satisfies the given condition:

LITTLE $f(u) = 6e^{u} + 10$, BIG $F(0) = -6$

formula $\int e^{f(x)} f'(x)\,dx = e^{f(x)} + C$

$$\int f(u)\,du = \int (6e^{u} + 10)\,du$$

$$F(u) = \int (6e^{u}(1) + 10)\,du$$

$$F(u) = 6e^{u} + 10u + C$$

$$F(0) = 6e^{(0)} + 10(0) + C = -6$$

$$6(1) + 0 + C = -6$$

$$6 + C = -6$$

$$\cancel{6} + C - \cancel{6} = -6 - 6$$

$$C = -12$$

$$F(u) = 6e^{u} + 10u - 12$$

(104) Find the solution of the initial value problems.

$f'(x) = 4x - 7$, $f(0) = 8$

$\int f'(x)\,dx = \int (4x-7)\,dx$

$f(x) = \frac{4x^{1+1}}{1+1} - 7x + C$

$f(x) = \frac{4x^2}{2} - 7x + C$

$f(x) = 2x^2 - 7x + C$

$f(0) = 2(0)^2 - 7(0) + C = 8$

$2(0)(0) - 7(0) + C = 8$

$0 - 0 + C = 8$

$C = 8$

$\boxed{f(x) = 2x^2 - 7x + 8}$

formula

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$\int a\,dx = ax + C$

(105) Given the velocity function of an object moving along a line, find the position function with the given initial position

$V(t) = 4t + 5$, $s(0) = 0$

formula

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int V(t)\,dt = \int (4t+5)\,dt$

$\int a\,dx = ax + C$

$s(t) = \frac{4t^{1+1}}{1+1} + 5t + C$

$(s(t))' = V(t)$

$s(t) = \frac{4t^2}{2} + 5t + C$

$s(t) = 2t^2 + 5t + C$

$s(0) = 2(0)^2 + 5(0) + C = 0$

$0 + 0 + C = 0$

$C = 0$

$s(t) = 2t^2 + 5t + 0$

$\boxed{s(t) = 2t^2 + 5t}$

Next Page please

(106) Given the velocity function of an object moving along a line, find the position function with the given initial position.

$V(t) = 9t^2 + 2t - 7$ $\quad$ $s(0) = 0$

formula

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int a\,dx = ax + C$

$(s(t))' = v(t)$

$\int v(t)\,dt = \int (9t^2 + 2t - 7)\,dt$

$s(t) = \frac{9t^{2+1}}{2+1} + \frac{2t^{1+1}}{1+1} - 7t + C$

$s(t) = \frac{9t^3}{3} + \frac{2t^2}{2} - 7t + C$

$s(t) = 3t^3 + t^2 - 7t + C$

$s(0) = 3(0)^3 + (0)^2 - 7(0) + C = 0$

$0 + 0 - 0 + C = 0$

$C = 0$

$s(t) = 3t^3 + t^2 - 7t + 0$

$\boxed{s(t) = 3t^3 + t^2 - 7t}$

106. Given the following velocity function of an object moving along a line, find the position function with the given initial position.

$v(t) = 9t^2 + 2t - 7$; $s(0) = 0$

The position function is $s(t) =$ ______. $3t^3 + t^2 - 7t$

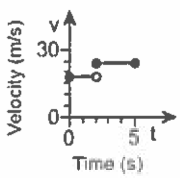
Answer: $3t^3 + t^2 - 7t$

---

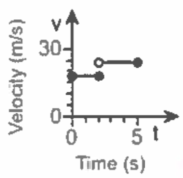
107. Suppose an object moves along a line at 18 m/s for $0 \le t \le 2$ s and at 24 m/s for $2 < t \le 5$ s. Sketch the graph of the velocity function and find the displacement of the object for $0 \le t \le 5$.

Sketch the graph of the velocity function. Choose the correct graph below.

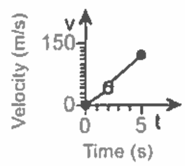
○ A.



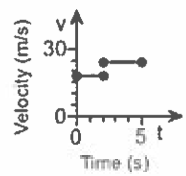
○ B.



○ C.

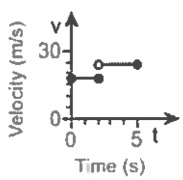


○ D.



The displacement of the object for $0 \le t \le 5$ is ______ m. (Simplify your answer.)

Answers

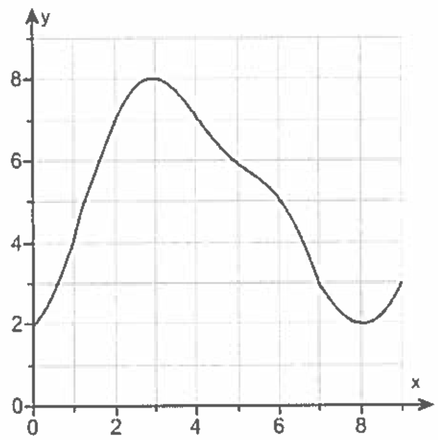


B.

108

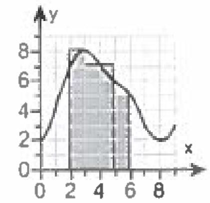
108.

Approximate the area of the region bounded by the graph of f(x) (shown below) and the x-axis by dividing the interval [0,4] into n = 4 subintervals. Use a right and left Riemann sum to obtain two different approximations. Draw the approximating rectangles.

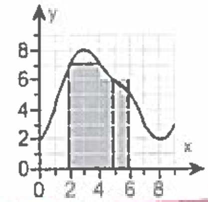


In which graph below are the selected points the right endpoints of the 4 approximating rectangles?

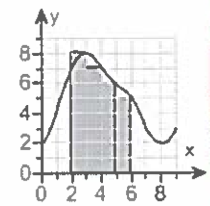
A.



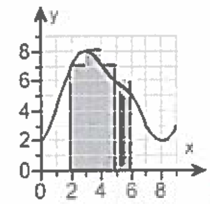
B.



C.



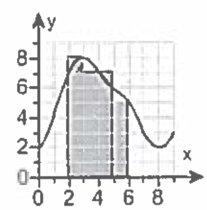
D.



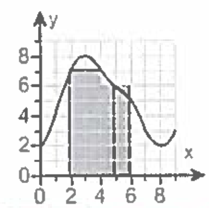
Using the specified rectangles, approximate the area.

In which graph below are the selected points the left endpoints of the 4 approximating rectangles?

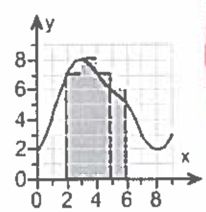
A.



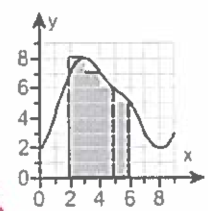
B.



C.



D.



Using the specified rectangles, approximate the area.

109. Evaluate the following expressions.

a. $\sum_{k=1}^{14} k$ b. $\sum_{k=1}^{6} (5k+1)$ c. $\sum_{k=1}^{7} k^2$ d. $\sum_{n=1}^{8} (1+n^2)$

e. $\sum_{m=1}^{3} \frac{5m+5}{7}$ f. $\sum_{j=1}^{3} (5j-6)$ g. $\sum_{k=1}^{9} k(8k+3)$ h. $\sum_{n=0}^{7} \sin\frac{n\pi}{2}$

use graphing calculator

a. $\sum_{k=1}^{14} k = \boxed{105}$ (Type an integer or a simplified fraction.)

b. $\sum_{k=1}^{6} (5k+1) = \boxed{111}$ (Type an integer or a simplified fraction.)

c. $\sum_{k=1}^{7} k^2 = \boxed{140}$ (Type an integer or a simplified fraction.)

d. $\sum_{n=1}^{8} (1+n^2) = \boxed{212}$ (Type an integer or a simplified fraction.)

e. $\sum_{m=1}^{3} \frac{5m+5}{7} = \boxed{\frac{45}{7}}$ (Type an integer or a simplified fraction.)

f. $\sum_{j=1}^{3} (5j-6) = \boxed{12}$ (Type an integer or a simplified fraction.)

g. $\sum_{k=1}^{9} k(8k+3) = \boxed{2415}$ (Type an integer or a simplified fraction.)

h. $\sum_{n=0}^{7} \sin\frac{n\pi}{2} = \boxed{0}$ (Type an integer or a simplified fraction.)

Answers 105

111

140

212

$\frac{45}{7}$

12

2415

0

110.

The functions f and g are integrable and $\int_3^7 f(x)dx = 6$, $\int_3^7 g(x)dx = 4$, and $\int_5^7 f(x)dx = 3$. Evaluate the integral below or state that there is not enough information.

$$-\int_7^3 3f(x)dx$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $-\int_7^3 3f(x)dx =$ 18 (Simplify your answer.)

B. There is not enough information to evaluate $-\int_7^3 3f(x)dx$.

Answer: A. $-\int_7^3 3f(x)dx =$ 18 (Simplify your answer.)

$-\int_7^3 3f(x)dx =$

$\int_3^7 3f(x)dx =$

$3\int_3^7 f(x)dx =$

$3(6) =$

$18 =$

111.

Evaluate $\frac{d}{dx}\int_a^x f(t)\,dt$ and $\frac{d}{dx}\int_a^b f(t)\,dt$, where a and b are constants.

$\frac{d}{dx}\int_a^x f(t)\,dt =$ f(x) (Simplify your answer.)

$\frac{d}{dx}\int_a^b f(t)\,dt =$ 0 (Simplify your answer.)

Answers f(x)

0

(112) $\int_0^1 (x^2-3x+5)\,dx$

Formula

$\int x^n dx$

$\frac{x^{n+1}}{n+1}+c$

$\int a\,dx =$

$ax+c=$

$\left(\frac{x^{2+1}}{2+1}-\frac{3x^{1+1}}{1+1}+5x\right)\Big|_0^1$

$\left(\frac{x^3}{3}-\frac{3x^2}{2}+5x\right)\Big|_0^1$

$\left(\frac{(1)^3}{3}-\frac{3(1)^2}{2}+5(1)\right)-\left(\frac{(0)^3}{3}-\frac{3(0)^2}{2}+5(0)\right)=$

$\left(\frac{1}{3}-\frac{3}{2}+\frac{5}{1}\right)-\left(\frac{0}{3}-\frac{0}{2}+0\right)=$

$\left(\frac{1}{3}\left(\frac{2}{2}\right)-\frac{3}{2}\left(\frac{3}{3}\right)+\frac{5}{1}\left(\frac{6}{6}\right)\right)-(0-0+0)=$

$\left(\frac{2}{6}-\frac{9}{6}+\frac{30}{6}\right)-(0)=$

$\left(\frac{2-9+30}{6}\right)-(0)=$

$\left(\frac{23}{6}\right)-(0)=$

$\frac{23}{6}=$

(113) $\int_{-2}^{5}(x^2-3x-10)\,dx$

formula

$\int x^n\,dx$

$\frac{x^{n+1}}{n+1}+C$

$\int a\,dx = ax+C$

$\left(\frac{x^{2+1}}{2+1}-\frac{3x^{1+1}}{1+1}-10x\right)\Big|_{-2}^{5}=$

$\left(\frac{x^3}{3}-\frac{3x^2}{2}-10x\right)\Big|_{-2}^{5}=$

$\left(\frac{(5)^3}{3}-\frac{3(5)^2}{2}-10(5)\right)-\left(\frac{(-2)^3}{3}-\frac{3(-2)^2}{2}-10(-2)\right)=$

$\left(\frac{(5)(5)(5)}{3}-\frac{3(5)(5)}{2}-10(5)\right)-\left(\frac{(-2)(-2)(-2)}{3}-\frac{3(-2)(-2)}{2}+20\right)=$

$\left(\frac{125}{3}-\frac{75}{2}-\frac{50}{1}\right)-\left(\frac{-8}{3}-\frac{12}{2}+\frac{20}{1}\right)=$

$\left(\frac{125}{3}\left(\frac{2}{2}\right)-\frac{75}{2}\left(\frac{3}{3}\right)-\frac{50}{1}\left(\frac{6}{6}\right)\right)-\left(\frac{-8}{3}\left(\frac{2}{2}\right)-\frac{12}{2}\left(\frac{3}{3}\right)+\frac{20}{1}\left(\frac{6}{6}\right)\right)$

$\left(\frac{250}{6}-\frac{225}{6}-\frac{300}{6}\right)-\left(\frac{-16}{6}-\frac{36}{6}+\frac{120}{6}\right)$

$\left(\frac{-275}{6}\right)-\left(\frac{68}{6}\right)=$

$\frac{-275}{6}-\frac{68}{6}=$

$\frac{-343}{6}=$

(114) $\int_2^3 (3x^3+6)\,dx =$

formula

$\int x^n\,dx =$

$\frac{x^{n+1}}{n+1} + C =$

$\int a\,dx =$

$ax + C =$

$\left(\frac{3x^{3+1}}{3+1} + 6x\right)\Big|_2^3 =$

$\left(\frac{3x^4}{4} + 6x\right)\Big|_2^3 =$

$\left(\frac{3(3)^4}{4} + 6(3)\right) - \left(\frac{3(2)^4}{4} + 6(2)\right) =$

$\left(\frac{3(3)(3)(3)(3)}{4} + 18\right) - \left(\frac{3(2)(2)(2)(2)}{4} + 12\right) =$

$\left(\frac{243}{4} + \frac{18}{1}\right) - \left(\frac{48}{4} + \frac{12}{1}\right) =$

$\left(\frac{243}{4} + \frac{18}{1}\left(\frac{4}{4}\right)\right) - \left(\frac{48}{4} + \frac{12}{1}\left(\frac{4}{4}\right)\right) =$

$\left(\frac{243}{4} + \frac{72}{4}\right) - \left(\frac{48}{4} + \frac{48}{4}\right)$

$\frac{315}{4} - \frac{96}{4} =$

$\frac{315-96}{4} =$ $\frac{219}{4}$

(115) $\int_1^6 (5x^3+7x)\,dx =$

$\left(\frac{5x^{3+1}}{3+1}+\frac{7x^{1+1}}{1+1}\right)\Big|_1^6 =$

$\left(\frac{5x^4}{4}+\frac{7x^2}{2}\right)\Big|_1^6 =$

$\left(\frac{5(6)^4}{4}+\frac{7(6)^2}{2}\right)-\left(\frac{5(1)^4}{4}+\frac{7(1)^2}{2}\right)=$

$\left(\frac{5(1296)}{4}+\frac{7(36)}{2}\right)-\left(\frac{5(1)}{4}+\frac{7(1)}{2}\right)=$

$\left(\frac{6480}{4}+\frac{252}{2}\right)-\left(\frac{5}{4}+\frac{7}{2}\right) =$

$\left(\frac{6480}{4}+\frac{252}{2}\left(\frac{2}{2}\right)\right)-\left(\frac{5}{4}+\frac{7}{2}\left(\frac{2}{2}\right)\right) =$

$\left(\frac{6480}{4}+\frac{504}{4}\right)-\left(\frac{5}{4}+\frac{14}{4}\right) =$

$\left(\frac{6480+504}{4}\right)-\left(\frac{5+14}{4}\right)=$

$\frac{6984}{4}-\frac{19}{4} =$

$\frac{6984-19}{4} = \frac{6965}{4}$

~~6965~~

Formula

$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C =$

$\int a\,dx = ax+C =$

(116) $\int_9^{12} (9-x)(x-12)\,dx$

$\int_9^{12} (9x - 108 - x^2 + 12x)\,dx$

$\int_9^{12} (-x^2 + 21x - 108)\,dx$

$\left(\frac{-x^{2+1}}{2+1} + \frac{21x^{1+1}}{1+1} - 108x\right)\Big|_9^{12} =$

$\left(\frac{-x^3}{3} + \frac{21x^2}{2} - 108x\right)\Big|_9^{12} =$

$\left(-\frac{(12)^3}{3} + \frac{21(12)^2}{2} - 108(12)\right) - \left(-\frac{(9)^3}{3} + \frac{21(9)^2}{2} - 108(9)\right) =$

$\left(\frac{-1728}{3} + \frac{21(144)}{2} - 1296\right) - \left(\frac{-729}{3} + \frac{21(81)}{2} - 972\right) =$

$(-576 + 1512 - 1296) - \left(-243 + \frac{1701}{2} - 972\right) =$

$\cancel{-1872}$

$(-360) - \left(-1215 + \frac{1701}{2}\right) =$

$-360 + 1215 - \frac{1701}{2} =$

$855 - \frac{1701}{2} =$

$\frac{855}{1}\left(\frac{2}{2}\right) - \frac{1701}{2} =$

$\frac{1710}{2} - \frac{1701}{2} =$

$\frac{1710 - 1701}{2} =$

$\frac{9}{2} =$

formula

$\int x^n\,dx$

$\frac{x^{n+1}}{n+1} + C$

$\int a\,dx =$

$ax + C =$

(117) $\int_{-7}^{7} (49 - x^2)\,dx$

formula

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C =$

$\int a\,dx = ax + C =$

$49x - \frac{x^{2+1}}{2+1} \Big|_{-7}^{7} =$

$\left(49x - \frac{x^3}{3}\right) \Big|_{-7}^{7} =$

$\left(49(7) - \frac{(7)^3}{3}\right) - \left(49(-7) - \frac{(-7)^3}{3}\right) =$

$\left(49(7) - \frac{(7)(7)(7)}{3}\right) - \left(49(-7) - \frac{(-7)(-7)(-7)}{3}\right) =$

$\left(343 - \frac{343}{3}\right) - \left(-343 - \frac{-343}{3}\right) =$

$\left(343 - \frac{343}{3}\right) - \left(-343 + \frac{343}{3}\right) =$

$343 - \frac{343}{3} + 343 - \frac{343}{3} =$

$686 - \frac{686}{3} =$

$\frac{686}{1}\left(\frac{3}{3}\right) - \frac{686}{3} =$

$\frac{2058}{3} - \frac{686}{3} =$

$\frac{2058 - 686}{3} =$

$\frac{1372}{3}$

118

$$= \int_1^4 (x^2 - 25)\,dx$$

since $\int_1^4 (0) - (x^2 - 25)\,dx$

$\int_1^4 0 - x^2 + 25\,dx$

$-\int_1^4 x^2 - 25\,dx$

$$-\left(\frac{x^{2+1}}{2+1} - 25x\right)\Big|_1^4 =$$

$$-\left(\frac{x^3}{3} - 25x\right)\Big|_1^4 =$$

$$-\left(\left(\frac{(4)^3}{3} - 25(4)\right) - \left(\frac{(1)^3}{3} - 25(1)\right)\right) =$$

$$-\left(\left(\frac{(4)(4)(4)}{3} - 25(4)\right) - \left(\frac{(1)(1)(1)}{3} - 25(1)\right)\right) =$$

$$-\left(\left(\frac{64}{3} - 100\right) - \left(\frac{1}{3} - 25\right)\right) =$$

$$-\left(\left(\frac{64}{3} - \frac{100}{1}\left(\frac{3}{3}\right)\right) - \left(\frac{1}{3} - \frac{25}{1}\left(\frac{3}{3}\right)\right)\right) =$$

$$-\left(\left(\frac{64}{3} - \frac{300}{3}\right) - \left(\frac{1}{3} - \frac{75}{3}\right)\right) =$$

$$-\left(\left(\frac{-236}{3}\right) - \left(\frac{-74}{3}\right)\right) =$$

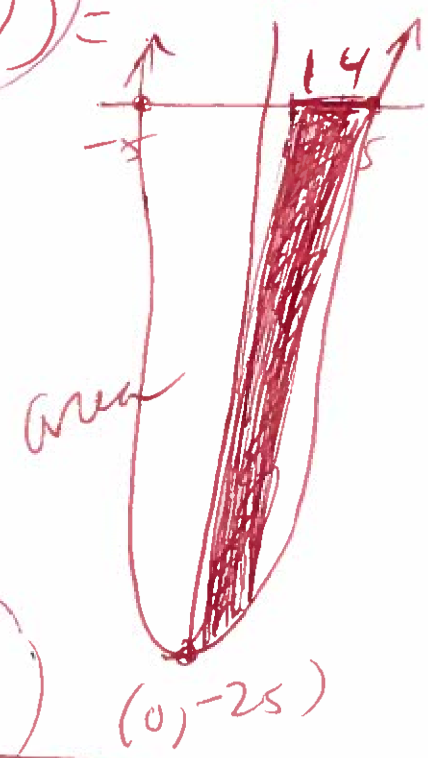
$$-\left(\frac{-236}{3} + \frac{74}{3}\right) =$$

$$-\left(\frac{-162}{3}\right) =$$

$$-(-54) =$$

$$54 =$$

$\int_1^4 f(x) - g(x)\,dx$



formula

$\int x^n\,dx =$

$\frac{x^{n+1}}{n+1} + C =$

$\int a\,dx =$

$ax + C =$

(119)

formula

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C =$$

$$\int a\,dx = ax + C =$$

$$\frac{d}{dx}\int_2^x (2t^2 + t + 8)\,dt$$

$$\frac{d}{dx}\left(\frac{2t^{2+1}}{2+1} + \frac{t^{1+1}}{1+1} + 8t\right)\Big|_2^x$$

$$\frac{d}{dx}\left(\frac{2t^3}{3} + \frac{t^2}{2} + 8t\right)\Big|_2^x$$

$$\frac{d}{dx}\left(\left(\frac{2(x)^3}{3} + \frac{(x)^2}{2} + 8(x)\right) - \left(\frac{2(2)^3}{3} + \frac{(2)^2}{2} + 8(2)\right)\right) =$$

$$\frac{d}{dx}\left(\left(\frac{2x^3}{3} + \frac{x^2}{2} + 8x\right) - \left(\frac{16}{3} + \frac{4}{2} + 16\right)\right) =$$

$$\frac{d}{dx}\left(\left(\frac{2x^3}{3} + \frac{x^2}{2} + 8x\right) - \left(\frac{16}{3}\left(\frac{2}{2}\right) + \frac{4}{2}\left(\frac{3}{3}\right) + \frac{16}{1}\left(\frac{6}{6}\right)\right)\right) =$$

$$\frac{d}{dx}\left(\left(\frac{2x^3}{3} + \frac{x^2}{2} + 8x\right) - \left(\frac{32}{6} + \frac{12}{6} + \frac{96}{6}\right)\right) =$$

$$\frac{d}{dx}\left(\left(\frac{2x^3}{3} + \frac{x^2}{2} + 8x\right) - \left(\frac{32+12+96}{6}\right)\right)$$

$$\frac{d}{dx}\left(\left(\frac{2x^3}{3} + \frac{x^2}{2} + 8x\right) - \left(\frac{140}{6}\right)\right)$$

$$\frac{d}{dx}\left(\frac{2x^3}{3} + \frac{x^2}{2} + 8x - \frac{140}{6}\right)$$

$$\frac{d}{dx}\left(\frac{2x^3}{3} + \frac{x^2}{2} + 8x - 0\right) =$$

$$\cancel{2\left(\frac{3x^2}{3}\right)} + \frac{2x}{2} + 8 - 0 =$$

$$2x^2 + x + 8$$

(120) find the average value of the following function

$f(x) = x(x-1) \qquad [3, 5]$ with $a = 3$, $b = 5$

formula

$\int x^n dx = \frac{x^{n+1}}{n+1} + C =$

$\int a\,dx = ax + C =$

$\frac{1}{b-a}\int_a^b f(x)\,dx =$

$\frac{1}{(5)-(3)}\int_3^5 x(x-1)\,dx =$

$\frac{1}{5-3}\int_3^5 (x^2-x)\,dx =$

$\frac{1}{2}\int_3^5 (x^2-x)\,dx =$

$\frac{1}{2}\left(\frac{x^{2+1}}{2+1} - \frac{x^{1+1}}{1+1}\right)\Big|_3^5 =$

$\frac{1}{2}\left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_3^5 =$

$\frac{1}{2}\left[\left(\frac{(5)^3}{3} - \frac{(5)^2}{2}\right) - \left(\frac{(3)^3}{3} - \frac{(3)^2}{2}\right)\right] =$

$\frac{1}{2}\left[\left(\frac{125}{3} - \frac{25}{2}\right) - \left(\frac{27}{3} - \frac{9}{2}\right)\right] =$

$\frac{1}{2}\left[\frac{125}{3}\left(\frac{2}{2}\right) - \frac{25}{2}\left(\frac{3}{3}\right)\right] - \left(\frac{27}{3}\left(\frac{2}{2}\right) - \frac{9}{2}\left(\frac{3}{3}\right)\right) =$

$\frac{1}{2}\left[\left(\frac{250}{6} - \frac{75}{6}\right) - \left(\frac{54}{6} - \frac{27}{6}\right)\right] =$

$\frac{1}{2}\left[\left(\frac{250-75}{6}\right) - \left(\frac{54-27}{6}\right)\right] =$

$\frac{1}{2}\left[\frac{175}{6} - \frac{27}{6}\right] =$

$\frac{1}{2}\left[\frac{175-27}{6}\right] =$

$\frac{1}{2}\left[\frac{148}{6}\right] =$

$\frac{148}{12} =$

$\frac{4(37)}{4(3)} = \frac{37}{3}$

(121) $f(x) = x^3 - 5x^2 + 2$ $\quad [0, 4]$ (a = 0, b = 4)

Find average value

$$\frac{1}{b-a}\int_a^b (f(x))\,dx$$ average value

formula

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C =$$

$$\int a\,dx = ax + C =$$

$$\frac{1}{(4)-(0)}\int_0^4 (x^3 - 5x^2 + 2)\,dx$$

$$\frac{1}{4-0}\left(\frac{x^{3+1}}{3+1} - \frac{5x^{2+1}}{2+1} + 2x\right)\Big|_0^4 =$$

$$\frac{1}{4}\left(\frac{x^4}{4} - \frac{5x^3}{3} + 2x\right)\Big|_0^4 =$$

$$\left(\frac{1}{4}\left(\frac{(4)^4}{4} - \frac{5(4)^3}{3} + 2(4)\right)\right) - \left(\frac{1}{4}\left(\frac{(0)^4}{4} - \frac{5(0)^3}{3} + 2(0)\right)\right) =$$

$$\left(\frac{1}{4}\left(\frac{256}{4} - \frac{5(64)}{3} + \frac{8}{1}\right)\right) - \left(\frac{1}{4}(0 - 0 + 0)\right) =$$

$$\left(\frac{1}{4}\left(64 - \frac{5(64)}{3} + 8\right)\right) - \left(\frac{1}{4}(0)\right)$$

$$\left(\frac{1}{4}\left(72 - \frac{320}{3}\right)\right) - (0)$$

$$\left(\frac{1}{4}\left(\frac{72}{1}\left(\frac{3}{3}\right) - \frac{320}{3}\right)\right) - (0)$$

$$\left(\frac{1}{4}\left(\frac{216}{3} - \frac{320}{3}\right)\right) - (0)$$

$$\left(\frac{1}{4}\left(\frac{-104}{3}\right)\right) - (0)$$

$$\frac{1}{\cancel{4}}\left(\frac{\cancel{-104}}{3}\right) =$$

$$\left(\frac{1}{1}\right)\cdot\left(\frac{-26}{3}\right) =$$

$$-\frac{26}{3} =$$

(122) find the points at which the function $f(x) = 7 - 8x$ equals the average value on the interval $[0, 8]$.

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$ average value

$$\frac{1}{(8)-(0)}\int_0^8 (7-8x)\,dx =$$

$$\frac{1}{8-0}\int_0^8 (7-8x)\,dx =$$

$$\frac{1}{8}\left(7x - \frac{8x^2}{2}\right)\Big|_0^8$$

$$\frac{1}{8}(7x - 4x^2)\Big|_0^8$$

$$\left(\frac{1}{8}(7(8) - 4(8)^2)\right) - \left(\frac{1}{8}(7(0) - 4(0)^2)\right) =$$

$$\left(\frac{1}{8}(56 - 4(64))\right) - \left(\frac{1}{8}(0 - 0)\right) =$$

$$\left(\frac{1}{8}(56 - 256)\right) - (0)$$

$$\left(\frac{1}{8}(-200)\right) =$$

$$\frac{-200}{8} =$$

$-25 =$ average value $[0, 8]$

Set average value

$$7 - 8x = -25$$

$$\cancel{7} - 8x - \cancel{7} = -25 - 7$$

$$-8x = -32$$

$$\frac{-8x}{-8} = \frac{-32}{-8}$$

$$x = 4$$

formula

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C =$$

$$\int a\,dx = ax + C =$$

(123) $\int 2x(x^2+11)^4\,dx =$

$\int (x^2+11)^4 (2x)\,dx =$

$\dfrac{(x^2+11)^{4+1}}{4+1} + C =$

$\dfrac{(x^2+11)^5}{5} + C =$

OR

$\frac{1}{5}(x^2+11)^5 + C =$

formula

$\int (f(x))^N \cdot f'(x)\,dx =$

$\dfrac{(f(x))^{N+1}}{N+1} + C$

124 $\int -14x \sin(7x^2-3)\,dx =$

$\int -\sin(7x^2-3)(14x)\,dx =$

$\cos(7x^2-3) + C =$

formula

$\int -\sin(f(x)) \cdot f'(x)\,dx =$

$\cos(f(x)) + C =$

125

$$\int (4x+4)\sqrt{2x^2+4x}\,dx =$$

$$\int \sqrt{2x^2+4x}\,(4x+4)\,dx =$$

$$\int (2x^2+4x)^{1/2}(4x+4)\,dx =$$

$$\frac{(2x^2+4x)^{1/2+1}}{\frac{1}{2}+1} + C =$$

$$\frac{(2x^2+4x)^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C =$$

$$\frac{(2x^2+4x)^{\frac{1+2}{2}}}{\frac{1+2}{2}} + C =$$

$$\frac{(2x^2+4x)^{3/2}}{3/2} + C =$$

$$\frac{2}{3}(2x^2+4x)^{3/2} + C =$$

Formula

$$\int (f(x))^N \cdot f'(x)\,dx =$$

$$\frac{(f(x))^{N+1}}{N+1} + C =$$

126

$$\int e^{9x-4}\,dx =$$

$$\frac{1}{9}\int e^{9x-4}(9)\,dx =$$

$$\frac{1}{9}e^{9x-4} + C =$$

$$\int e^{f(x)} \cdot f'(x)\,dx =$$

$$e^{f(x)} + C =$$

127

$$\int x^{10} e^{x^{11}} dx =$$

$$\int e^{x^{11}} (x^{10}) dx =$$

$$\frac{1}{11} \int e^{x^{11}} (11x^{10}) =$$

$$\frac{1}{11} e^{x^{11}} + C =$$

~~formula~~

$$\int e^{f(x)} \cdot f'(x) dx =$$

$$e^{f(x)} + C =$$

128

$$\int (x^5+x^2)^{15}(5x^4+2x)\,dx =$$

$$\frac{(x^5+x^2)^{15+1}}{15+1}+C=$$

$$\frac{(x^5+x^2)^{16}}{16}+C=$$

formula

$$\int (f(x))^N \cdot f'(x)\,dx =$$

$$\frac{(f(x))^{N+1}}{N+1}+C=$$

(129) $\int \frac{e^{2x}}{e^{2x}+2}\,dx =$

Formula

$\int \frac{f'(x)}{f(x)}\,dx =$

$\ln|f(x)| + C =$

$\frac{1}{2}\int \frac{e^{2x}(2)}{e^{2x}+2}\,dx =$

$\frac{1}{2}\ln|e^{2x}+2| + C =$

(12)

$$\int_0^{\frac{\pi}{30}} \cos(5x)\,dx =$$

formula

$$\int \cos(f(x)) \cdot f'(x)\,dx = \sin(f(x)) + C =$$

$$\frac{1}{5}\int_0^{\frac{\pi}{30}} \cos(5x)(5)\,dx =$$

$$\left(\frac{1}{5}\sin(5x)\right)\Big/_0^{\frac{\pi}{30}} =$$

$$\frac{1}{5}\sin\left(5\left(\frac{\pi}{30}\right)\right) - \frac{1}{5}\sin(5(0)) =$$

$$\frac{1}{5}\sin\left(\frac{\pi}{6}\right) - \frac{1}{5}\sin(0) =$$

$$\frac{1}{5}\left(\frac{1}{2}\right) - \frac{1}{5}(0) =$$

$$\frac{1}{10} - 0 =$$

$$\frac{1}{10} =$$

(131) $\int_0^3 4e^{2x}\,dx =$

$4\int_0^3 e^{2x}\,dx =$

$(4)\left(\frac{1}{2}\right)\int_0^3 e^{2x}(2)\,dx =$

$\frac{4}{2}\int_0^3 e^{2x}(2)\,dx =$

$2\int_0^3 e^{2x}(2)\,dx =$

$2e^{2x}\Big|_0^3 =$

$2e^{2(3)} - 2e^{2(0)} =$

$2e^{6} - 2e^{0} =$

$2e^{6} - 2(1) =$

$2e^{6} - 2 =$

$\int e^{f(x)} f'(x)\,dx =$

$e^{f(x)} + C =$

132

$$\int_0^1 7x^6(8-x^7)\,dx$$

$$\int_0^1 (8-x^7)(7x^6)\,dx$$

$$-\int_0^1 (8-x^7)(-7x^6)\,dx$$

$$-\frac{(8-x^7)^{1+1}}{1+1}\Big|_0^1 =$$

$$-\frac{(8-x^7)^2}{2}\Big|_0^1 =$$

$$\left(-\frac{(8-(1)^7)^2}{2}\right)-\left(-\frac{(8-(0)^7)^2}{2}\right)=$$

$$\left(-\frac{(8-1)^2}{2}\right)-\left(-\frac{(8-0)^2}{2}\right)=$$

$$\left(-\frac{(7)^2}{2}\right)-\left(-\frac{(8)^2}{2}\right)=$$

$$\left(-\frac{49}{2}\right)-\left(-\frac{64}{2}\right)=$$

$$-\frac{49}{2}+\frac{64}{2}=$$

$$\frac{15}{2}$$

formula

$$\int (f(x))^N \cdot f'(x)\,dx = \frac{(f(x))^{N+1}}{N+1}+C$$

133 $\int_0^1 \frac{2x}{(x^2+1)^3}\,dx =$

$\int_0^1 (x^2+1)^{-3}(2x)\,dx =$

$\frac{(x^2+1)^{-3+1}}{-3+1}\Big/_0^1 =$

$\frac{(x^2+1)^{-2}}{-2}\Big/_0^1 =$

$\frac{1}{-2(x^2+1)^2}\Big/_0^1$

$\left(\frac{1}{-2((1)^2+1)^2}\right) - \left(\frac{1}{-2((0)^2+1)^2}\right) =$

$\left(\frac{1}{-2(1+1)^2}\right) - \left(\frac{1}{-2(0+1)^2}\right) =$

$\left(\frac{1}{-2(2)^2}\right) - \left(\frac{1}{-2(1)^2}\right) =$

$\left(\frac{1}{-2(2)(2)}\right) - \left(\frac{1}{-2(1)(1)}\right) =$

$\left(\frac{1}{-8}\right) - \left(\frac{1}{-2}\right) =$

formula

$\int (f(x))^N \cdot f'(x)\,dx = \frac{(f(x))^{N+1}}{N+1} + C =$

$\frac{1}{-8} + \frac{1}{2} =$

$\frac{1}{-8} + \frac{1}{2}\left(\frac{4}{4}\right) =$

$-\frac{1}{8} + \frac{4}{8} =$

$\frac{-1+4}{8} =$

$\frac{3}{8} =$

(134)

$$\int_0^{\frac{\pi}{4}} 2\tan(x)\sec^2(x)\,dx =$$

$$2\int_0^{\frac{\pi}{4}} (\tan(x))^1 \sec^2(x)\,dx =$$

$$2\,\frac{(\tan(x))^{1+1}}{1+1}\Big/_0^{\frac{\pi}{4}} =$$

$$\cancel{2}\,\frac{(\tan(x))^2}{\cancel{2}}\Big/_0^{\frac{\pi}{4}} =$$

$$(\tan(x))^2\Big/_0^{\frac{\pi}{4}} =$$

$$\left(\tan\left(\tfrac{\pi}{4}\right)\right)^2 - (\tan(0))^2 =$$

$$(1)^2 - (0)^2 =$$

$$(1)(1) - (0)(0) =$$

$$1 - 0 =$$

$$1 =$$

Formula

$$\int (f(x))^N \cdot f'(x)\,dx = \frac{(f(x))^{N+1}}{N+1} + C =$$

135 $\int_0^3 \frac{x^2}{x^3+7}\,dx =$

For mula

$\int \frac{f'(x)}{f(x)}\,dx =$

$\ln|f(x)| + c =$

$\frac{1}{3}\int_0^3 \frac{3x^2}{x^3+7}\,dx =$

$\frac{1}{3}\ln|x^3+7|\Big|_0^3 =$

$\frac{1}{3}\ln|(3)^3+7| - \frac{1}{3}\ln|(0)^3+7| =$

$\frac{1}{3}\ln|27+7| - \frac{1}{3}\ln|0+7| =$

$\frac{1}{3}\ln|34| - \frac{1}{3}\ln|7| =$

$\frac{1}{3}\left(\ln|34| - \ln|7|\right) =$

$\frac{1}{3}\left(\ln\left|\frac{34}{7}\right|\right) =$

$\frac{1}{3}\ln\left(\frac{34}{7}\right) =$

(136) $\int \frac{dx}{x^2-2x+17}$

$\int \frac{dx}{x^2-2x+17+\left(\frac{1}{2}(-2)\right)^2-\left(\frac{1}{2}(-2)\right)^2} =$ complete square

$\int \frac{dx}{x^2-2x+17+(-1)^2-(-1)^2} =$

$\int \frac{dx}{x^2-2x+17+1-(1)} =$

$\int \frac{dx}{x^2-2x+1+17-1} =$ rewrite

$\int \frac{dx}{(x-1)^2+16} =$

formula

$\int \frac{f'(x)}{(a)^2+(f(x))^2}\,dx =$

$\frac{1}{a}\tan^{-1}\left(\frac{f(x)}{a}\right)+C =$

$\int \frac{dx}{16+(x-1)^2} =$

$\int \frac{dx}{(4)^2+(x-1)^2} =$

$\frac{1}{4}\tan^{-1}\left(\frac{x-1}{4}\right)+C =$

(137) Use integration by parts

$$\int 13t \cdot e^t \, dt$$

	D	I
+	$13t$	e^t
−	13	e^t

formula

$$\int e^{f(x)} f'(x)\,dx$$

$$e^{f(x)} + C$$

$$13te^t - \int 13e^t \, dt =$$

$$13te^t - 13e^t + C =$$

(138) Use integration by parts

$$\int 16x \cdot \ln(8x) \, dx$$

	D	I
+	$\ln(8x)$	$16x$
−	$\frac{1}{x}$	$\frac{16x^2}{2}$ or $8x^2$

formula

$$\int x^n dx$$

$$\frac{x^{n+1}}{n+1} + C$$

$$\ln(8x) \cdot (8x^2) - \int \frac{1}{x}(8x^2)\,dx =$$

$$8x^2 \ln(8x) - \int \frac{8x^2}{x} dx =$$

$$8x^2 \ln(8x) - \int 8x \, dx =$$

$$8x^2 \ln(8x) - \frac{8x^2}{2} + C =$$

$$8x^2 \ln(8x) - 4x^2 + C =$$

$$\boxed{8x^2 \ln(8x) - 4x^2 + C}$$

(139) If the general solution of a differential equation is $y(t) = Ce^{-5t} + 4$, what is the solution that satisfies the initial condition

$y(0) = 7$

$$y(t) = Ce^{-5t} + 4$$

$$y(0) = Ce^{-5(0)} + 4 = 7$$

$$Ce^{0} + 4 = 7$$

$$C(1) + 4 = 7$$

$$C + 4 = 7$$

$$C + \cancel{4} - \cancel{4} = 7 - 4$$

$$C = 3$$

$$y(t) = 3e^{-5t} + 4$$