calmath241314sPSLp 11-24-18 11-28-18 11-29-18 11-30-18 12-04-18 12-08-18

Student: ____________	**Instructor:** Alfredo Alvarez	**Assignment:**
Date: ____________	**Course:** 2413 Cal I	calmath2413alvarez145nextp

1. The function s(t) represents the position of an object at time t moving along a line. Suppose s(1) = 114 and s(6) = 254. Find the average velocity of the object over the interval of time [1,6].

The average velocity over the interval [1,6] is v_{av} = [ ]. (Simplify your answer.)

Answer: 28

$$\frac{s(6)-s(1)}{6-1} = \frac{254-114}{6-1} = \frac{140}{5} = 28$$

2. The position of an object moving along a line is given by the function $s(t) = -8t^2 + 80t$. Find the average velocity of the object over the following intervals.

(a) [1, 4]
(b) [1, 3]
(c) [1, 2]
(d) [1, 1 + h] where h > 0 is any real number.

(a) The average velocity of the object over the interval [1, 4] is 40.

(b) The average velocity of the object over the interval [1, 3] is 48.

(c) The average velocity of the object over the interval [1, 2] is 56.

(d) The average velocity of the object over the interval [1, 1 + h] is $-8h+64$.

$s(t) = -8t^2 + 80t$

Answers 40

48

56

$-8h+64$

$$\frac{s(4)-s(1)}{4-1} = 40$$

$$\frac{s(3)-s(1)}{3-1} = 48$$

$$\frac{s(2)-s(1)}{2-1} = 56$$

② ① $s(t) = -8t^2 + 80t$

$$\frac{s(1+h) - s(1)}{(1+h)-(1)} =$$

$$\frac{\left(-8(1+h)^2 + 80(1+h)\right) - \left(-8(1)^2 + 80(1)\right)}{(1+h)-(1)} =$$

$$\frac{\left(-8(1+h)(1+h) + 80 + 80h\right) - \left(-8(1)(1) + 80(1)\right)}{(1+h)-(1)} =$$

$$\frac{\left(-8(1 + 1h + 1h + h^2) + 80 + 80h\right) - (-8 + 80)}{1+h-1} =$$

$$\frac{\left(-8(1+2h+h^2) + 80 + 80h\right) - (72)}{h} =$$

$$\frac{\left(-8 - 16h - 8h^2 + 80 + 80h\right) - (72)}{h} =$$

$$\frac{\left(-8h^2 + 72 + 64h\right) - (72)}{h} =$$

$$\frac{-8h^2 + \cancel{72} + 64h - \cancel{72}}{h} =$$

$$\frac{-8h^2 + 64h}{h} =$$

$$\frac{\cancel{h}(-8h + 64)}{\cancel{h}} =$$

$-8h + 64 =$

3. For the position function $s(t) = -16t^2 + 102t$, complete the following table with the appropriate average velocities. Then make a conjecture about the value of the instantaneous velocity at $t = 1$.

Time Interval	[1, 2]	[1, 1.5]	[1, 1.1]	[1, 1.01]	[1, 1.001]
Average Velocity	___	___	___	___	___

Complete the following table.

Time Interval	[1, 2]	[1, 1.5]	[1, 1.1]	[1, 1.01]	[1, 1.001]
Average Velocity	54	62	68.4	69.84	69.984

(Type exact answers. Type integers or decimals.)

The value of the instantaneous velocity at t = 1 is 70.
(Round to the nearest integer as needed.)

$s(t) = -16t^2 + 102t$

Answers 54

62

68.4

69.84

69.984

70

$$\frac{s(2) - s(1)}{(2) - (1)} = 54$$

$$\frac{s(1.5) - s(1)}{(1.5) - (1)} = 62$$

$$\frac{s(1.1) - s(1)}{(1.1) - (1)} = 68.4$$

$$\frac{s(1.01) - s(1)}{(1.1) - (1)} = 69.84$$

$$\frac{s(1.001) - s(1)}{(1.001) - (1)} = 69.984$$

4. For the function $f(x) = 8x^3 - x$, make a table of slopes of secant lines and make a conjecture about the slope of the tangent line at $x = 1$.

Complete the table.

(Round the final answer to three decimal places as needed. Round all intermediate values to four decimal places as needed.)

Interval	Slope of secant line
[1, 2]	55.000
[1, 1.5]	37.000
[1, 1.1]	25.480
[1, 1.01]	23.240
[1, 1.001]	23.010

An accurate conjecture for the slope of the tangent line at $x = 1$ is 23.
(Round to the nearest integer as needed.)

$f(x) = 8x^3 - x$

Answers 55.000

37.000

25.480

23.240

23.000

23

$$\frac{f(2) - f(1)}{(2) - (1)} = 55.000$$

$$\frac{f(1.5) - f(1)}{(1.5) - (1)} = 37.000$$

$$\frac{f(1.1) - f(1)}{(1.1) - (1)} = 25.480$$

$$\frac{f(1.01) - f(1)}{(1.01) - (1)} = 23.240$$

$$\frac{f(1.001) - f(1)}{(1.001) - (1)} = 23.000$$

5.

Let $f(x) = \frac{x^2-4}{x+2}$. **(a)** Calculate f(x) for each value of x in the following table. **(b)** Make a conjecture about the value of $\lim_{x \to -2} \frac{x^2-4}{x+2}$.

(a) Calculate f(x) for each value of x in the following table.

x	−1.9	−1.99	−1.999	−1.9999
$f(x) = \frac{x^2-4}{x+2}$	−3.9	−3.99	−3.999	−3.9999
x	−2.1	−2.01	−2.001	−2.0001
$f(x) = \frac{x^2-4}{x+2}$	−4.1	−4.01	−4.001	−4.0001

(Type an integer or decimal rounded to four decimal places as needed.)

(b) Make a conjecture about the value of $\lim_{x \to -2} \frac{x^2-4}{x+2}$.

$\lim_{x \to -2} \frac{x^2-4}{x+2} =$ −4 (Type an integer or a decimal.)

$f(x) = \frac{x^2-4}{x+2}$

OR

$\lim_{x \to -2} \frac{x^2-4}{x+2} =$

$\lim_{x \to -2} \frac{(x^2-(2)^2}{x+2} =$

$\lim_{x \to -2} \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}} =$

$\lim_{x \to -2} (x-2) =$

$-2-2 =$

-4

Answers −3.9

−3.99

−3.999

−3.9999

−4.1

−4.01

−4.001

−4.0001

−4

$f(-1.9) = \frac{(-1.9)^2-4}{(-1.9)+2} = -3.9$

$f(-1.99) = \frac{(-1.99)^2+4}{(-1.99)+2} = -3.99$

$f(-1.999) = \frac{(-1.999)^2+4}{(-1.999)+2} = -3.999$

$f(-1.9999) = \frac{(-1.9999)^2+4}{(-1.9999)+2} = -3.9999$

$f(-2.1) = \frac{(-2.1)^2-4}{(-2.1)+2} = -4.1$

$f(-2.01) = \frac{(-2.01)^2-4}{(-2.01)+2} = -4.01$

$f(-2.001) = \frac{(-2.001)^2-4}{(-2.001)+2} = -4.001$

$f(-2.0001) = \frac{(-2.0001)^2-4}{(-2.0001)+2} = -4.0001$

6. Let $g(t) = \frac{t-25}{\sqrt{t}-5}$.

OR

a. Make two tables, one showing the values of g for t = 24.9, 24.99, and 24.999 and one showing values of g for t = 25.1, 25.01, and 25.001.

b. Make a conjecture about the value of $\lim_{t \to 25} \frac{t-25}{\sqrt{t}-5}$.

$\lim_{t \to 25} \frac{t-25}{\sqrt{t}-5} =$

$\lim_{t \to 25} \left(\frac{t-25}{\sqrt{t}-5}\right)\left(\frac{\sqrt{t}+5}{\sqrt{t}+5}\right) =$

$\lim_{t \to 25} \frac{(t-25)(\sqrt{t}+5)}{(\sqrt{t})^2+5\cancel{\sqrt{t}}-\cancel{5\sqrt{t}}-25}$

$\lim_{t \to 25} \frac{\cancel{(t-25)}(\sqrt{t}+5)}{\cancel{(t-25)}}$

$\lim_{t \to 25} (\sqrt{t}+5) =$

$\sqrt{25}+5 =$

$5+5 =$

$10 =$

a. Make a table showing the values of g for t = 24.9, 24.99, and 24.999.

t	24.9	24.99	24.999
g(t)	9.9900	9.9990	9.9999

(Round to four decimal places.)

Make a table showing the values of g for t = 25.1, 25.01, and 25.001.

t	25.1	25.01	25.001
g(t)	10.0100	10.0010	10.0001

(Round to four decimal places.)

b. Make a conjecture about the value of $\lim_{t \to 25} \frac{t-25}{\sqrt{t}-5}$. Select the correct choice below and fill in any answer boxes in your choice.

○ **A.** $\lim_{t \to 25} \frac{t-25}{\sqrt{t}-5} =$ 10 (Simplify your answer.)

○ **B.** The limit does not exist.

$g(t) = \frac{t-25}{\sqrt{t}-5}$

Answers 9.9900

9.9990

9.9999

10.0100

10.0010

10.0001

A. $\lim_{t \to 25} \frac{t-25}{\sqrt{t}-5} =$ **10** (Simplify your answer.)

$g(24.9) = \frac{(24.9)-25}{\sqrt{(24.9)}-5} = 9.9900$

$g(24.99) = \frac{(24.99)-25}{\sqrt{(24.99)}-5} = 9.9990$

$g(24.999) = \frac{(24.999)-25}{\sqrt{24.999}-5} = 9.999$

$g(25.1) = \frac{(25.1)-25}{\sqrt{25.1}-5} = 10.0100$

$g(25.01) = \frac{(25.01)-25}{\sqrt{25.01}-5} = 10.0010$

$g(25.001) = \frac{25.001-25}{\sqrt{25.001}-5} = 10.001$

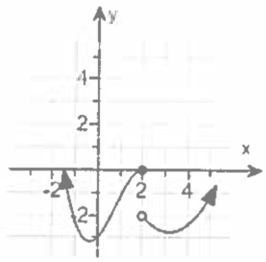
7. Use the graph to find the following limits and function value.

a. $\lim_{x \to 2^-} f(x)$

b. $\lim_{x \to 2^+} f(x)$

c. $\lim_{x \to 2} f(x)$

d. f(2)



a. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 2^-} f(x) =$ 0 (Type an integer.)

B. The limit does not exist.

b. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 2^+} f(x) =$ -2 (Type an integer.)

B. The limit does not exist.

c. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 2} f(x) =$ ______ (Type an integer.)

B. The limit does not exist.

d. Find the function value. Select the correct choice below and fill in any answer boxes in your choice.

A. f(2) = 0 (Type an integer.)

B. The answer is undefined.

Answers A. $\lim_{x \to 2^-} f(x) =$ 0 (Type an integer.)

A. $\lim_{x \to 2^+} f(x) =$ -2 (Type an integer.)

B. The limit does not exist.

A. f(2) = 0 (Type an integer.)

8.

Explain why $\lim_{x \to 7} \frac{x^2 - 10x + 21}{x - 7} = \lim_{x \to 7} (x - 3)$, and then evaluate $\lim_{x \to 7} \frac{x^2 - 10x + 21}{x - 7}$.

Choose the correct answer below.

- A. Since each limit approaches 7, it follows that the limits are equal.
- B. The numerator of the expression $\frac{x^2 - 10x + 21}{x - 7}$ simplifies to $x - 3$ for all x, so the limits are equal.
- C. The limits $\lim_{x \to 7} \frac{x^2 - 10x + 21}{x - 7}$ and $\lim_{x \to 7} (x - 3)$ equal the same number when evaluated using direct substitution.
- D. Since $\frac{x^2 - 10x + 21}{x - 7} = x - 3$ whenever $x \neq 7$, it follows that the two expressions evaluate to the same number as x approaches 7.

Now evaluate the limit.

$\lim_{x \to 7} \frac{x^2 - 10x + 21}{x - 7} =$ 4 (Simplify your answer.)

Answers D.

Since $\frac{x^2 - 10x + 21}{x - 7} = x - 3$ whenever $x \neq 7$, it follows that the two expressions evaluate to the same number as x approaches 7.

4

$$\lim_{x \to 7} \frac{x^2 - 10x + 21}{x - 7} =$$

$$\lim_{x \to 7} \frac{(x-3)\cancel{(x-7)}}{\cancel{(x-7)}} =$$

$$\lim_{x \to 7} (x-3) =$$

$$7 - 4 =$$

$$4 =$$

9. Assume $\lim_{x\to 2} f(x) = 20$ and $\lim_{x\to 2} h(x) = 2$. Compute the following limit and state the limit laws used to justify the computation.

$$\lim_{x\to 2} \frac{f(x)}{h(x)}$$

$\lim_{x\to 2} \frac{f(x)}{h(x)} =$ ☐ (Simplify your answer.)

Select each limit law used to justify the computation.

- ☐ A. Difference
- ☐ B. Root
- ☑ C. Quotient
- ☐ D. Product
- ☐ E. Power
- ☐ F. Sum
- ☐ G. Constant multiple

Answers 10

C. Quotient

$\lim_{x\to 2} f(x) = 20$ $\lim_{x\to 2} h(x) = 2$

$\lim_{x\to 2} \frac{f(x)}{h(x)} = \frac{\lim_{x\to 2} f(x)}{\lim_{x\to 2} h(x)} = \frac{20}{2} = 10 =$

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10. Find the following limit or state that it does not exist.

$$\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25}$$

Simplify the given limit.

$\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25} = \lim_{x\to 25}$ $\boxed{\frac{1}{\sqrt{x}+5}}$ (Simplify your answer.)

Evaluate the limit, if possible. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

✓ A. $\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25} = \frac{1}{10}$ (Type an exact answer.)

○ B. The limit does not exist.

Answers $\frac{1}{\sqrt{x}+5}$

A. $\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25} = \boxed{\frac{1}{10}}$ (Type an exact answer.)

$\lim_{x\to 25} \frac{(\sqrt{x}-5)}{(x-25)} \cdot \frac{(\sqrt{x}+5)}{(\sqrt{x}+5)} =$ Mult

$\lim_{x\to 25} \frac{(\sqrt{x})^2 + \cancel{5\sqrt{x}} - \cancel{5\sqrt{x}} - 25}{(x-25)(\sqrt{x}+5)} =$

$\lim_{x\to 25} \frac{(\sqrt{x})^2 - 25}{(x-25)(\sqrt{x}+5)} =$

$\lim_{x\to 25} \frac{\cancel{(x-25)}}{\cancel{(x-25)}(\sqrt{x}+5)} =$

$\lim_{x\to 25} \frac{1}{\sqrt{x}+5} =$

$\frac{1}{\sqrt{25}+5} =$

$\frac{1}{5+5} =$

$\frac{1}{10} =$

11. Determine the following limit at infinity.

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} \text{ if } f(x)\to 200{,}000 \text{ and } g(x)\to\infty \text{ as } x\to\infty$$

Find the limit. Choose the correct answer below.

- A. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$
- B. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 200{,}000$
- C. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{1}{200{,}000}$
- D. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$

Answer: D. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$

$\lim_{x\to\infty} f(x) = 200{,}000$ $\lim_{x\to\infty} (g(x)) = \infty$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} =$$

$$\frac{\lim_{x\to\infty} f(x)}{\lim_{x\to\infty} g(x)} =$$

$$\frac{200{,}000}{\infty} =$$

$$0 =$$

12. Determine the following limit at infinity.

$$\lim_{x\to\infty} \frac{8+2x+6x^3}{x^3}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $\lim_{x\to\infty} \frac{8+2x+6x^3}{x^3} =$ ______
- B. The limit does not exist and is neither $-\infty$ nor ∞.

Answer: A. $\lim_{x\to\infty} \frac{8+2x+6x^3}{x^3} = \boxed{6}$

$$\lim_{x\to\infty} \left(\frac{8+2x+6x^3}{x^3}\right) \frac{\frac{1}{x^3}}{\frac{1}{x^3}} =$$

$$\lim_{x\to\infty} \frac{\frac{8}{x^3}+\frac{2x}{x^3}+\frac{6x^3}{x^3}}{\frac{x^3}{x^3}} =$$

$$\lim_{x\to\infty} \frac{\frac{8}{x^3}+\frac{2}{x^2}+6}{1} =$$

$$\frac{0+0+6}{1} =$$

$$\frac{6}{1} =$$

$$6 =$$

(13) $\lim_{w \to \infty} \frac{6w^2+3w+5}{\sqrt{4w^4+2w^3}} =$

$\lim_{w \to \infty} \left(\frac{6w^2+3w+5}{\sqrt{4w^4+2w^3}}\right)\left(\frac{\sqrt{\frac{1}{w^4}}}{\sqrt{\frac{1}{w^4}}}\right) =$ Mult

$\lim_{w \to \infty} \frac{(6w^2+3w+5)\left(\frac{1}{w^2}\right)}{\left(\sqrt{4w^4+2w^3}\right)\sqrt{\frac{1}{w^4}}} =$

$\lim_{w \to \infty} \frac{\frac{6w^2}{w^2}+\frac{3w}{w^2}+\frac{5}{w^2}}{\sqrt{\frac{4w^4}{w^4}+\frac{2w^3}{w^4}}} =$

formula

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to \infty} \frac{1}{x^n} = 0$

$\lim_{w \to \infty} \frac{6+\frac{3}{w}+\frac{5}{w^2}}{\sqrt{4+\frac{2}{w}}} =$

$\frac{6+0+0}{\sqrt{4+0}} =$

$\frac{6}{\sqrt{4}} =$

$\frac{6}{2} =$

$3 =$

(14) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{36x^2+x}}{x} =$

$\lim\limits_{x \to -\infty} \dfrac{\sqrt{36x^2+x}}{x} \cdot \dfrac{\sqrt{\frac{1}{x^2}}}{\sqrt{\frac{1}{x^2}}} =$

$\lim\limits_{x \to -\infty} \dfrac{\sqrt{36x^2+x}}{x} \cdot \dfrac{\sqrt{\frac{1}{x^2}}}{\frac{1}{x}} =$

$\lim\limits_{x \to -\infty} \dfrac{\sqrt{\frac{36x^2}{x^2} + \frac{x}{x^2}}}{\frac{x}{x}} =$

$\lim\limits_{x \to -\infty} \dfrac{\sqrt{36 + \frac{1}{x}}}{1} =$

$- \dfrac{\sqrt{36+0}}{1} =$

$- \sqrt{36} =$

$-6 =$

15 $\lim_{x\to\infty} \frac{5x}{35x+1} =$

$\lim_{x\to\infty} \left(\frac{5x}{35x+1}\right) \frac{\frac{1}{x}}{\frac{1}{x}} =$ mult

$\lim_{x\to\infty} \frac{\frac{5x}{x}}{\frac{35x}{x}+\frac{1}{x}} =$

$\lim_{x\to\infty} \frac{5}{35+\frac{1}{x}} =$

$\frac{5}{35+0} =$

$\frac{5}{35} =$

$\frac{\cancel{5}(1)}{\cancel{5}(7)} =$

$\frac{1}{7} =$

formula

$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to\infty} \frac{1}{x^n} = 0$

The function has one horizontal asymptote

$y = \frac{1}{7}$

(16) $\lim_{x\to\infty} \dfrac{4x^2-9x+8}{2x^2+1} =$

$\lim_{x\to\infty} \left(\dfrac{4x^2-9x+8}{2x^2+1}\right) \dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} =$ mult

Formula

$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to\infty} \frac{1}{x^n} = 0$

$\lim_{x\to\infty} \dfrac{\frac{4\cancel{x^2}}{\cancel{x^2}} - \frac{9x}{x^2} + \frac{8}{x^2}}{\frac{2\cancel{x^2}}{\cancel{x^2}} + \frac{1}{x^2}} =$

$\lim_{x\to\infty} \dfrac{4 - \frac{9}{x} + \frac{8}{x^2}}{2 + \frac{1}{x^2}} =$

$\dfrac{4-0+0}{2+0} =$

$\dfrac{4}{2} =$

$2 =$

The horizontal asymptote is

$y = 2$

17. $\lim_{x\to\infty} \dfrac{28x^8-5}{4x^8-7x^7} =$

$\lim_{x\to\infty} \left(\dfrac{28x^8-5}{4x^8-7x^7}\right)\dfrac{\frac{1}{x^8}}{\frac{1}{x^8}} =$ mult

$\lim_{x\to\infty} \dfrac{\frac{28x^8}{x^8}-\frac{5}{x^8}}{\frac{4x^8}{x^8}-\frac{7x^7}{x^8}} =$

$\lim_{x\to\infty} \dfrac{28-\frac{5}{x^8}}{4-\frac{7}{x}} =$

formula

$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to\infty} \frac{1}{x^n} = 0$

$\dfrac{28-0}{4-0} =$

$\dfrac{28}{4} =$

$7 =$

The function has a horizontal asymptote at $y=7$

(18.) $\lim_{x\to\infty} \frac{2x^5-9}{x^6+4x^4} =$

$\lim_{x\to\infty} \left(\frac{2x^5-9}{x^6+4x^4}\right)\left(\frac{\frac{1}{x^6}}{\frac{1}{x^6}}\right) =$ Mult

$\lim_{x\to\infty} \left(\frac{\frac{2x^5}{x^6} - \frac{9}{x^6}}{\frac{x^6}{x^6} + \frac{4x^4}{x^6}}\right) =$

formula

$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to\infty} \frac{1}{x^n} = 0$

$\lim_{x\to\infty} \frac{\frac{2}{x} - \frac{9}{x^6}}{1 - \frac{4}{x^2}} =$

$\frac{0-0}{1-0} =$

$\frac{0}{1} =$

$0 =$

The function has an horizontal asymptote

$y=0$

(19) find all asymptotes

$$f(x) = \frac{8x^2+16}{2x^2+9x-5}$$

$$\lim_{x\to\infty} \frac{8x^2+16}{2x^2+9x-5} =$$

$$\lim_{x\to\infty} \frac{(8x^2+16)}{(2x^2+9x-5)} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} =$$ mult

$$\lim_{x\to\infty} \frac{\frac{8\cancel{x^2}}{\cancel{x^2}} + \frac{16}{x^2}}{\frac{2\cancel{x^2}}{\cancel{x^2}} + \frac{9x}{x^2} - \frac{5}{x^2}} =$$

$$\lim_{x\to\infty} \frac{8+\frac{16}{x^2}}{2+\frac{9}{x}-\frac{5}{x^2}} =$$

$$\frac{8+0}{2+0-0} =$$

$$\frac{8}{2} =$$

$$4 =$$

Since Powers are Same

$$\frac{8x^2}{2x^2}$$ TOP Bottom

NO Slant asymptotes

The function has one horizontal asymptote $y=4$

find vertical asymptotes

$$f(x) = \frac{8x^2+16}{2x^2+9x-5}$$

let $2x^2+9x-5=0$

$(2x-1)(x+5)=0$

$2x-1=0$ OR $x+5=0$

$2x-1+1=0+1$ OR $x+5-5=0-5$

$2x=1$ OR $x=-5$

$\frac{2x}{2}=\frac{1}{2}$

$x=\frac{1}{2}$ the function has two vertical asymptotes

$x=\frac{1}{2}$ OR $x=-5$

20. Determine whether the following function is continuous at a. Use the continuity checklist to justify your answer.

$$f(x) = \frac{5x^2 + 16x + 3}{x^2 + 7x}, \ a = -7$$

Select all that apply.

- [ ] **A.** The function is continuous at $a = -7$.
- [x] **B.** The function is not continuous at $a = -7$ because $f(-7)$ is undefined.
- [x] **C.** The function is not continuous at $a = -7$ because $\lim_{x \to -7} f(x)$ does not exist.
- [x] **D.** The function is not continuous at $a = -7$ because $\lim_{x \to -7} f(x) \neq f(-7)$.

Answer: B. The function is not continuous at $a = -7$ because $f(-7)$ is undefined. , C. The function is not continuous at $a = -7$ because $\lim_{x \to -7} f(x)$ does not exist. , D. The function is not continuous at $a = -7$ because $\lim_{x \to -7} f(x) \neq f(-7)$.

21. Determine whether the following function is continuous at a. Use the continuity checklist to justify your answer.

$$f(x) = \begin{cases} \dfrac{x^2 - 196}{x - 14} & \text{if } x \neq 14 \\ 1 & \text{if } x = 14 \end{cases} ; \ a = 14$$

Select all that apply.

- [ ] **A.** The function is continuous at $a = 14$.
- [ ] **B.** The function is not continuous at $a = 14$ because $f(14)$ is undefined.
- [ ] **C.** The function is not continuous at $a = 14$ because $\lim_{x \to 14} f(x)$ does not exist.
- [x] **D.** The function is not continuous at $a = 14$ because $\lim_{x \to 14} f(x) \neq f(14)$.

Answer: D. The function is not continuous at $a = 14$ because $\lim_{x \to 14} f(x) \neq f(14)$.

22. Determine the intervals on which the following function is continuous.

set $x^2 - 16 = 0$

$(x)^2 - (4)^2 = 0$

$(x+4)(x-4) = 0$

$x+4=0 \qquad x-4=0$

$x+4-4 = 0-4 \qquad x-4+4 = 0+4$

$x = -4 \qquad x = 4$

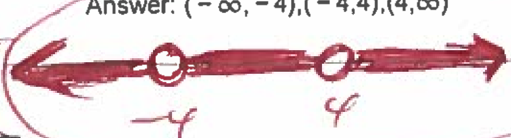
$$f(x) = \frac{x^2 - 6x + 8}{x^2 - 16}$$

On what interval(s) is f continuous?

$(-\infty,-4)\cup(-4,4)\cup(4,\infty)$

(Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)

Answer: $(-\infty,-4),(-4,4),(4,\infty)$



OR $(-\infty,-4)\cup(-4,4)\cup(4,\infty)$

23 eval

$$\lim_{x \to 4} \sqrt{x^2+9} =$$

because x^2+9 is continuous for all x and the square root function is continuous for all $x \geq 0$

$$\sqrt{(4)^2+9} =$$

$$\sqrt{16+9} =$$

$$\sqrt{25} =$$

$$5 =$$

24. Suppose x lies in the interval $(3,7)$ with $x \neq 5$. Find the smallest positive value of δ such that the inequality $0 < |x-5| < \delta$ is true for all possible values of x.

$$0 < |x-5| < \delta$$

$$|x-5| < \delta$$

$$-\delta < x-5 < \delta$$

$$-\delta + 5 < x - \cancel{5} + \cancel{5} < \delta + 5$$

$$-\delta + 5 < x < \delta + 5$$

$-\delta + 5 = 3$ OR $\delta + 5 = 7$

$-\delta + \cancel{5} - \cancel{5} = 3 - 5$ OR $\delta + \cancel{5} - \cancel{5} = 7 - 5$

$-\delta = -2$ OR $\delta = 2$

$-1(-\delta) = -1(-2)$

$\delta = 2$

$$\delta = 2$$

27 use the precise definition of a limit to prove the following limit. Specify a relationship between e and δ that guarantees the limit exists.

$$\lim_{x\to 0}(4x+2)=2$$

$$|(4x+2)-2|<e$$

$$|4x+2-2|<e$$

$$|4x|<e$$

$$|4|\cdot|x|<e$$

$$4|x|<e$$

$$\frac{\cancel{4}|x|}{\cancel{4}}<\frac{e}{4}$$

$$|x|<\frac{e}{4}$$

$|x-0|<\frac{e}{4}$ rewrite

let $\delta=\frac{e}{4}$

$|(4x+2)-2|<e$ whenever $0<|x-0|<\delta$

or $0<|x-0|<\frac{e}{4}$

28 Find the average velocity

$f(x) = y = \frac{3}{x-2}$ $\quad [4, 7]$ ($a = 4$, $b = 7$)

$$\frac{f(b) - f(a)}{b - a} = \text{average velocity}$$

$$\frac{f(7) - f(4)}{(7) - (4)} =$$

$$\frac{\left(\frac{3}{(7)-2}\right) - \left(\frac{3}{(4)-2}\right)}{7-4} =$$

$$\frac{\frac{3}{7-2} - \frac{3}{4-2}}{7-4} =$$

$$\frac{\frac{3}{5} - \frac{3}{2}}{3} =$$

$$\frac{\frac{3}{5}\left(\frac{2}{2}\right) - \frac{3}{2}\left(\frac{5}{5}\right)}{3} =$$

$$\frac{\frac{6}{10} - \frac{15}{10}}{3} =$$

$$\frac{\frac{6-15}{10}}{3} =$$

$$\frac{-\frac{9}{10}}{\frac{3}{1}} =$$

$$-\frac{9}{10} \cdot \frac{1}{3} =$$

$$-\frac{3\cancel{(3)}}{10} \cdot \frac{1}{\cancel{(3)}} =$$

$$-\frac{3}{10} =$$

(29) find all vertical asymptotes

$$g(x) = \frac{x+11}{x^2-49x}$$

set $x^2-49x=0$

$x(x-49)=0$

$x=0$ OR $x-49=0$

OR $x-\cancel{49}+\cancel{49}=0+49$

OR $x=49$

(30) find the limit at infinity

formula

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} \frac{\sqrt[3]{x} - 4x + 3}{-2x + x^{2/3} + 7} =$$

$$\lim_{x\to\infty} \frac{x^{1/3} - 4x^{3/3} + 3}{-2x^{3/3} + x^{2/3} + 7} =$$ rewrite

$$\lim_{x\to\infty} \left(\frac{x^{1/3} - 4x^{3/3} + 3}{-2x^{3/3} + x^{2/3} + 7}\right) \frac{\frac{1}{x^{3/3}}}{\frac{1}{x^{3/3}}} =$$ mult

$$\lim_{x\to\infty} \left(\frac{\frac{x^{1/3}}{x^{3/3}} - \frac{4\cancel{x^{3/3}}}{\cancel{x^{3/3}}} + \frac{3}{x^{3/3}}}{\frac{-2\cancel{x^{3/3}}}{\cancel{x^{3/3}}} + \frac{x^{2/3}}{x^{3/3}} + \frac{7}{x^{3/3}}}\right) =$$

$$\lim_{x\to\infty} \left(\frac{\frac{1}{x^{3/3 - 1/3}} - 4 + \frac{3}{x}}{-2 + \frac{1}{x^{3/3 - 2/3}} + \frac{7}{x}}\right) =$$

$$\lim_{x\to\infty} \left(\frac{\frac{1}{x^{2/3}} - 4 + \frac{3}{x}}{-2 + \frac{1}{x^{1/3}} + \frac{7}{x}}\right) =$$

$$\frac{0 - 4 + 0}{-2 + 0 + 0} = \frac{-4}{-2} =$$

$$2 =$$

(32) find value of the derivative of the function at the given point

$f(x) = 4x^2 - 2x$ $\qquad$ $(-1, 6)$

$f'(x) = 8x - 2$

$f'(-1) = 8(-1) - 2$

$f'(-1) = -8 - 2$

$f'(-1) = -10$

formula

$y = x^n$

$y' = nx^{n-1}$

$y = ax$

$y' = a$

33 (a) use defin to $m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ to find slope of the line tangent to the graph of f at P.

(b) determine an equation of the tange line at P.

$f(x) = x^2 - 4$ $\quad P(-5, 21)$ $\quad x_1 \; y_1$

$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} =$

$\lim_{h \to 0} \frac{((a+h)^2 - 4) - ((a)^2 - 4)}{h} =$

$\lim_{h \to 0} \frac{((a+h)(a+h) - 4) - (a^2 - 4)}{h} =$

$\lim_{h \to 0} \frac{(a^2 + ah + ah + h^2 - 4) - (a^2 - 4)}{h} =$

$\lim_{h \to 0} \frac{\cancel{a^2} + 2ah + h^2 \cancel{-4} \cancel{- a^2} \cancel{+4}}{h} =$

$\lim_{h \to 0} \frac{2ah + h^2}{h} =$

$\lim_{h \to 0} \frac{\cancel{h}(2a + h)}{\cancel{h}} =$

$\lim_{h \to 0} 2a + h =$

$2a + (0)$

$2a$

$f'(a) = 2a$ = slope = m

$f'(a) = 2a$

$f'(-5) = 2(-5)$

$f'(-5) = -10$

Slope m

Point Slope formula

$y - y_1 = m(x - x_1)$

$y - (21) = -10(x - (-5))$

$y - 21 = -10(x + 5)$

$y - 21 = -10x - 50$

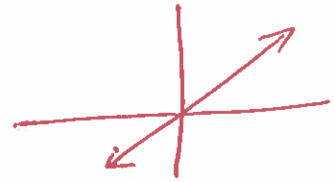
$y - \cancel{21} + \cancel{21} = -10x - 50 + 21$

$y = -10x - 29$

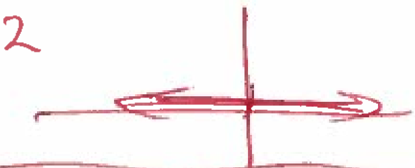
(34) Match the graph of the function on the right with its derivative

Example

$y = 2x$

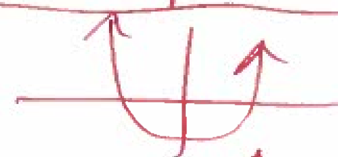


$y' = 2$

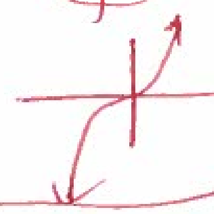


example

$y = x^4$



$y' = 4x^3$



example

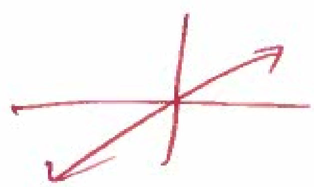
$y = x^3$

$y' = 3x^2$

example

$y = x^2$

$y' = 2x$



35 a line perpendicular to another line or to a tangent line is called a normal line. Find an equation of the line perpendicular to the line that is tangent to the curve at the given point P.

$y = 5x + 7$ $\qquad P(-1, 2)$

$\qquad\qquad\qquad\qquad\quad x_1 \;\; y_1$

$m_{\perp} = -\frac{1}{5}$ slope

$$y - y_1 = m(x - x_1)$$

$$y - (2) = -\frac{1}{5}(x - (-1))$$

$$y - 2 = -\frac{1}{5}(x + 1)$$

$$y - 2 = -\frac{1}{5}x - \frac{1}{5}$$

$$y \cancel{-2} \cancel{+2} = -\frac{1}{5}x - \frac{1}{5} + 2$$

$$y = -\frac{1}{5}x - \frac{1}{5} + \frac{2}{1}\left(\frac{5}{5}\right)$$

$$y = -\frac{1}{5}x - \frac{1}{5} + \frac{10}{5}$$

$$y = -\frac{1}{5}x + \frac{-1+10}{5}$$

$$y = -\frac{1}{5}x + \frac{9}{5}$$

(36) evaluate the derivative using the limit definition of the derivative

$f(x) = x^2 + 3x - 8$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \to 0} \frac{((x+h)^2 + 3(x+h) - 8) - (x^2 + 3x - 8)}{h} =$$

$$\lim_{h \to 0} \frac{((x+h)(x+h) + 3x + 3h - 8) - (x^2 + 3x - 8)}{h} =$$

$$\lim_{h \to 0} \frac{((x^2 + xh + xh + h^2) + 3x + 3h - 8) - (x^2 + 3x - 8)}{h} =$$

$$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - 8 - x^2 - 3x + 8}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h \cancel{- 8} \cancel{- x^2} \cancel{- 3x} \cancel{+ 8}}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2 + 3h}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + h + 3)}{\cancel{h}} =$$

$$\lim_{h \to 0} 2x + h + 3 =$$

$$2x + (0) + 3 =$$

$$f'(x) = 2x + 3$$

(37)

$$y = \frac{x-1}{4x-5}$$

$$y = \frac{f}{g}$$

$$y' = \frac{f'g - fg'}{(g)^2}$$

$$y' = \frac{(x-1)'(4x-5) - (x-1)(4x-5)'}{(4x-5)^2}$$

$$y' = \frac{(1-0)(4x-5) - (x-1)(4-0)}{(4x-5)^2}$$

$$y' = \frac{(1)(4x-5) - (x-1)(4)}{(4x-5)^2}$$

$$y' = \frac{(4x-5) - (4x-4)}{(4x-5)^2}$$

$$y' = \frac{\cancel{4x} - 5 - \cancel{4x} + 4}{(4x-5)^2}$$

$$y' = \frac{-1}{(4x-5)^2}$$

$$y' = -\frac{1}{(4x-5)^2}$$

(38) use Quotient Rule to find $g'(1)$

$$y = \frac{f}{g}$$

$$y' = \frac{f'g - fg'}{(g)^2}$$

$$g(x) = \frac{2x^2}{3x+2}$$

$$g'(x) = \frac{(2x^2)'(3x+2) - (2x^2)(3x+2)'}{(3x+2)^2}$$

$$g'(x) = \frac{(4x)(3x+2) - (2x^2)(3+0)}{(3x+2)^2}$$

$$g'(x) = \frac{(4x)(3x+2) - (2x^2)(3)}{(3x+2)^2}$$

$$g'(x) = \frac{(12x^2+8x) - (6x^2)}{(3x+2)^2}$$

$$g'(x) = \frac{12x^2+8x-6x^2}{(3x+2)^2}$$

$$g'(x) = \frac{6x^2+8x}{(3x+2)^2}$$

$$g'(1) = \frac{6(1)^2+8(1)}{(3(1)+2)^2}$$

$$g'(1) = \frac{6(1)(1)+8(1)}{(3+2)^2}$$

$$g'(1) = \frac{6+8}{(5)^2}$$

$$g'(1) = \frac{14}{25}$$

(39) $f(x) = (x-3)(5x+3)$

$f'(x) = (x-3)'(5x+3) + (x-3)(5x+3)'$

$f'(x) = (1-0)(5x+3) + (x-3)(5+0)$

$f'(x) = (1)(5x+3) + (x-3)(5)$

$f'(x) = (5x+3) + (5x-15)$

$f'(x) = 5x+3+5x-15$

$f'(x) = 10x-12$

$y = f \cdot g$
$y' = f'g + fg'$

$y = x^n$
$y' = nx^{n-1}$

$y = ax$
$y' = a$

$y = a$
$y' = 0$

OR

$f(x) = (x-3)(5x+3)$

$f(x) = 5x^2 + 3x - 15x - 9$

$f(x) = 5x^2 - 12x - 9$

$f'(x) = 10x - 12 - 0$

$f'(x) = 10x - 12$

(40) $h(w) = \frac{3w^4 - w}{w}$

$h'(w) = \frac{(3w^4-w)'(w) - (3w^4-w)(w)'}{(w)^2}$

$h'(w) = \frac{(12w^3-1)(w) - (3w^4-w)(1)}{(w)^2}$

$h'(w) = \frac{(12w^4-w) - (3w^4-w)}{(w)^2}$

$h'(w) = \frac{12w^4 - \cancel{w} - 3w^4 + \cancel{w}}{(w)^2}$

$h'(w) = \frac{9w^4}{w^2}$

$h'(w) = 9w^{4-2}$

$h'(w) = 9w^2$ ✓

formula

$y = \frac{f}{g}$

$y' = \frac{f'g - fg'}{(g)^2}$

$y = x^n$

$y' = nx^{n-1}$

$y = ax$

$y' = a$

$y = a$

$y' = 0$

OR

$h(w) = \frac{3w^4 - w}{w}$

$h(w) = \frac{3w^4}{w^1} - \frac{w^1}{w^1}$

$h(w) = 3w^{4-1} - 1$

$h(w) = 3w^3 - 1$

$h'(w) = 9w^{3-1} - 0$

$\rightarrow h'(w) = 9w^2$

(41) $f(x) = \sin(x)$ find $f'(\frac{\pi}{2})$

$f'(x) = \cos(x) \cdot (x)'$

$f'(x) = \cos(x)(1)$

$f'(x) = \cos x$

$f'(\frac{\pi}{2}) = \cos(\frac{\pi}{2})$

$f'(\frac{\pi}{2}) = 0$

formula

$y = \sin f(x)$

$y' = \cos(f(x)) \cdot f'(x)$

4c

$$\lim_{x\to 0} \frac{\sin(6x)}{\sin(5x)}$$

formula

$$\lim_{x\to 0} \frac{\sin(x)}{(x)} = 1$$

$$\lim_{x\to 0} \frac{\sin(mx)}{(mx)} = 1$$

$$\lim_{x\to 0} \frac{\frac{\sin(6x)}{1}}{\frac{\sin(5x)}{1}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} =$$ Mul

$$\lim_{x\to 0} \frac{\frac{\sin(6x)}{x}}{\frac{\sin(5x)}{x}} =$$

$$\lim_{x\to 0} \frac{\frac{\sin(6x)}{x}\left(\frac{6}{6}\right)}{\frac{\sin(5x)}{x} \cdot \frac{5}{5}} =$$ Mult

$$\lim_{x\to 0} \frac{\frac{6\sin(6x)}{6x}}{\frac{5\sin(5x)}{5x}} =$$

$$\frac{\lim_{x\to 0} \frac{6\sin(6x)}{6x}}{\lim_{x\to 0} \frac{5\sin(5x)}{5x}} =$$

$$\frac{6 \lim_{x\to 0} \frac{\sin(6x)}{6x}}{5 \lim_{x\to 0} \frac{\sin(5x)}{5x}} =$$

$$\frac{6(1)}{5(1)} =$$

$$\frac{6}{5}$$

(43)

$$y = 2\sin(x) + 8\cos(x)$$

$$y' = 2\cos(x)(x)' + (-8\sin(x)(x)')$$

$$y' = 2\cos(x)(1) + (-8\sin(x)(1))$$

$$y' = 2\cos(x) - 8\sin(x)$$

OR

$$\frac{dy}{dx} = 2\cos(x) - 8\sin(x)$$

formula

$$y = \sin(f(x))$$

$$y' = \cos(f(x)) \cdot f'(x)$$

$$y = \cos(f(x))$$

$$y' = -\sin(f(x))\, f'(x)$$

$$y = ax$$

$$y' = a$$

(44) $y = e^{-x}\sin(x)$

$y' = (e^{-x})'(\sin(x)) + (e^{-x})(\sin(x))'$

$y' = (e^{-x}(-x)')(\sin(x)) + (e^{-x})(\cos(x)(x)')$

$y' = (e^{-x}(-1))(\sin(x)) + (e^{-x})(\cos(x)(1))$

$y' = (-e^{-x})(\sin(x)) + (e^{-x})(\cos(x))$

$y' = -e^{-x}\sin(x) + e^{-x}\cos(x)$

$y' = e^{-x}\cos x - e^{-x}\sin(x)$ rewrite

$y' = e^{-x}(\cos(x) - \sin(x))$ rewrite

OR

$\frac{dy}{dx} = e^{-x}(\cos(x) - \sin(x))$

Formula

$y = e^{f(x)}$
$y' = e^{f(x)} \cdot f'(x)$

$y = \sin(f(x))$
$y' = \cos(f(x)) \cdot f'(x)$

$y = \cos(f(x))$
$y' = -\sin(f(x)) \cdot f'(x)$

(45.) find a equation of the line tangent to the curve at point

$y = -15x^2 + 8\sin(x)$ $\quad P(0, 0)$ $\quad x_1 \; y_1$

$y' = -30x + 8\cos(x)(x)'$

$y' = -30x + 8\cos(x)(1)$

$y' = -30x + 8\cos(x)$

$y'(0) = -30(0) + 8\cos(0)$

$y'(0) = 0 + 8(1)$

$y'(0) = 0 + 8$

$y'(0) = 8 = \text{Slope} = m$

$y - y_1 = m(x - x_1)$ ← Point Slope formula

$y - (0) = 8(x - (0))$

$y - 0 = 8(x - 0)$

$y = 8(x)$

$y = 8x$

46 $h(x) = f(g(x))$ $p(x) = g(f(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$p'(x) = g'(f(x)) \cdot f'(x)$

find $h'(2)$

$p'(4)$

x	1	2	3	4
$f(x)$	1	2	3	4
$f'(x)$	-5	-2	-3	-7
$g(x)$	2	1	4	3
$g'(x)$	$\frac{1}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	$\frac{4}{7}$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(2) = f'(g(2)) \cdot g'(2)$

$h'(2) = f'(1) \cdot \left(\frac{5}{7}\right)$

$h'(2) = (-5) \cdot \left(\frac{5}{7}\right)$

$h'(2) = -\frac{25}{7}$

$p'(x) = g'(f(x)) \cdot f'(x)$

$p'(4) = g'(f(4)) \cdot f'(4)$

$p'(4) = g'(4) \cdot (-7)$

$p'(4) = \left(\frac{4}{\cancel{7}}\right) \cdot (-\cancel{7})$

$p'(4) = -4$

(47)

$$y = (4x+9)^8$$

$$y' = 8(4x+9)^{8-1} \cdot (4x+9)'$$

$$y' = 8(4x+9)^7(4+0)$$

$$y' = 8(4x+9)^7(4)$$

$$y' = 32(4x+9)^7$$

or

$$\frac{dy}{dx} = 32(4x+9)^7$$

formula

$$y = (f(x))^N$$

$$y' = N(f(x))^{N-1} \cdot f'(x)$$

$$y = x^n$$

$$y' = nx^{n-1}$$

$$y = ax$$

$$y' = a$$

$$y = a$$

$$y' = 0$$

48

$$y = 8(8x^2+1)^{-2}$$

$$y' = 8\,(-2)\,(8x^2+1)^{-2-1} \cdot (8x^2+1)'$$

$$y' = -16\,(8x^2+1)^{-3}(16x+0)$$

$$y' = -16\,(8x^2+1)^{-3}(16x)$$

$$y' = -256x\,(8x^2+1)^{-3}$$

$$y' = \frac{-256x}{(8x^2+1)^3}$$

OR

$$\frac{dy}{dx} = \frac{-256x}{(8x^2+1)^3}$$

formula

$$y = (f(x))^N$$

$$y' = N(f(x))^{N-1} \cdot f'(x)$$

$$y = x^n$$

$$y' = nx^{n-1}$$

$$y = a$$

$$y' = 0$$

$$y = ax$$

$$y' = a$$

(49)

$y = \sin(13t + 20)$

$y' = \cos(13t+20) \cdot (13t+20)'$

$y' = \cos(13t+20) \cdot (13+0)$

$y' = \cos(13t+20)(13)$

$y' = 13\cos(13t+20)$

OR

$\frac{dy}{dt} = 13\cos(13t+20)$

formula

$y = \sin(f(x))$

$y' = \cos(f(x)) \cdot f'(x)$

$y = x^n$

$y' = n x^{n-1}$

$y = ax$

$y' = a$

$y = a$

$y' = 0$

50 $y = \tan(e^x)$

$y' = \sec^2(e^x)\cdot(e^x)'$

$y' = \sec^2(e^x)\cdot(e^x(x)')$

$y' = \sec^2(e^x)\cdot(e^x(1))$

$y' = \sec^2(e^x)\cdot(e^x)$

$y' = e^x \sec^2(e^x)$ rewrite

OR

$\frac{dy}{dx} = e^x \sec^2(e^x)$

formula

$y = \tan(f(x))$

$y' = \sec^2(f(x))\cdot f'(x)$

$y = e^{f(x)}$

$y' = e^{f(x)}\cdot f'(x)$

$y = ax$

$y' = a$

(57.) $y = \sin(9\cos x))$

$y' = \cos(9\cos(x)) \cdot (9\cos x))'$

$y' = \cos(9\cos(x)) \cdot (-9\sin(x) \cdot (x)')$

$y' = \cos(9\cos x)) \cdot (-9\sin(x) \cdot (1))$

$y' = \cos(9\cos(x)) \cdot (-9\sin(x))$

$y' = -9\cos(9\cos(x)) \cdot \sin(x)$

rewrite

OR

$$\frac{dy}{dx} = -9\cos(9\cos(x)) \cdot \sin(x)$$

Formula

$y = \sin(f(x))$

$y' = \cos(f(x)) \cdot f'(x)$

$y = \cos(f(x))$

$y' = -\sin(f(x)) \cdot f'(x)$

52

use implicit differentiation

$2x = y^2$

$2 = 2y^{2-1} \cdot y'$

$2 = 2y \cdot y'$

$\frac{2}{2y} = \frac{2y \cdot y'}{2y}$

$\frac{2}{2y} = y'$

$\frac{\cancel{2}(1)}{\cancel{2}(y)} = y'$

$\frac{1}{y} = y'$

OR

$\frac{1}{y} = \frac{dy}{dx}$

53

$\cos(y) + 10 = x$

use implicit differentation

$-\sin(y) \cdot y' + 0 = 1$

$-\sin(y) \cdot y' = 1$

$\frac{\cancel{-\sin(y)} \cdot y'}{\cancel{-\sin(y)}} = \frac{1}{-\sin(y)}$

divide

$y' = \frac{1}{-\sin(y)}$

formula

$\csc(x) = \frac{1}{\sin(x)}$

$y' = -\frac{1}{\sin(y)}$

$y' = -\csc(y)$

$\frac{dy}{dx} = -\csc(y)$

54

$$x = y^{19}$$

$$1 = 19y^{19-1} \cdot y'$$

$$1 = 19y^{18}y'$$

$$\frac{1}{19y^{18}} = \frac{19\cancel{y^{18}}y'}{\cancel{19y^{18}}}$$

$$\frac{1}{19y^{18}} = y'$$

$$y' = \frac{1}{19y^{18}}$$

$$y' = \frac{1}{19}y^{-18}$$

$$y'' = \frac{1}{19}\left(-18y^{-18-1} \cdot y'\right)$$

$$y'' = \frac{-18}{19}y^{-19} \cdot y'$$

$$y'' = \frac{-18}{19y^{19}}y'$$

Subst

$$y'' = \frac{-18}{19y^{19}} \cdot \left(\frac{1}{19y^{18}}\right)$$

$$y'' = -\frac{18}{361y^{37}}$$

$$y' = \frac{dy}{dx} = \frac{1}{19y^{18}}$$

$$y'' = \frac{d^2y}{dx^2} = \frac{-18}{361y^{37}}$$

55

(a) find $\frac{dy}{dx}$

use implicit differentiation

(b) find the slope of the curve at the given point

$x^3 + y^3 = 19$ $\quad(-2, 3)$

$3x^2 + 3y^{3-1} \cdot y' = 0$

$3x^2 + 3y^2 y' = 0$

$3y^2 y' = -3x^2$ rewrite

$\frac{\cancel{3y^2} y'}{\cancel{3y^2}} = \frac{-3x^2}{3y^2}$

$y' = \frac{-1x^2}{y^2}$ OR $\frac{dy}{dx} = \frac{-1x^2}{y^2}$

$y'(\underset{x}{-2}, \underset{y}{3}) = \frac{-1(-2)^2}{(3)^2}$

$y'(-2, 3) = \frac{-1(-2)(-2)}{(3)(3)}$

$y'(-2, 3) = \frac{-4}{9}$

$y'(-2, 3) = \frac{-4}{9}$

(56) $\sin(y) + \cos(x) = 5y$

use implicit differentiation

$$\cos(y) \cdot y' - \sin(x) = 5y'$$

$$-\sin(x) = 5y' - \cos(y) \cdot y'$$

$$-\sin(x) = y'(5 - \cos(y))$$

$$\frac{-\sin(x)}{(5-\cos(y))} = \frac{y'\cancel{(5-\cos(y))}}{\cancel{(5-\cos(y))}}$$

$$\frac{-\sin(x)}{5-\cos(y)} = y'$$

$$-\frac{\sin(x)}{5-\cos(y)} = y'$$

OR

$$-\frac{\sin(x)}{5-\cos(y)} = \frac{dy}{dx}$$

57

use implicit differentiation.

$$4\sin(xy) = 7x + 9y$$

$$4\cos(xy)\cdot(xy)' = 7 + 9y'$$

$$4\cos(xy)\cdot\left((x)'(y) + (x)(y)'\right) = 7 + 9y'$$

$$4\cos(xy)\cdot\left((1)(y) + (x)(y')\right) = 7 + 9y'$$

$$4\cos(xy)\cdot(y + xy') = 7 + 9y'$$

$$4\cos(xy)\cdot y + 4\cos(xy)(xy') = 7 + 9y'$$

$$4y\cos(xy) + 4\cos(xy)(xy') = 7 + 9y'$$

$$4y\cos(xy) + 4x\cos(xy)\cdot y' = 7 + 9y'$$

$$4x\cos(xy)\cdot y' - 9y' = 7 - 4y\cos(xy)$$

$$y'(4x\cos(xy) - 9) = 7 - 4y\cos(xy)$$

$$\frac{y'\cancel{(4x\cos(xy) - 9)}}{\cancel{(4x\cos(xy) - 9)}} = \frac{7 - 4y\cos(xy)}{(4x\cos(xy) - 9)}$$

$$y' = \frac{7 - 4y\cos(xy)}{4x\cos(xy) - 9} = \frac{dy}{dx}$$

58) $e^{3xy} = 4y$

Use implicit differentiation

$e^{(3xy)} \cdot (3xy)' = 4y'$

$e^{(3xy)} \cdot \left((3x)'(y) + (3x)(y)' \right) = 4y'$

$e^{3xy} \cdot \left((3)(y) + (3x)(y') \right) = 4y'$

$e^{3xy} \cdot (3y + 3xy') = 4y'$

$3ye^{3xy} + 3xy'e^{3xy} = 4y'$

$3ye^{3xy} = 4y' - 3xy'e^{3xy}$

$3ye^{3xy} = y'(4 - 3e^{3xy})$

$\frac{3ye^{3xy}}{4-3e^{3xy}} = \frac{y'\cancel{(4-3e^{3xy})}}{\cancel{(4-3e^{3xy})}}$

$\frac{3ye^{3xy}}{4-3e^{3xy}} = y' = \frac{dy}{dx}$

59. $5x^4 + 8y^4 = 13xy$ use implicit differentation

$$20x^3 + 32y^{4-1} \cdot y' = (13x)'(y) + (13x)(y)'$$

$$20x^3 + 32y^3 \cdot y' = (13)(y) + (13x)(y')$$

$$20x^3 + 32y^3 \cdot y' = 13y + 13xy'$$

$$20x^3 - 13y = 13xy' - 32y^3y'$$

$$20x^3 - 13y = y'(13x - 32y^3)$$

$$\frac{20x^3 - 13y}{(13x - 32y^3)} = \frac{y'\cancel{(13x - 32y^3)}}{\cancel{(13x - 32y^3)}}$$

$$\frac{20x^3 - 13y}{13 - 32y^3} = y' = \frac{dy}{dx}$$

(60) $y = \ln\sqrt{x^2+1}$

$y = \ln (x^2+1)^{1/2}$

$y = \frac{1}{2}\ln(x^2+1)$

$y' = \frac{1}{2}\frac{(x^2+1)'}{(x^2+1)}$

$y' = \frac{1}{2}\frac{(2x+0)}{(x^2+1)}$

$y' = \frac{1}{2}\frac{2x}{(x^2+1)}$

$y' = \frac{1}{\cancel{2}}\frac{\cancel{2}x}{(x^2+1)}$

$y' = \frac{x}{x^2+1}$

OR

$\frac{dy}{dx} = \frac{x}{x^2+1}$

formula

$y = \ln\sqrt{Ax+B}$

$y = \ln(Ax+B)^{1/2}$

$y = \frac{1}{2}\ln(Ax+B)$

$y = \ln(f(x))$

$y' = \frac{f'(x)}{f(x)}$

(61) express the function

$f(x) = g(x)^{h(x)}$ in terms of the natural logarithmic and natural exponential functions (base e)

$$f(x) = g(x)^{h(x)}$$

$$f(x) = e^{\ln\left(g(x)\right)^{h(x)}}$$

$$f(x) = e^{h(x)\ln(g(x))}$$

formula

$$e^{\ln(x)} = x$$

$$e^{\ln(f(x))} = f(x)$$

$$e^{\ln(f(x))^{N}} = (f(x))^{N}$$

$$e^{\ln(f(x))^{m(x)}} =$$

$$e^{m(x)\ln f(x)}$$

6a

$$y = \ln(5x^2+2)$$

$$y' = \frac{(5x^2+2)'}{(5x^2+2)}$$

$$y' = \frac{10x+0}{5x^2+2}$$

$$y' = \frac{10x}{5x^2+2}$$

formula

$$y = \ln(f(x))$$

$$y' = \frac{f'(x)}{f(x)}$$

$$y = x^n$$

$$y' = nx^{n-1}$$

$$y = ax$$

$$y' = a$$

$$\cancel{y = a}$$

$$\cancel{y' = 0}$$

63

$$y = 4x^{2\pi}$$

$$y' = 4(2\pi)x^{2\pi-1}$$

$$y' = 8\pi x^{2\pi-1}$$

formula

$$y = x^{n}$$

$$y' = nx^{n-1}$$

64

$y = 2^x$

$y' = 2^x \cdot (x)' \ln(2)$

$y' = 2^x (1) \ln(2)$

$y' = 2^x \ln(2)$

or

$\frac{dy}{dx} = 2^x \ln(2)$

Formula

$y = 2^{f(x)}$

$y' = 2^{f(x)} \cdot f'(x) \ln(2)$

(65) $y = 3\log_2(x^3-8)$

formula

$$y = \log_b(f(x))$$

$$y' = \frac{f'(x)}{f(x)\cdot\ln(b)}$$

$$y' = 3\,\frac{(x^3-8)'}{(x^3-8)\ln(2)}$$

$$y' = 3\,\frac{(3x^2-0)}{(x^3-8)\ln(2)}$$

$$y' = 3\,\frac{(3x^2)}{(x^3-8)\ln(2)}$$

$$y' = \frac{9x^2}{(x^3-8)\ln(2)}$$

OR

$$\frac{dy}{dx} = \frac{9x^2}{(x^3-8)\ln(2)}$$

(66) use logarithmic differentiation to evaluate $f'(x)$

$$f(x) = \frac{(x+3)^{12}}{(3x-6)^{10}}$$

$$\ln(f(x)) = \ln \frac{(x+3)^{12}}{(3x-6)^{10}}$$

$$\ln(f(x)) = \ln (x+3)^{12} - \ln (3x-6)^{10}$$

$$\ln(f(x)) = 12 \ln (x+3) - 10 \ln (3x-6)$$

$$\frac{f'(x)}{f(x)} = 12 \frac{(x+3)'}{(x+3)} - 10 \frac{(3x-6)'}{(3x-6)}$$

$$\frac{f'(x)}{f(x)} = 12 \frac{(1+0)}{(x+3)} - 10 \frac{(3-0)}{(3x-6)}$$

$$\frac{f'(x)}{f(x)} = 12 \frac{1}{(x+3)} - 10 \frac{3}{3x-6}$$

$$\frac{f'(x)}{f(x)} = \frac{12}{(x+3)} - \frac{10(\cancel{3})}{\cancel{3}(x-2)}$$

$$\frac{f'(x)}{f(x)} = \frac{12}{x+3} - \frac{10}{(x-2)}$$

$$\cancel{f(x)} \cdot \frac{f'(x)}{\cancel{f(x)}} = f(x) \left[\frac{12}{(x+3)} - \frac{10}{(x-2)} \right]$$

$$f'(x) = \frac{(x+3)^{12}}{(3x-6)^{10}} \left[\frac{12}{x+3} - \frac{10}{x-2} \right] = \frac{dy}{dx}$$

67.

$y = \sin^{-1}(f(x))$

$y' = \dfrac{f'(x)}{\sqrt{1-(f(x))^2}}$ for $-1 < x < 1$

OR

$y = \sin^{-1}(x)$

$y' = \dfrac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$

$y = \tan^{-1}(f(x))$

$y' = \dfrac{f'(x)}{1+(f(x))^2}$ $\quad -\infty < x < \infty$

OR

$y = \tan^{-1}(x)$

$y' = \dfrac{1}{1+x^2}$ $\quad -\infty < x < \infty$

$y = \sec^{-1}(f(x))$

$y' = \dfrac{f'(x)}{|f(x)|\sqrt{(f(x))^2-1}}$ for $|x| > 1$

OR

$y = \sec^{-1}(x)$

$y' = \dfrac{1}{|x|\sqrt{x^2-1}}$ for $|x| > 1$

68

$$f(x) = \sin^{-1}(4x^4)$$

$$f'(x) = \frac{(4x^4)'}{\sqrt{1-(4x^4)^2}}$$

$$f'(x) = \frac{16x^3}{\sqrt{1-(4x^4)(4x^4)}}$$

$$f'(x) = \frac{16x^3}{\sqrt{1-16x^8}}$$

formula

$$y = \sin^{-1}(f(x))$$

$$y' = \frac{f'(x)}{\sqrt{1-(f(x))^2}}$$

(69) $y = 6 \cdot \tan^{-1}(3x)$

$$y' = 6 \cdot \frac{(3x)'}{1+(3x)^2}$$

$$y' = 6 \cdot \frac{(3)}{1+(3x)^2}$$

$$y' = \frac{18}{1+(3x)^2}$$

OR

$$y' = \frac{18}{1+(3x)(3x)}$$

OR

$$y' = \frac{18}{1+9x^2}$$

formula

$$y = \tan^{-1}(f(x))$$

$$y' = \frac{f'(x)}{1+(f(x))^2}$$

$y = ax$

$y' = a$

(70)

$$f(s) = \cot^{-1}(e^{s})$$

$$f'(s) = -\frac{(e^{s})'}{1+(e^{s})^2}$$

$$f'(s) = -\frac{e^{s}(s)'}{1+(e^{s})(e^{s})}$$

$$f'(s) = -\frac{e^{s}(1)}{1+e^{s+s}}$$

$$f'(s) = -\frac{e^{s}}{1+e^{2s}}$$

Formula

$$y = \cot^{-1}(f(x))$$

$$y' = -\frac{(f(x))'}{1+(f(x))^2}$$

$$y = e^{f(x)}$$

$$y' = e^{f(x)} \cdot f'(x)$$

71. The sides of a square increase in length at a rate of 4 m/sec. $= \frac{ds}{dt}$

a. At what rate is the area of the square changing when the sides are 11 m long?
b. At what rate is the area of the square changing when the sides are 25 m long?

a. Write an equation relating the area of a square, A, and the side length of the square, s.

$A = s^2$

Differentiate both sides of the equation with respect to t.

$\frac{dA}{dt} = (2s^1)\frac{ds}{dt}$

$\frac{dA}{dt} = 2(11)(4) = 22(4) = 88$

The area of the square is changing at a rate of 88 (1) m^2/s when the sides are 11 m long.

b. The area of the square is changing at a rate of 200 (2) m^2/s when the sides are 25 m long.

(1) ○ m^3/s ○ m/s ○ m ○ m^2/s

(2) ○ m^2/s ○ m^3/s ○ m ○ m/s

$\frac{dA}{dt} = 2(25)(4) = 50(4) = 200$

$A = s^2$

$\frac{dA}{dt} = 2s^1 \frac{ds}{dt}$

Answers $A = s^2$

2s

88

(1) m^2/s

200

(2) m^2/s

72. The area of a circle increases at a rate of 2 cm^2 / s. $= \frac{dA}{dt}$

a. How fast is the radius changing when the radius is 3 cm?
b. How fast is the radius changing when the circumference is 1 cm?

a. Write an equation relating the area of a circle, A, and the radius of the circle, r.

$A = \pi r^2$

(Type an exact answer, using π as needed.)

Differentiate both sides of the equation with respect to t.

$\frac{dA}{dt} = (2\pi r)\frac{dr}{dt}$

(Type an exact answer, using π as needed.)

$r = 3$

When the radius is 3 cm, the radius is changing at a rate of $\frac{1}{3\pi}$ (1) cm/s
(Type an exact answer, using π as needed.)

b. When the circumference is 1 cm, the radius is changing at a rate of 2 (2) cm/s
(Type an exact answer, using π as needed.)

(1) ○ cm^2 / s. ○ cm. ○ cm / s. ○ cm^3 / s.

(2) ○ cm / s. ○ cm^2 / s. ○ cm. ○ cm^3 / s.

Answers $A = \pi r^2$

$2\pi r$

$\frac{1}{3\pi}$

(1) cm / s.

2

(2) cm / s.

a

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$2 = 2\pi(3)\frac{dr}{dt}$

$2 = 6\pi \frac{dr}{dt}$

$\frac{2}{6\pi} = \frac{\cancel{6\pi}\frac{dr}{dt}}{\cancel{6\pi}}$

$\frac{2}{6\pi} = \frac{dr}{dt}$

$\frac{2(1)}{2(3\pi)} = \frac{dr}{dt}$

$\frac{1}{3\pi} = \frac{dr}{dt}$

b

$C = 2\pi r$

$1 = 2\pi r$

$\frac{1}{2\pi} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$

$\frac{1}{2\pi} = r$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$2 = 2\pi\left(\frac{1}{2\pi}\right)\frac{dr}{dt}$

$2 = 1\frac{dr}{dt}$

$2 = \frac{dr}{dt}$

73. The edges of a cube increase at a rate of 3 cm / s. How fast is the volume changing when the length of each edge is 50 cm?

Write an equation relating the volume of a cube, V, and an edge of the cube, a.

$\boxed{V=a^3}$

edge = a

Differentiate both sides of the equation with respect to t.

$\frac{dV}{dt} = (\boxed{3a^2})\frac{da}{dt}$ (Type an expression using a as the variable.)

The rate of change of the volume is $\boxed{22,500}$ (1) $\boxed{cm^3/sec}$
(Simplify your answer.)

$V=a^3$

$\frac{dV}{dt} = 3a^2\frac{da}{dt}$

$\frac{dV}{dt} = 3(50)^2(3)$

$\frac{dV}{dt} = 3(50)(50)(3)$

$\frac{dV}{dt} = 22,500$

(1) ○ cm. ○ cm^3.
○ cm^2 / sec. ○ cm / sec.
○ cm^2.
● cm^3 / sec.

Answers $V=a^3$

$3a^2$

22,500

(1) cm^3 / sec.

---

74. Find an equation for the tangent to the curve at the given point.

$y = x^2 - 2$, (2,2)

○ A. $y = 4x - 12$
○ B. $y = 4x - 10$
○ C. $y = 2x - 6$
○ D. $y = 4x - 6$

Answer: D. $y = 4x - 6$

$y' = 2x$

$y'(2) = 2(2)$

$y'(2) = 4$ = slope = m

$y - y_1 = m(x - x_1)$

$y - (2) = 4(x - (2))$

$y - 2 = 4(x-2)$

$y - 2 = 4x - 8$

(2, 2)
x_1 y_1

$y - 2 + 2 = 4x - 8 + 2$

$y = 4x - 6$

---

75. At time t, the position of a body moving along the s-axis is $s = t^3 - 18t^2 + 60t$ m. Find the displacement of the body from t = 0 to t = 3.

$s(3) - s(0)$

$((3)^3 - 18(3)^2 + 60(3)) - ((0)^3 - 18(0)^2 + 60(0)) =$

$((3)(3)(3) - 18(3)(3) + 60(3)) - ((0)(0)(0) - 18(0)(0) + 60(0)) =$

$(27 - 162 + 180) - (0 - 0 + 0) =$

$(45) - (0) =$

$45 - 0 =$

$45 =$

○ A. 105 m
○ B. 56 m
○ C. 67 m
○ D. 45 m

Answer: D. 45 m

(16)

$xy + x = 2$

use implicit differentiation

$(x)'(y) + (x)(y)' + 1 = 0$

$(1)(y) + (x)(y') + 1 = 0$

$y + xy' + 1 = 0$

$xy' = -1 - y$

$\frac{\cancel{x}y'}{\cancel{x}} = \frac{-1-y}{x}$

$y' = \frac{-1-y}{x}$

$y' = -\frac{1+y}{x}$

OR

$\frac{dy}{dx} = -\frac{1+y}{x}$

(77.)

$y = \log_7 \sqrt{3x+6}$

$y = \log_7 (3x+6)^{1/2}$

$y = \frac{1}{2} \log_7 (3x+6)$

$y' = \frac{1 \cdot (3x+6)'}{2 \cdot (3x+6) \ln(7)}$

$y' = \frac{1 \ (3+0)}{2 \ (3x+6) \ln(7)}$

$y' = \frac{1 \ (3)}{2 \ (3x+6) \ln(7)}$

$y' = \frac{3}{2 \ln(7) (3x+6)}$

OR

$\frac{dy}{dx} = \frac{3}{2 \ln(7) (3x+6)}$

formula

$y = \log_b \sqrt{Ax+B}$

$y = \log_b (Ax+B)^{1/2}$

$y = \frac{1}{2} \log_b (Ax+B)$

$y = \log_b (f(x))$

$y' = \frac{f'(x)}{f(x) \cdot \ln(b)}$

$y = ax$

$y' = a$

$y = a$

$y' = 0$

78. Boyle's law states that if the temperature of a gas remains constant, then PV = c, where P = pressure, V = volume, and c is a constant. Given a quantity of gas at constant temperature, if V is decreasing at a rate of 9 in^3/sec, at what rate is P increasing when P = 70 lb/in^2 and V = 90 in^3? (Do not round your answer.)

$\frac{dV}{dt}$

A. 7 lb / in^2 per sec

B. $\frac{81}{7}$ lb / in^2 per sec

C. 700 lb / in^2 per sec

D. $\frac{49}{81}$ lb / in^2 per sec

Answer: A. 7 lb / in^2 per sec

Formula

$y = fg$

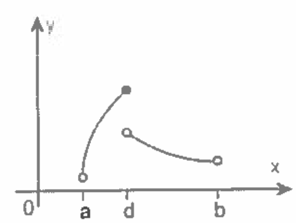
$y' = f'g + fg'$

$y = a$

$y' = 0$

$$PV = c$$
$$(PV)' = 0$$
$$P'V + PV' = 0$$
$$\frac{dP}{dt}V + P\frac{dV}{dt} = 0$$
$$\frac{dP}{dt}(90) + (70)(-9) = 0$$
$$90\frac{dP}{dt} - 630 = 0$$
$$90\frac{dP}{dt} = 630$$
$$\frac{\cancel{90}\frac{dP}{dt}}{\cancel{90}} = \frac{630}{90}$$
$$\frac{dP}{dt} = 7 \text{ lb/in}^2 \text{ per sec}$$

79. Determine from the graph whether the function has any absolute extreme values on [a, b].



Where do the absolute extreme values of the function occur on [a, b]?

A. There is no absolute maximum and there is no absolute minimum on [a, b].

B. The absolute maximum occurs at x = d and the absolute minimum occurs at x = a on [a, b].

C. There is no absolute maximum and the absolute minimum occurs at x = a on [a, b].

D. The absolute maximum occurs at x = d and there is no absolute minimum on [a, b].

Answer: D. The absolute maximum occurs at x = d and there is no absolute minimum on [a, b].

(80) Find the critical points

$f(x) = 3x^2 - 5x + 2$

$f'(x) = 6x - 5 + 0$

$f'(x) = 6x - 5$

set $6x - 5 = 0$

$6x - \cancel{5} + \cancel{5} = 0 + 5$

$6x = 5$

$\frac{6x}{6} = \frac{5}{6}$

$x = 5/6$ critical point

81. Find the critical points

$$f(x) = -\frac{x^3}{3} + 36x$$

$$f'(x) = -\frac{1}{3}(3x^2) + 36$$

$$f'(x) = -1x^2 + 36$$

set $-1x^2 + 36 = 0$

$$-1x^2 = -36$$

$$\frac{-1x^2}{-1} = \frac{-36}{-1}$$

$$x^2 = 36$$

$$\sqrt{x^2} = \pm\sqrt{36}$$

$$x = \pm 6$$

$x = -6$ or $x = 6$

Critical points

(82) determine the location and value of the absolute extreme value of f on the interval

$f(x) = -x^2 + 8$ on $[-3, 4]$

$f'(x) = -2x + 0$

$f'(x) = -2x$

set $-2x = 0$

$\frac{-2x}{-2} = \frac{0}{-2}$

$x = 0$ critical point

$f(x) = -x^2 + 8$

$f(0) = -(0)^2 + 8$

$f(0) = -(0)(0) + 8$

$f(0) = 0 + 8$

$f(0) = 8$

$f(x) = -x^2 + 8$

$f(-3) = -(-3)^2 + 8$

$f(-3) = -(-3)(-3) + 8$

$f(-3) = -9 + 8$

$f(-3) = -1$

$f(x) = -x^2 + 8$

$f(4) = -(4)^2 + 8$

$f(4) = -(4)(4) + 8$

$f(4) = -16 + 8$

$f(4) = -8$

absolute maximum is 8 at $x = 0$

absolute minimum is -8 at $x = 4$

(83) Find max

$s = -16t^2 + 64t + 336 \qquad 0 \le t \le 7$

$s' = -32t + 64 + 0$

$s' = -32t + 64$

Set $-32t + 64 = 0$

$-32t + \cancel{64} - \cancel{64} = 0 - 64$

$-32t = -64$

$\frac{-32t}{-32} = \frac{-64}{-32}$

$t = 2$ Critical point

$s(t) = -16t^2 + 64t + 336$

$s(2) = -16(2)^2 + 64(2) + 336$

$s(2) = -16(2)(2) + 64(2) + 336$

$s'(2) = -256 + 128 + 336$

$s'(2) = 208$

Max at $t = 2$

(84) find max

(a) $P(n) = n(45 - 0.5n) - 90$

$P(n) = 45n - 0.5n^2 - 90$

$P(n) = -0.5n^2 + 45n - 90$

$P'(n) = -0.5(2n) + 45 - 0$

$P'(n) = -1n + 45$

set $-1n + 45 = 0$

$-1n + \cancel{45} - \cancel{45} = 0 - 45$

$-1n = -45$

$\frac{-1n}{-1} = \frac{-45}{-1}$

$n = 45$

$P'(n) = -1n + 45$

$P'(44) = -1(44) + 45 = -44 + 45 = 1 > 0$ increasing

$f'(46) = -1(46) + 45 = -46 + 45 = -1 < 0$ decreasing

Maximum at $n = 45$

$P(45) = -0.5(45)^2 + 45(45) - 90$

$P(45) = 922.5$ max

(b) If the bus holds a maximum of 38 people the guide should take 38 people

(85.) $f(x) = x^3 \quad [-8, 8]$

at what points c does the conclusion of the MEAN VALUE theorem hold

$f'(x) = 3x^2$

$\frac{f(b)-f(a)}{b-a} = f'(c)$

Mean VALUE Theorem

$\frac{f(8)-f(-8)}{(8)-(-8)} = 3x^2$

$\frac{(8)^3-(-8)^3}{8+8} = 3x^2$

$\frac{(512)-(-512)}{16} = 3x^2$

$\frac{512+512}{16} = 3x^2$

$\frac{1024}{16} = 3x^2$

$64 = 3x^2$

$\frac{64}{3} = \frac{\cancel{3}x^2}{\cancel{3}}$

$\frac{64}{3} = x^2$

$\pm\sqrt{\frac{64}{3}} = \sqrt{x^2}$

$\pm\frac{\sqrt{64}}{\sqrt{3}} = x$

$\pm\frac{8}{\sqrt{3}} = x$

$\frac{\pm 8\sqrt{3}}{\sqrt{3}\sqrt{3}} = x$

$\frac{\pm 8\sqrt{3}}{\sqrt{9}} = x$

$\frac{\pm 8\sqrt{3}}{3} = x$

$x = \frac{8\sqrt{3}}{3}$ or $x = -\frac{8\sqrt{3}}{3}$

86. determine whether the Mean Value theorem applies to the function

$f(x) = -7 + x^2$ on interval $[-1, 2]$
a b

$f'(x) = 0 + 2x$

$f'(x) = 2x$

$$\frac{f(b)-f(a)}{b-a} = 2x = f'(c)$$

Mean Value theorem

$$\frac{f(2)-f(-1)}{(2)-(-1)} = 2x$$

$$\frac{(-7+(2)^2)-(-7+(-1)^2)}{(2)-(-1)} = 2x$$

$$\frac{(-7+4)-(-7+1)}{2+1} = 2x$$

$$\frac{(-3)-(-6)}{3} = 2x$$

$$\frac{-3+6}{3} = 2x$$

$$\frac{3}{3} = 2x$$

$1 = 2x$

$$\frac{1}{2} = \frac{2x}{2}$$

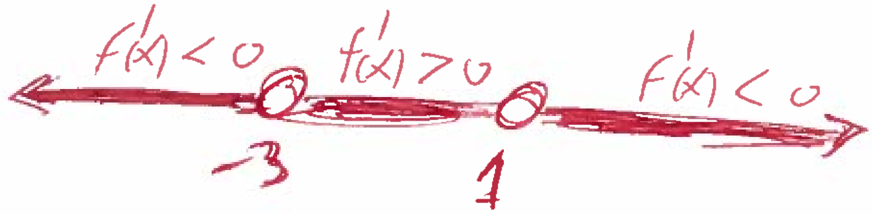
$$\frac{1}{2} = x$$

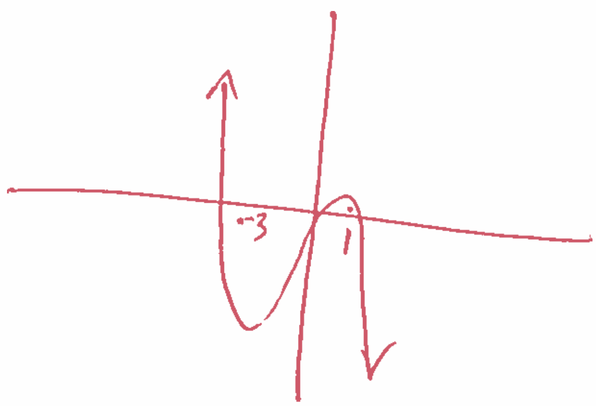
(87) Sketch a function that is continuous on $(-\infty, \infty)$ and has the following properties.

$f'(x) < 0$ on $(-\infty, -3)$

$f'(x) > 0$ on $(-3, 1)$

$f'(x) < 0$ on $(1, \infty)$





(88) Find the intervals on which f is increasing and intervals on which f is decreasing.

$f(x) = -5 + x^2$

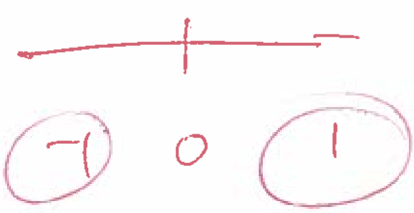
$f'(x) = 0 + 2x$

$f'(x) = 2x$

set $2x = 0$

$\frac{2x}{2} = \frac{0}{2}$

$x = 0$



$f'(x) = 2x$

$f'(-1) = 2(-1) = -2 < 0$ decreasing $(-\infty, 0)$

$f'(1) = 2(1) = 2 > 0$ increasing $(0, \infty)$

(89) find intervals on which f is increasing and intervals on which f is decreasing.

$$f(x) = -4 + x - 2x^2$$

$$f'(x) = 0 + 1 - 4x$$

$$f'(x) = 1 - 4x$$

set $1 - 4x = 0$

$$\cancel{1} - 4x \cancel{-1} = 0 - 1$$

$$-4x = -1$$

$$\frac{-4x}{-4} = \frac{-1}{-4}$$

$$x = \frac{1}{4}$$

0     $\frac{1}{4}$ 1

$$f'(x) = 1 - 4x$$

$f'(0) = 1 - 4(0) = 1 - 0 = 1 > 0$ increasing $(-\infty, \frac{1}{4})$

$f'(1) = 1 - 4(1) = 1 - 4 = -3 < 0$ decreasing on $(\frac{1}{4}, \infty)$

(90) locate the critical points then use the second derivative test to determine whether they correspond to local maxima or local minima or neither.

$f(x) = x^3 - 6x^2$

$f'(x) = 3x^2 - 12x$

set $3x^2 - 12x = 0$

$3x(x-4) = 0$

$3x = 0$ or $x - 4 = 0$

$\frac{3x}{3} = \frac{0}{3}$ or $x - \cancel{4} + \cancel{4} = 0 + 4$

$x = 0$ OR $x = 4$ Critical points

$f'(x) = 3x^2 - 12x$

$f''(x) = 6x - 12$

$f''(0) = 6(0) - 12 = 0 - 12 = -12 < 0$ concave down max

$f''(4) = 6(4) - 12 = 24 - 12 = 12 > 0$ concave up min

Local maximum at $x = 0$

Local minimum at $x = 4$

(91) Find critical points, max, min.

$f(x) = 3 - 4x^2$

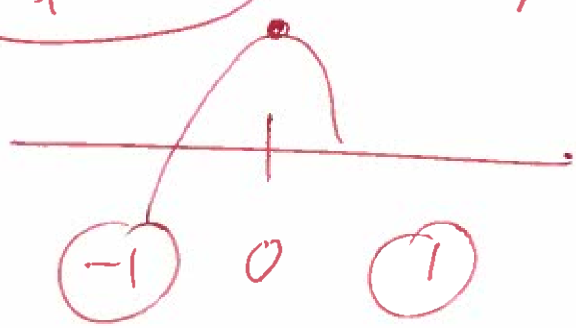
$f'(x) = 0 - 8x$

$f'(x) = -8x$

set $-8x = 0$

$$\frac{-8x}{-8} = \frac{0}{-8}$$

$x = 0$ critical point(s)



$f'(x) = -8x$

$f'(-1) = -8(-1) = 8 > 0$ increasing on $(-\infty, 0)$

$f'(1) = -8(1) = -8 < 0$ decreasing on $(0, \infty)$

Maximum at $x = 0$

(92) Locate the critical points at max or min.

$f(x) = 3x^3 + 9x^2 + 16$

$f'(x) = 9x^2 + 18x + 0$

$f'(x) = 9x^2 + 18x$

set $9x^2 + 18x = 0$

$9x(x+2) = 0$

$9x = 0$ or $x + 2 = 0$

$\frac{9x}{9} = \frac{0}{9}$ or $x + 2 - 2 = 0 - 2$

$x = 0$ or $x = -2$ Critical points

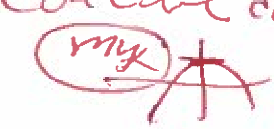
$f'(x) = 9x^2 + 18x$

$f''(x) = 18x + 18$

$f''(x) = 18x + 18$

$f''(0) = 18(0) + 18 = 0 + 18 > 0$ concave up

$f''(-2) = 18(-2) + 18 = -36 + 18 = -18 < 0$ concave down



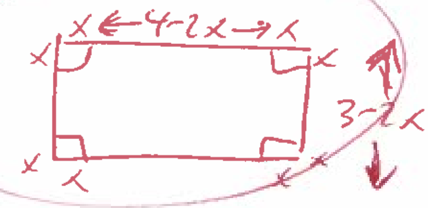
Local minimum at $x = 0$

Local maximum at $x = -2$

93 (a) Part

Find Max

$V(x) = x(4-2x)(3-2x)$



$V(x) = x(12 - 8x - 6x + 4x^2)$

$V(x) = x(12 - 14x + 4x^2)$

$V(x) = 12x - 14x^2 + 4x^3$

$V(x) = 4x^3 - 14x^2 + 12x$

$V'(x) = 12x^2 - 28x + 12$

$V''(x) = 24x - 28$

set $12x^2 - 28x + 12 = 0$

$a = 12, \quad b = -28, \quad c = 12$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-28) \pm \sqrt{(-28)^2 - 4(12)(12)}}{2(12)}$

$x = \dfrac{28 \pm \sqrt{784 - 576}}{24}$

$x = \dfrac{28 \pm \sqrt{208}}{24}$

$x = \dfrac{28 + \sqrt{208}}{24}$ or $x = \dfrac{28 - \sqrt{208}}{24}$

$x = (28 + \sqrt{208})/(24)$ OR $x = (28 - \sqrt{208})/(24)$

$x = 1.767591879$ OR $x = .5657414541$

Critical Value

MAX at $x = .5657$

$V(x) = x(4-2x)(3-2x)$

$V(.5657) = (.5657)(4 - 2(.5657))(3 - 2(.5657))$

$V(.5657) = 3.03230$

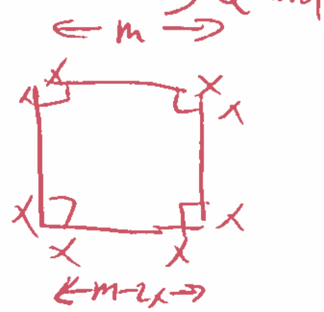
MAX

$V''(1.7675) = 24(1.7675) - 28 = 14.42 > 0$ Concave up Min

$V''(.5657) = 24(.5657) - 28 = -14.4232 < 0$ Concave down (Max)

Find max   let ($s = m$)

93 (b) part 2



$V(x) = x(m-2x)(m-2x)$

$V(x) = x(m^2 - 2mx - 2mx + 4x^2)$

$V(x) = x(m^2 - 4mx + 4x^2)$

$V(x) = m^2x - 4mx^2 + 4x^3$

$V(x) = 4x^3 - 4mx^2 + m^2x$

$V'(x) = 12x^2 - 8mx + m^2$

$a = 12, \quad b = -8m, \quad c = m^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8m) \pm \sqrt{(-8m)^2 - 4(12)(m^2)}}{2(12)}$$

$$x = \frac{8m \pm \sqrt{64m^2 - 48m^2}}{24}$$

$$x = \frac{8m \pm \sqrt{16m^2}}{24}$$

$$x = \frac{8m \pm 4m}{24}$$

$$x = \frac{8m + 4m}{24} \quad \text{or} \quad x = \frac{8m - 4m}{24}$$

$$x = \frac{12m}{24} \qquad x = \frac{4m}{24}$$

$$x = \frac{\cancel{12}(1)m}{\cancel{12}(2)} = \frac{m}{2} \qquad x = \frac{\cancel{4}(m)}{\cancel{4}(6)}$$

$$x = \frac{m}{6}$$

$V''(x) = 24x - 8m$

$V''\left(\frac{m}{2}\right) = 24\left(\frac{m}{2}\right) - 8m$

$= 12m - 8m$

$= 4m > 0$ concave up

min

$V''\left(\frac{m}{6}\right) = 24\left(\frac{m}{6}\right) - 8m$

$= 4m - 8m$

$= -4m < 0$ concave down

MAX at $x = \frac{m}{6}$ max

$V(x) = x(m-2x)(m-2x)$

$V\left(\frac{m}{6}\right) = \frac{m}{6}\left(m - 2\left(\frac{m}{6}\right)\right)\left(m - 2\left(\frac{m}{6}\right)\right)$

$V\left(\frac{m}{6}\right) = \frac{m}{6}\left(m - \frac{m}{3}\right)\left(m - \frac{m}{3}\right)$

$V\left(\frac{m}{6}\right) = \frac{m}{6}\left(\frac{3m}{3} - \frac{m}{3}\right)\left(\frac{3m}{3} - \frac{m}{3}\right)$

$= \frac{m}{6}\left(\frac{2m}{3}\right)\left(\frac{2m}{3}\right) = \frac{4m^3}{54} = \frac{\cancel{2}(2)m^3}{\cancel{2}(27)} = \frac{2m^3}{27}$ max

(94) use a linear approximation to estimate the quantity. Choose a value of a to produce a small error

$\ln(0.97) =$

$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

$f(x) = \ln(x)$

$f(1) = \ln(1)$

$f(1) = 0$

$L(x) = f(a) + f'(a)(x - a)$

$L(0.97) = f(1) + f'(1)(0.97 - 1)$

$L(0.97) = \ln(1) + \frac{1}{(1)}(0.97 - 1)$

$L(0.97) = 0 + 1(0.97 - 1)$

$L(0.97) = 0 + 1(-0.03)$

$L(0.97) = 0 - 0.03$

$L(0.97) = -0.03$ ✓

(95) Consider the function and express the relationship between a small change in x and the corresponding change in y in the form $dy = f'(x)\,dx$

$f(x) = 2x^3 - 6x$

$f'(x) = 6x^2 - 6$

$dy = f'(x)\,dx$

$\frac{dy}{dx} = 6x^2 - 6$

$\frac{dy}{\cancel{dx}}(\cancel{dx}) = (6x^2 - 6)\,dx$

$dy = (6x^2 - 6)\,dx$

96 Consider the function and express the relationship between a small change in x and the corresponding change in y in the form

Ex $dy = f'(x)\,dx$

$f(x) = \cot(10x)$

$f'(x) = -\csc^2(10x) \cdot (10x)'$

$f'(x) = -\csc^2(10x)\,(10)$

$f'(x) = -10\csc^2(10x)$

$\frac{dy}{dx} = -10\csc^2(10x)$

$\frac{dy}{\cancel{dx}}(\cancel{dx}) = \left(-10\csc^2(10x)\right)dx$

$dy = \left(-10\csc^2(10x)\right)dx$

97 evaluate use L'Hopital Rule

$$\lim_{x \to 0} \frac{2\sin(9x)}{7x} =$$

$$\lim_{x \to 0} \frac{2\cos(9x) \cdot (9x)'}{(7x)'} =$$

$$\lim_{x \to 0} \frac{2\cos(9x)(9)}{7} =$$

$$\lim_{x \to 0} \frac{18\cos(9x)}{7} =$$

$$\frac{18\cos(9(0))}{7} =$$

$$\frac{18\cos(0)}{7} =$$

$$\frac{18(1)}{7} =$$

$$\frac{18}{7} =$$

98. Use a calculator or program to compute the first 10 iterations of Newton's method when they are applied to the following function with the given initial approximation.

$f(x) = x^2 - 8;\ x_0 = 3$ $f'(x) = 2x$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 3 - \frac{f(3)}{f'(3)}$$

$$x_1 = 3 - \frac{(3)^2 - 8}{2(3)}$$

$$x_1 = 3 - ((3)^2 - 8) \div (2(3))$$

$$x_1 = 2.833333$$

k	x_k	k	x_k
0	3.000000	6	2.828427
1	2.833333	7	2.828427
2	2.828431	8	2.828427
3	2.828427	9	2.828427
4	2.828427	10	2.828427
5	2.828427		

(Round to six decimal places as needed.)

Answers 3.000000

2.828427

2.833333

2.828427

2.828431

2.828427

2.828427

2.828427

2.828427

2.828427

2.828427

99. Use a calculator or program to compute the first 10 iterations of Newton's method for the given function and initial approximation.

$f(x) = 3\sin x + 4x - 1,\ x_0 = 1.6$ $f'(x) = 3\cos(x) + 4$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

Complete the table.
(Do not round until the final answer. Then round to six decimal places as needed.)

k	x_k	k	x_k
1	-0.546692	6	0.143066
2	0.176536	7	0.143066
3	0.143026	8	0.143066
4	0.143066	9	0.143066
5	0.143066	10	0.143066

$x_1 = 1.6 - \frac{f(1.6)}{f'(1.6)}$

Answers −0.546692

0.143066

0.176536

0.143066

0.143026

0.143066

0.143066

0.143066

0.143066

0.143066

$x_1 = 1.6 - \frac{3\sin(1.6) + 4(1.6) - 1}{3\cos(1.6) + 4}$

$x_1 = -0.546692$

100. Determine the following indefinite integral. Check your work by differentiation.

$$\int \left(11x^{21} - 5x^9\right) dx$$

$\int \left(11x^{21} - 5x^9\right) dx =$ ______ (Use C as the arbitrary constant.)

Answer: $\frac{x^{22}}{2} - \frac{x^{10}}{2} + C$

$\int (11x^{21} - 5x^{9})\,dx =$

$\frac{11x^{21+1}}{21+1} - \frac{5x^{9+1}}{9+1} + C =$

$\frac{11x^{22}}{22} - \frac{5x^{10}}{10} + C =$

formula

$\int x^n\,dx =$

$x^{n+1} + C =$

$\frac{\cancel{11}x^{22}}{\cancel{11}(2)} - \frac{\cancel{5}x^{10}}{\cancel{5}(2)} + C =$

$\frac{x^{22}}{2} - \frac{x^{10}}{2} + C =$

(101) $\int \left(\frac{3}{\sqrt{x}} + 3\sqrt{x}\right)dx =$

$\int \left(\frac{3}{x^{1/2}} + 3x^{1/2}\right)dx =$

$\int \left(3x^{-1/2} + 3x^{1/2}\right)dx =$

$\frac{3x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{3x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C =$

$\frac{3x^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}} + \frac{3x^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C =$

$\frac{3x^{\frac{-1+2}{2}}}{\frac{-1+2}{2}} + \frac{3x^{\frac{1+2}{2}}}{\frac{1+2}{2}} + C =$

$\frac{3x^{1/2}}{\frac{1}{2}} + \frac{3x^{3/2}}{\frac{3}{2}} + C =$

$\frac{2}{1}\,3x^{1/2} + \cancel{\frac{2}{3}}\left(\cancel{3}x^{3/2}\right) + C =$

$6x^{1/2} + 2x^{3/2} + C =$

$6\sqrt{x} + 2x^{3/2} + C =$

formula

$\int x^n dx = \frac{x^{n+1}}{n+1} + C =$

102

$\int (3x+2)^2 \, dx =$

$\frac{1}{3} \int (3x+2)^2 (3) \, dx =$

$\frac{1}{3} \frac{(3x+2)^{2+1}}{2+1} + C =$

$\frac{1}{3} \frac{(3x+2)^3}{3} + C =$

$\frac{1}{9} (3x+2)^3 + C =$

OR

$\int (3x+2)^2 \, dx =$

$\int (3x+2)(3x+2) \, dx =$

$\int (9x^2 + 6x + 6x + 4) \, dx =$

$\int (9x^2 + 12x + 4) \, dx =$

$\frac{9x^{2+1}}{2+1} + \frac{12x^{1+1}}{1+1} + 4x + C =$

$\frac{9x^3}{3} + \frac{12x^2}{2} + 4x + C =$

$3x^3 + 6x^2 + 4x + C =$

Same

formula

$\int x^n \, dx =$

$\frac{x^{n+1}}{n+1} + C =$

$\int a \, dx =$

$ax + C =$

(103)

$$\int 4m(7m^2 - 10m)\,dm =$$

$$\int (28m^3 - 40m^2)\,dm =$$

$$\frac{28m^{3+1}}{3+1} - \frac{40m^{2+1}}{2+1} + C =$$

$$\frac{28m^4}{4} - \frac{40m^3}{3} + C =$$

$$7m^4 - \frac{40m^3}{3} + C =$$

formula

$$\int x^n\,dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

(104)

$$\int \left(4x^{\frac{1}{3}} + 2x^{-\frac{2}{3}} + 12\right) dx =$$

$$\frac{4x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + \frac{2x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + 12x + c =$$

$$\frac{4x^{\frac{1}{3}+\frac{3}{3}}}{\frac{1}{3}+\frac{3}{3}} + \frac{2x^{-\frac{2}{3}+\frac{3}{3}}}{-\frac{2}{3}+\frac{3}{3}} + 12x + c =$$

$$\frac{4x^{\frac{1+3}{3}}}{\frac{1+3}{3}} + \frac{2x^{\frac{-2+3}{3}}}{\frac{-2+3}{3}} + 12x + c =$$

$$\frac{4x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{2x^{\frac{1}{3}}}{\frac{1}{3}} + 12x + c =$$

$$\frac{3}{\cancel{4}} \cdot \cancel{4}x^{\frac{4}{3}} + \frac{3}{1} \cdot 2x^{\frac{1}{3}} + 12x + c =$$

$$3x^{\frac{4}{3}} + 6x^{\frac{1}{3}} + 12x + c =$$

Formula

$$\int a\, dx = ax + c =$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c =$$

105

$$\int 3\sqrt[8]{x}\,dx =$$

$$\int 3x^{1/8}\,dx =$$

$$\frac{3x^{\frac{1}{8}+1}}{\frac{1}{8}+1} + C =$$

$$\frac{3x^{\frac{1}{8}+\frac{8}{8}}}{\frac{1}{8}+\frac{8}{8}} + C =$$

$$\frac{3x^{\frac{1+8}{8}}}{\frac{1+8}{8}} + C =$$

$$\frac{3x^{\frac{9}{8}}}{9/8} + C =$$

$$\frac{8}{9} \cdot 3x^{9/8} + C =$$

$$\frac{8\cancel{(3)}}{(3)\cancel{(3)}} x^{9/8} + C =$$

$$\frac{8}{3} x^{9/8} + C =$$

106

$$\int (6x+5)(7-x)\,dx =$$

$$\int (42x - 6x^2 + 35 - 5x)\,dx =$$

$$\int (-6x^2 + 37x + 35)\,dx =$$

$$\frac{-6x^{2+1}}{2+1} + \frac{37x^{1+1}}{1+1} + 35x + C =$$

$$\frac{-6x^3}{3} + \frac{37x^2}{2} + 35x + C =$$

$$-2x^3 + \frac{37x^2}{2} + 35x + C =$$

Formula

$$\int a x\,dx$$

$$\frac{a x^{1+1}}{1+1} + C$$

$$\int x^n\,dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

$$\int a\,dx =$$

$$ax + c =$$

107

$$\int \frac{4x^4+6x^3}{x}\,dx =$$

$$\int \left(\frac{4x^4}{x^1}+\frac{6x^3}{x^1}\right)dx =$$

$$\int \left(4x^{4-1}+6x^{3-1}\right)dx =$$

$$\int \left(4x^3+6x^2\right)dx =$$

$$\frac{4x^{3+1}}{3+1}+\frac{6x^{2+1}}{2+1}+C =$$

$$\frac{4x^4}{4}+\frac{6x^3}{3}+C =$$

$$x^4+2x^3+C =$$

formula

$$\int x^n\,dx$$

$$\frac{x^{n+1}}{n+1}+C$$

$$\int a\,dx =$$

$$ax+c =$$

(108) For f find the antiderivative F that satisfies the given condition.

$$f(x) = 6x^3 + 9\sin(x), \quad F(0) = 4$$

$$\int f(x)\,dx = \int \left(6x^3 + 9\sin(x)\right)dx$$

$$F(x) = \frac{6x^{3+1}}{3+1} - 9\cos(x) + C$$

$$F(x) = \frac{6x^4}{4} - 9\cos(x) + C$$

$$F(0) = \frac{6(0)^4}{4} - 9\cos(0) + C = 4$$

$$\frac{6(0)(0)(0)(0)}{4} - 9\cos(0) + C = 4$$

$$0 - 9(1) + C = 4$$

$$0 - 9 + C = 4$$

$$-9 + C = 4$$

$$\cancel{-9} + C \cancel{+9} = 4 + 9$$

$$C = 13$$

$$F(x) = \frac{3x^4}{2} - 9\cos(x) + 13$$

(109) for f find the antiderivative F that satisfies the given condition

$f(u) = 9e^u + 9$ $\quad F(0) = 4$

$$\int f(u)\,du = \int (9e^u + 9)\,du$$

$$F(u) = 9e^u + 9u + C$$

$$F(0) = 9e^{(0)} + 9(0) + C = 4$$

$$9e^0 + 9(0) + C = 4$$

$$9(1) + 0 + C = 4$$

$$9 + 0 + C = 4$$

$$9 + C = 4$$

$$\cancel{9} + C \cancel{-9} = 4 - 9$$

$$C = -5$$

$$F(u) = 9e^u + 9u - 5$$

(110) given the velocity function of an object moving along a line, find the position function with the given initial position

$$V(t) = 9t^2 + 2t - 5 \qquad s(0) = 0$$

$$\int V(t)\,dt = \int (9t^2 + 2t - 5)\,dt$$

$$s(t) = \frac{9t^{2+1}}{2+1} + \frac{2t^{1+1}}{1+1} - 5t + C$$

$$s(t) = \frac{9x^3}{3} + \frac{2t^2}{2} - 5t + C$$

$$s(t) = 3t^3 + t^2 - 5t + C$$

$$s(0) = 3(0)^3 + (0)^2 - 5(0) + C = 0$$

$$3(0)(0)(0) + (0)(0) - 5(0) + C = 0$$

$$\cancel{0} + \cancel{0} - \cancel{0} + C = 0$$

$$C = 0$$

$$s(t) = 3t^3 + t^2 - 5t$$

111. Find the absolute extreme values of the function on the interval

$F(x) = \sqrt[3]{x}$ $\quad -8 \le x \le 27$

$\qquad\qquad\qquad a \qquad\quad b$

$F(x) = x^{1/3}$

$F'(x) = \frac{1}{3}x^{1/3 - 1}$

$F'(x) = \frac{1}{3}x^{\frac{1}{3} - \frac{3}{3}}$

$F'(x) = \frac{1}{3}x^{\frac{1-3}{3}}$

$F'(x) = \frac{1}{3}x^{-2/3}$

$F'(x) = \dfrac{1}{3x^{2/3}}$ undefined if $x = 0$

Critical point

$F(x) = \sqrt[3]{x}$

$F(0) = \sqrt[3]{0} = 0$

absolute maximum is 3 at $x = 27$

absolute minimum is -2 at $x = -8$

$F(x) = \sqrt[3]{x}$

$F(-8) = \sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

$F(x) = \sqrt[3]{x}$

$F(27) = \sqrt[3]{27} = \sqrt[3]{(3)^3} = 3$

---

(112) Find the value c that satisfy the equation $\frac{f(b)-f(a)}{b-a} = f'(c)$ in the conclusion of the Mean Value theorem, for the function and interval

$f(x) = x^2 + 4x + 2$ $\quad [-3, 2]$ ($a = -3$, $b = 2$)

$f(x) = x^2 + 4x + 2$

$f'(x) = 2x + 4 + 0$

$f'(x) = 2x + 4$

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

$$\frac{f(2) - f(-3)}{(2) - (-3)} = 2x + 4$$

$$\frac{((2)^2 + 4(2) + 2) - ((-3)^2 + 4(-3) + 2)}{2 + 3} = 2x + 4$$

$$\frac{(4 + 8 + 2) - (9 - 12 + 2)}{2 + 3} = 2x + 4$$

$$\frac{(14) - (-1)}{2 + 3} = 2x + 4$$

$$\frac{14 + 1}{5} = 2x + 4$$

$$\frac{15}{5} = 2x + 4$$

$$3 = 2x + 4$$

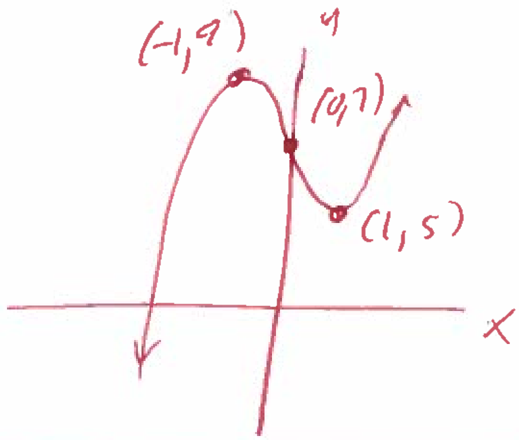
$$3 - 4 = 2x + 4 - 4$$

$$-1 = 2x$$

$$\frac{-1}{2} = \frac{2x}{2}$$

$$-\frac{1}{2} = x$$

113



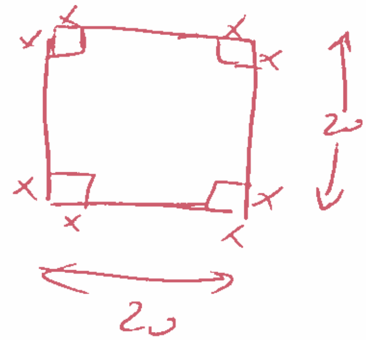
Local minimum at $x=1$

Local Maximum at $x=-1$

Concave up on $(0,\infty)$

Concave down on $(-\infty, 0)$

(114) find max value



$V(x) = x(20-2x)(20-2x)$

$V(x) = x(400 - 40x - 40x + 4x^2)$

$V(x) = x(400 - 80x + 4x^2)$

$V(x) = 400x - 80x^2 + 4x^3$

$V(x) = 4x^3 - 80x^2 + 400x$

$V'(x) = 12x^2 - 160x + 400$

$V'(x) = 4(3x^2 - 40x + 100)$

$V'(x) = 4(3x-10)(x-10)$

$V'(x) = 4(3x-10)(x-10) = 0$

$\cancel{4 = 0}$

$3x - 10 = 0$

$3x - 10 + 10 = 0 + 10$

$3x = 10$

$\frac{3x}{3} = \frac{10}{3}$

$x = \frac{10}{3}$

$x - 10 = 0$

$x - 10 + 10 = 0 + 10$

$x = 10$

$V'(x) = 12x^2 - 160x + 400$

$V''(x) = 24x - 160 + 0$

$V''(x) = 24x - 160$

$V''\left(\frac{10}{3}\right) = 24\left(\frac{10}{3}\right) - 160 = 8(10) - 160 = 80 - 160 = -80 < 0$

max concave down $\cap$

$V''(10) = 24(10) - 160 = 240 - 160 = 80 > 0$ concave up $\cup$ min

$V(x) = \frac{10}{3}\left(20 - 2\left(\frac{10}{3}\right)\right)\left(20 - 2\left(\frac{10}{3}\right)\right) = 3.3(13.3)(13.3) = 592.6 \text{ in}^3$

115) Solve the initial value problem

$$\frac{ds}{dt} = \cos(t) - \sin(t), \qquad s\left(\frac{\pi}{2}\right) = 7$$

$$\int ds = \int \left(\cos(t) - \sin(t)\right) dt$$

$$s = \sin(t) + \cos(t) + C$$

$$s(t) = \sin(t) + \cos(t) + C$$

$$s\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) + C = 7$$

$$\sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) + C = 7$$

$$(1) + 0 + C = 7$$

$$1 + C = 7$$

$$\cancel{1} + C - \cancel{1} = 7 - 1$$

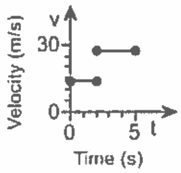
$$C = 6$$

$$s = \sin(t) + \cos(t) + 6$$

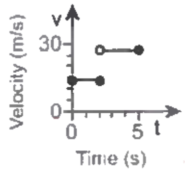
116. Suppose an object moves along a line at 14 m/s for $0 \le t \le 2$ s and at 27 m/s for $2 < t \le 5$ s. Sketch the graph of the velocity function and find the displacement of the object for $0 \le t \le 5$.

Sketch the graph of the velocity function. Choose the correct graph below.

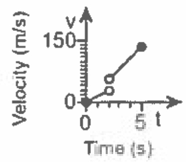
A.



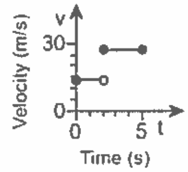
✓ B.



C.

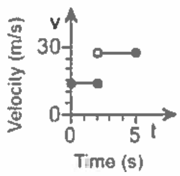


D.



The displacement of the object for $0 \le t \le 5$ is 109 m. (Simplify your answer.)

Answers



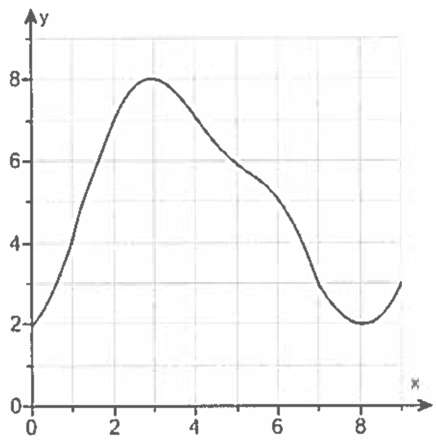
B.

109

117.

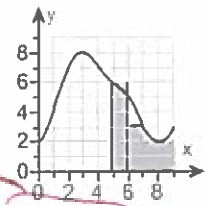
Put!

Approximate the area of the region bounded by the graph of f(x) (shown below) and the x-axis by dividing the interval [5,9] into n = 4 subintervals. Use a left and right Riemann sum to obtain two different approximations. Draw the approximating rectangles.

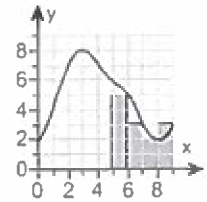


In which graph below are the selected points the left endpoints of the 4 approximating rectangles?

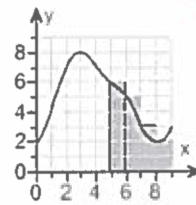
A.



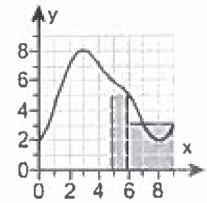
B.



C.



D.

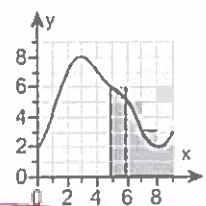


Using the specified rectangles, approximate the area.

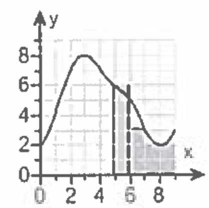
16

In which graph below are the selected points the right endpoints of the 4 approximating rectangles?

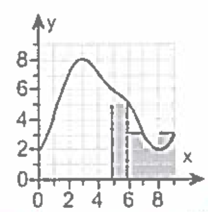
A.



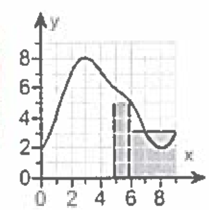
B.



C.



D.

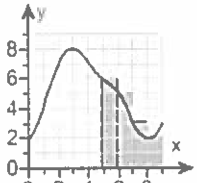


Using the specified rectangles, approximate the area.

13

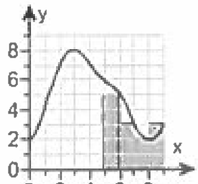
Answers

117 part 2



C.

16



C.

13

118. Evaluate the following expressions.

a. $\sum_{k=1}^{16} k$ b. $\sum_{k=1}^{9} (4k+1)$ c. $\sum_{k=1}^{7} k^2$ d. $\sum_{n=1}^{8} \left(1+n^2\right)$

e. $\sum_{m=1}^{3} \frac{3m+3}{8}$ f. $\sum_{j=1}^{3} (5j-7)$ g. $\sum_{k=1}^{9} k(5k+8)$ h. $\sum_{n=0}^{7} \sin\frac{n\pi}{2}$

a. $\sum_{k=1}^{16} k =$ 136 (Type an integer or a simplified fraction.)

b. $\sum_{k=1}^{9} (4k+1) =$ 189 (Type an integer or a simplified fraction.)

c. $\sum_{k=1}^{7} k^2 =$ 140 (Type an integer or a simplified fraction.)

d. $\sum_{n=1}^{8} \left(1+n^2\right) =$ 212 (Type an integer or a simplified fraction.)

e. $\sum_{m=1}^{3} \frac{3m+3}{8} =$ $\frac{27}{8}$ (Type an integer or a simplified fraction.)

f. $\sum_{j=1}^{3} (5j-7) =$ 9 (Type an integer or a simplified fraction.)

g. $\sum_{k=1}^{9} k(5k+8) =$ 1785 (Type an integer or a simplified fraction.)

h. $\sum_{n=0}^{7} \sin\frac{n\pi}{2} =$ 0 (Type an integer or a simplified fraction.)

Use Graphing Calculator

① Math
② Summation Σ

Answers 136

189

140

212

$\frac{27}{8}$

9

1785

0

(119) the functions f and g are integrable

$\int_1^7 f(x)\,dx = 8$, $\int_1^7 g(x)\,dx = 5$, $\int_5^7 f(x)\,dx = 2$

find

$$-\int_7^1 4f(x)\,dx =$$

$$\int_1^7 4f(x)\,dx =$$ rewrite

$$4\int_1^7 f(x)\,dx =$$

$$4(8) =$$

$$32 =$$

(120)

$$\frac{d}{dx}\int_a^x f(t)\,dt$$ and $$\frac{d}{dx}\int_a^b f(t)\,dt$$ when a and b are constants

$$\frac{d}{dx}\int_a^x f(t)\,dt =$$

$$f(x) =$$

$$\frac{d}{dx}\int_a^b f(t)\,dt =$$

$$0 =$$

(121) $\int_0^1 (x^2-2x+7)\,dx$

$\left(\frac{x^{2+1}}{2+1} - \frac{2x^{1+1}}{1+1} + 7x\right)\Big|_0^1 =$

$\left(\frac{x^3}{3} - \frac{2x^2}{2} + 7x\right)\Big|_0^1 =$

$\left(\frac{x^3}{3} - x^2 + 7x\right)\Big|_0^1 =$

$\left(\frac{(1)^3}{3} - (1)^2 + 7(1)\right) - \left(\frac{(0)^3}{3} - (0)^2 + 7(0)\right) =$

$\left(\frac{1}{3} - 1 + 7\right) - (0 - 0 + 0) =$

$\left(\frac{1}{3} + 6\right) - (0) =$

$\left(\frac{1}{3} + \frac{6}{1}\left(\frac{3}{3}\right)\right) - (0) =$

$\left(\frac{1}{3} + \frac{18}{3}\right) - (0) =$

$\left(\frac{1+18}{3}\right) - (0) =$

$\left(\frac{19}{3}\right) - (0) =$

$\frac{19}{3} - 0 =$ $\frac{19}{3}$

(122) $\int_{-\frac{\pi}{4}}^{7\pi/4} (\sin(x) + \cos(x))\, dx =$

$(-\cos(x) + \sin(x)) \Big|_{-\frac{\pi}{4}}^{\frac{7\pi}{4}} =$

$\left(-\cos\left(\frac{7\pi}{4}\right) + \sin\left(\frac{7\pi}{4}\right)\right) - \left(-\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)\right) =$

$(-.7071 \; -.7071) - (-.7071 - .7071) =$

$(-1.4142) - (-1.4142) =$

$-1.4142 + 1.4142 =$

$0 =$

(123)

$$\int_{-1}^{5} (x^2-4x-5)\,dx =$$

$$\left(\frac{x^{2+1}}{2+1} - \frac{4x^{1+1}}{1+1} - 5x\right)\Big|_{-1}^{5} =$$

$$\left(\frac{x^3}{3} - \frac{4x^2}{2} - 5x\right)\Big|_{-1}^{5} =$$

$$\left(\frac{x^3}{3} - 2x^2 - 5x\right)\Big|_{-1}^{5} =$$

$$\left(\frac{(5)^3}{3} - 2(5)^2 - 5(5)\right) - \left(\frac{(-1)^3}{3} - 2(-1)^2 - 5(-1)\right) =$$

$$\left(\frac{(5)(5)(5)}{3} - 2(5)(5) - 5(5)\right) - \left(\frac{(-1)(-1)(-1)}{3} - 2(-1)(-1) - 5(-1)\right) =$$

$$\left(\frac{125}{3} - 50 - 25\right) - \left(\frac{-1}{3} - 2 + 5\right) =$$

$$\left(\frac{125}{3} - 75\right) - \left(-\frac{1}{3} + 3\right) =$$

$$\left(\frac{125}{3} - \frac{75}{1}\left(\frac{3}{3}\right)\right) - \left(-\frac{1}{3} + \frac{3}{1}\left(\frac{3}{3}\right)\right) =$$

$$\left(\frac{125}{3} - \frac{225}{3}\right) - \left(-\frac{1}{3} + \frac{9}{3}\right) =$$

$$\left(\frac{125-225}{3}\right) - \left(\frac{-1+9}{3}\right) =$$

$$\left(-\frac{100}{3}\right) - \left(\frac{8}{3}\right) =$$

$$\frac{-100-8}{3} =$$

$$\frac{-108}{3} =$$

$$-36 =$$

12-4

$$\int_1^3 (4x^3+2)\,dx$$

$$\left(\frac{4x^{3+1}}{3+1}+2x\right)\Big|_1^3 =$$

$$\left(\frac{4x^4}{4}+2x\right)\Big|_1^3 =$$

$$\left(x^4+2x\right)\Big|_1^3$$

$$\left((3)^4+2(3)\right) - \left((1)^4+2(1)\right) =$$

$$\left((3)(3)(3)(3)+2(3)\right) - \left((1)(1)(1)(1)+2(1)\right) =$$

$$(81+6) - (1+2) =$$

$$(87) - (3) =$$

$$87 - 3 =$$

$$=$$

$$84$$

125.

$$\int_1^5 (1-x)(x-5)\,dx =$$

$$\int_1^5 (1x - 5 - x^2 + 5x)\,dx =$$

$$\int_1^5 (-x^2 + 6x - 5)\,dx =$$

$$\left(\frac{-x^{2+1}}{2+1} + \frac{6x^{1+1}}{1+1} - 5x\right)\Big|_1^5 =$$

$$\left(\frac{-x^3}{3} + \frac{6x^2}{2} - 5x\right)\Big|_1^5 =$$

$$\left(-\frac{x^3}{3} + 3x^2 - 5x\right)\Big|_1^5 =$$

$$\left(\frac{-(5)^3}{3} + 3(5)^2 - 5(5)\right) - \left(-\frac{(1)^3}{3} + 3(1)^2 - 5(1)\right) =$$

$$\left(-\frac{(5)(5)(5)}{3} + 3(5)(5) - 5(5)\right) - \left(-\frac{(1)(1)(1)}{3} + 3(1)(1) - 5(1)\right) =$$

$$\left(-\frac{125}{3} + 75 - 25\right) - \left(-\frac{1}{3} + 3 - 5\right) =$$

$$\left(-\frac{125}{3} + 50\right) - \left(-\frac{1}{3} - 2\right) =$$

$$\left(-\frac{125}{3} + \frac{50}{1}\left(\frac{3}{3}\right)\right) - \left(-\frac{1}{3} - \frac{2}{1}\left(\frac{3}{3}\right)\right) =$$

$$\left(\frac{-125}{3} + \frac{150}{3}\right) - \left(-\frac{1}{3} - \frac{6}{3}\right) =$$

$$\left(\frac{-125+150}{3}\right) - \left(\frac{-1-6}{3}\right) =$$

$$\left(\frac{25}{3}\right) - \left(-\frac{7}{3}\right) =$$

$$\frac{25}{3} + \frac{7}{3} =$$

$$\frac{25+7}{3} =$$

$$\frac{32}{3}$$

(126) find the area of the region bounded by the graph of f and the x-axis on the interval

$f(x) = x^2 - 20 \quad [-1, 1]$

$g(x) = 0$

$f(x) = x^2 - 20$

$\int_a^b (g(x) - f(x))\, dx$

$\int_{-1}^{1} ((0) - (x^2 - 20))\, dx =$

$\int_{-1}^{1} (0 - x^2 + 20)\, dx$

$\int_{-1}^{1} (-x^2 + 20)\, dx$

$\left(-\frac{x^{2+1}}{2+1} + 20x\right)\Big|_{-1}^{1}$

$\left(\frac{-x^3}{3} + 20x\right)\Big|_{-1}^{1}$

$\left(-\frac{(+1)^3}{3} + 20(+1)\right) - \left(\frac{-(-1)^3}{3} + 20(-1)\right) =$

$\left(-\frac{(1)(1)(1)}{3} + 20\right) - \left(-\frac{(-1)(-1)(-1)}{3} - 20\right) =$

$\left(-\frac{1}{3} + 20\right) - \left(\frac{1}{3} - 20\right) =$

$\left(-\frac{1}{3} + \frac{20}{1}\left(\frac{3}{3}\right)\right) - \left(\frac{1}{3} - \frac{20}{1}\left(\frac{3}{3}\right)\right) =$

$\left(-\frac{1}{3} + \frac{60}{3}\right) - \left(\frac{1}{3} - \frac{60}{3}\right)$

$\left(\frac{-1+60}{3}\right) - \left(\frac{1-60}{3}\right) =$

$\left(\frac{59}{3}\right) - \left(\frac{-59}{3}\right) =$

$\frac{59}{3} + \frac{59}{3} =$

$\frac{59+59}{3} =$

$\frac{118}{3} =$

127

$$\frac{d}{dx}\int_x^6 \sqrt{t^3+2}\,dt$$

$$\frac{d}{dx}(-1)\int_6^x \sqrt{t^3+2}\,dt$$

$$(-1)\sqrt{(x)^3+2} =$$

$$-\sqrt{x^3+2} =$$

128

$$\frac{d}{dx}\int_8^{x^3}\frac{dp}{p^2}$$

$$\frac{d}{dx}\int_8^{x^3}\frac{1}{p^2}\,dp$$

$$\frac{1}{(x^3)^2}\cdot(x^3)' =$$

$$\frac{1}{x^6}\cdot(3x^{3-1}) =$$

$$\frac{1}{x^6}\cdot 3x^2 =$$

$$\frac{3x^2}{x^6} =$$

$$\frac{3}{x^{6-2}} =$$

$$\frac{3}{x^4} =$$

(129)

$$\int_0^2 (x-3)^2 \, dx$$

$$\int_0^2 (x-3)^2 (1) \, dx$$

$$\frac{(x-3)^{2+1}}{2+1} \Big/_0^2$$

$$\frac{(x-3)^3}{3} \Big/_0^2$$

$$\left(\frac{(2-3)^3}{3}\right) - \left(\frac{(0-3)^3}{3}\right) =$$

$$\left(\frac{(-1)^3}{3}\right) - \left(\frac{(-3)^3}{3}\right) =$$

$$\left(\frac{(-1)(-1)(-1)}{3}\right) - \left(\frac{(-3)(-3)(-3)}{3}\right) =$$

$$\left(\frac{-1}{3}\right) - \left(\frac{-27}{3}\right) =$$

$$-\frac{1}{3} + \frac{27}{3} =$$

$$\frac{-1+27}{3} =$$

$$\frac{26}{3} =$$

formula

$$\int (f(x))^N \cdot f'(x) \, dx =$$

$$\frac{(f(x))^{N+1}}{N+1} + C =$$

(130) find the average value of the function over the interval

$f(x) = x(x-1) \qquad [\underset{a}{5}, \underset{b}{9}]$

$\frac{1}{b-a}\int_a^b f(x)\,dx =$

$\frac{1}{(9)-(5)}\int_5^9 x(x-1)\,dx =$

$\frac{1}{9-5}\int_5^9 (x^2-x)\,dx =$

$\frac{1}{4}\int_5^9 (x^2-x)\,dx =$

$\frac{1}{4}\left(\frac{x^{2+1}}{2+1}-\frac{x^{1+1}}{1+1}\right)\Big|_5^9 =$

$\frac{1}{4}\left(\frac{x^3}{3}-\frac{x^2}{2}\right)\Big|_5^9 =$

$\left(\frac{1}{4}\left(\frac{(9)^3}{3}-\frac{(9)^2}{2}\right)\right)-\left(\frac{1}{4}\left(\frac{(5)^3}{3}-\frac{(5)^2}{2}\right)\right) =$

$\left(\frac{1}{4}\left(\frac{729}{3}-\frac{81}{2}\right)\right)-\left(\frac{1}{4}\left(\frac{125}{3}-\frac{25}{2}\right)\right) =$

$\left(\frac{1}{4}\left(\frac{729}{3}\left(\frac{2}{2}\right)-\frac{81}{2}\left(\frac{3}{3}\right)\right)\right)-\left(\frac{1}{4}\left(\frac{125}{3}\left(\frac{2}{2}\right)-\frac{25}{2}\left(\frac{3}{3}\right)\right)\right) =$

$\left(\frac{1}{4}\left(\frac{1458}{6}-\frac{243}{6}\right)\right)-\left(\frac{1}{4}\left(\frac{250}{6}-\frac{75}{6}\right)\right) =$

$\left(\frac{1}{4}\left(\frac{1458-243}{6}\right)\right)-\left(\frac{1}{4}\left(\frac{250-75}{6}\right)\right) =$

$\left(\frac{1}{4}\left(\frac{1215}{6}\right)\right)-\left(\frac{1}{4}\left(\frac{175}{6}\right)\right) =$

$\frac{1215}{24}-\frac{175}{24} =$

$\frac{1215-175}{24} =$

$\frac{1040}{24} =$

$\frac{\cancel{8}(130)}{\cancel{8}(3)} =$

$\frac{130}{3}$

(131) the elevation of a path given by $f(x) = x^3 - 6x^2 + 8$ when x measures horizontal distance. find the average value for $[0, 4]$ OR $0 \le x \le 4$

a b

$$\frac{1}{b-a} \int_a^b f(x)\, dx =$$

$$\frac{1}{(4)-(0)} \int_0^4 (x^3 - 6x^2 + 8)\, dx =$$

$$\frac{1}{4-0} \int_0^4 (x^3 - 6x^2 + 8)\, dx =$$

$$\frac{1}{4}\left(\frac{x^{3+1}}{3+1} - \frac{6x^{2+1}}{2+1} + 8x\right)\Big|_0^4 =$$

$$\frac{1}{4}\left(\frac{x^4}{4} - \frac{6x^3}{3} + 8x\right)\Big|_0^4 =$$

$$\frac{1}{4}\left(\frac{x^4}{4} - 2x^3 + 8x\right)\Big|_0^4 =$$

$$\left(\frac{1}{4}\left(\frac{(4)^4}{4} - 2(4)^3 + 8(4)\right) - \left(\frac{1}{4}\left(\frac{(0)^4}{4} - 2(0)^2 + 8(0)\right)\right)\right) =$$

$$\left(\frac{1}{4}\left(\frac{256}{4} - 2(64) + 32\right)\right) - \left(\frac{1}{4}(0 - 0 + 0)\right) =$$

$$\left(\frac{1}{4}(64 - 128 + 32)\right) - \left(\frac{1}{4}(0)\right) =$$

$$\frac{1}{4}(-32) - \frac{1}{4}(0) =$$

$$\frac{-32}{4} - \frac{0}{4} =$$

$$-8 - 0 =$$

$$-8 =$$

(132) Find the points at which the function $f(x) = 8 - 6x$ equals its average value on the interval $[\underset{a}{0}, \underset{b}{6}]$.

$$\frac{1}{b-a}\int_a^b f(x)\,dx =$$

$$\frac{1}{(6)-(0)}\int_0^6 (8-6x)\,dx =$$

$$\frac{1}{6-0}\int_0^6 (8-6x)\,dx$$

$$\frac{1}{6}\int_0^6 (8-6x)\,dx$$

$$\frac{1}{6}\left(8x - \frac{6x^{1+1}}{1+1}\right)\Big|_0^6 =$$

$$\frac{1}{6}\left(8x - \frac{6x^2}{2}\right)\Big|_0^6 =$$

$$\frac{1}{6}\left(8x - 3x^2\right)\Big|_0^6 =$$

$$\frac{1}{6}\left(8(6) - 3(6)^2\right) - \frac{1}{6}\left(8(0) - 3(0)^2\right) =$$

$$\frac{1}{6}\left(48 - 3(6)(6)\right) - \frac{1}{6}(0 - 0) =$$

$$\frac{1}{6}(48 - 108) - \frac{1}{6}(0) =$$

$$\frac{1}{6}(-60) - 0 =$$

$$\frac{-60}{6} - 0 =$$

$$-10 - 0 =$$

$$-10 =$$

average value

$$8 - 6x = -10$$

$$\cancel{8} - 6x - \cancel{8} = -10 - 8$$

$$-6x = -18$$

$$\frac{-6x}{-6} = \frac{-18}{-6}$$

$$x = 3$$

133

$$\int 2x\,(x^2+8)^7\,dx =$$

$$\int (x^2+8)^7(2x)\,dx =$$

$$\frac{(x^2+8)^{7+1}}{7+1}+C=$$

$$\frac{(x^2+8)^8}{8}+C=$$

$$\frac{1}{8}(x^2+8)^8+C=$$

formula

$$\int (f(x))^N\cdot f'(x)\,dx =$$

$$\frac{(f(x))^{N+1}}{N+1}+C=$$

(134) $\int -10x \sin(5x^2+2)\, dx =$

$\int -\sin(5x^2+2)(10x)\, dx =$

$\cos(5x^2+2) + C =$

formula

$\int -\sin(f(x)) \cdot f'(x)\, dx =$

$\cos(f(x)) + C =$

135

$$\int (10x+4)\sqrt{5x^2+4x}\,dx =$$

$$\int \sqrt{5x^2+4x}\,(10x+4)\,dx =$$

$$\int (5x^2+4x)^{1/2}(10x+4)\,dx =$$

formula

$$\int (f(x))^{N} \cdot f'(x)\,dx = \frac{(f(x))^{N+1}}{N+1} + C =$$

$$\frac{(5x^2+4)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C =$$

$$\frac{(5x^2+4)^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C =$$

$$\frac{(5x^2+4)^{\frac{1+2}{2}}}{\frac{1+2}{2}} + C =$$

$$\frac{(5x^2+4)^{3/2}}{\frac{3}{2}} + C =$$

$$\frac{2}{3}(5x^2+4)^{3/2} + C =$$

136

$$\int \frac{e^{6x}}{e^{6x}+2}\,dx =$$

$$\frac{1}{6}\int \frac{e^{6x}(6)}{e^{6x}+2}\,dx =$$

$$\frac{1}{6}\ln\left|e^{6x}+2\right| + C =$$

formula

$$\int \frac{f'(x)}{f(x)}\,dx =$$

$$\ln|f(x)| + C =$$

$$y = e^{f(x)}$$

$$y' = e^{f(x)} \cdot f'(x)$$

(137) $\int_0^{\pi/4} \cos(6x)\,dx =$

$\frac{1}{6}\int_0^{\pi/4} \cos(6x)(6)\,dx =$

$\frac{1}{6}\sin(6x)\Big|_0^{\pi/4} =$

$\frac{1}{6}\sin\left(6\left(\frac{\pi}{4}\right)\right) - \frac{1}{6}\sin(6(0)) =$

$\frac{1}{6}\sin\left(\frac{6\pi}{4}\right) - \frac{1}{6}\sin(0) =$

$\frac{1}{6}\sin\left(\frac{\cancel{2}(3\pi)}{\cancel{2}(2)}\right) - \frac{1}{6}\sin(0) =$

$\frac{1}{6}\sin\left(\frac{3\pi}{2}\right) - \frac{1}{6}\sin(0) =$

$\frac{1}{6}(-1) - \frac{1}{6}(0) =$

$-\frac{1}{6} - 0 =$

$-\frac{1}{6}$

138

$$\int_0^5 8e^{2x}\,dx =$$

$$8\int_0^5 e^{2x}\,dx =$$

$$(8)\left(\tfrac{1}{2}\right)\int_0^5 e^{2x}(2)\,dx =$$

$$4\int_0^5 e^{2x}(2)\,dx =$$

$$4e^{2x}\Big|_0^5 =$$

$$4e^{2(5)} - 4e^{2(0)} =$$

$$4e^{10} - 4e^{0} =$$

$$4e^{10} - 4(1) =$$

$$4e^{10} - 4 =$$

(139)

$$\int_0^1 5x^4(1-x^5)\,dx$$

$$\int_0^1 (1-x^5)(5x^4)\,dx$$

$$-1\int_0^1 (1-x^5)(-5x^4)\,dx$$

$$-1\left.\frac{(1-x^5)^{1+1}}{1+1}\right|_0^1$$

$$-1\left.\frac{(1-x^5)^2}{2}\right|_0^1$$

$$\left(-1\left(\frac{1-(1)^2}{2}\right)\right)-\left(-1\left(\frac{1-(0)^2}{2}\right)\right)=$$

$$\left(-1\left(\frac{1-1}{2}\right)\right)-\left(-1\ \frac{(1-0)}{2}\right)=$$

$$\left(-1\ \frac{(0)}{2}\right)-\left(-1\left(\frac{1}{2}\right)\right)$$

$$(0)-\left(-\frac{1}{2}\right)=$$

$$0+\frac{1}{2}=$$

$$\frac{1}{2}=$$

formula

$$\int (f(x))^N \cdot f'(x)\,dx=$$

$$\frac{(f(x))^{N+1}}{N+1}+C=$$

140. use a finite approximation to estimate the area under the graph of the given function on the interval.

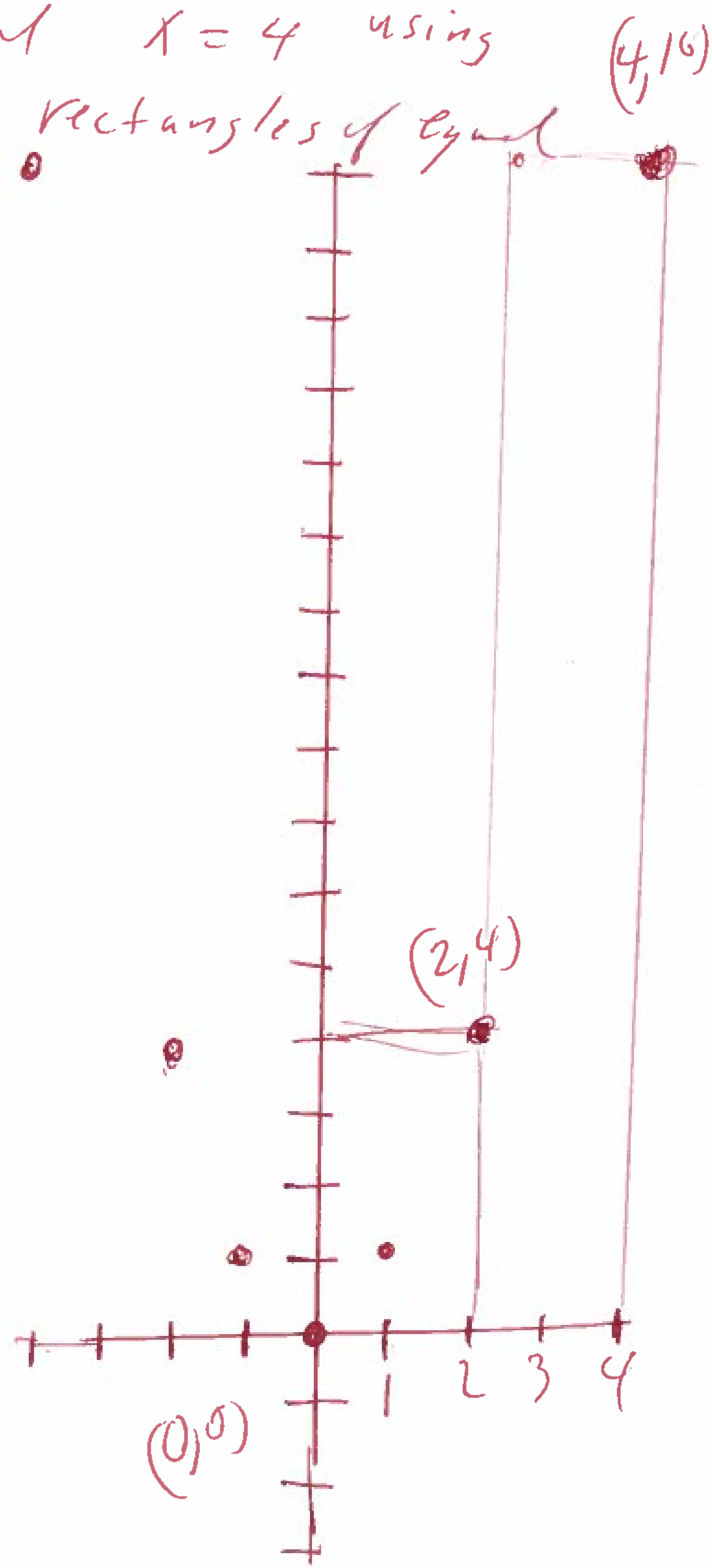
$f(x)=x^2$ between $x=0$ and $x=4$ using a right sum with two rectangles of equal width. area area

$A_1 + A_2 =$

$(2)(4) + (2)(16) =$

$8 + 32 =$

$40 =$



141. $\int_5^6 f(x)\,dx = -7$, $\int_5^6 4f(u)\,du$ and $\int_5^6 -f(u)\,du$

$$\int_5^6 4f(u)\,du =$$

$$4\int_5^6 f(u)\,du =$$

$$4(-7) =$$

$$-28 =$$

$$\int_5^6 -f(u)\,du =$$

$$-1\int_5^6 f(u)\,du =$$

$$-1(-7) =$$

$$7 =$$

(143) $\int x \cos(4x^2)\,dx =$

$\int \cos(4x^2)(x)\,dx =$

$\frac{1}{8}\int \cos(4x^2)(8x)\,dx =$

$\frac{1}{8}\sin(4x^2) + C =$

Formula

$\int \cos(f(x))\,f'(x)\,dx =$

$\sin(f(x)) + C =$

(144) $\int \frac{dx}{x^2-2x+101}$

$$\int \frac{dx}{x^2-2x+\left(\tfrac{1}{2}(-2)\right)^2+101-\left(\tfrac{1}{2}(-2)\right)^2}$$

Complete the square first

$$\int \frac{dx}{x^2-2x+(-1)^2+101-(-1)^2}$$

$$\int \frac{dx}{x^2-2x+1+101-1}$$

$$\int \frac{dx}{x^2-2x+1+100}$$

$$\int \frac{dx}{(x-1)(x-1)+100}$$

$$\int \frac{dx}{(x-1)^2+(10)^2}$$

$$\frac{1}{10}\tan^{-1}\left(\frac{x-1}{10}\right)+C =$$

(148) If the general solution of a differential equation is

$y(t) = Ce^{-4t} + 7$, what is the solution that satisfies the initial condition

$y(0) = 3$?

$$y(t) = Ce^{-4t} + 7$$

$$y(0) = Ce^{-4(0)} + 7 = 3$$

$$Ce^{-4(0)} + 7 = 3$$

$$Ce^{0} + 7 = 3$$

$$C(1) + 7 = 3$$

$$C + 7 = 3$$

$$C + \cancel{7} - \cancel{7} = 3 - 7$$

$$C = -4$$

$$y = -4e^{-4t} + 7$$