calmath2413147 step
120818
122618
12-12-18
12-14-18

① $s(2) = 143$ and $s(4) = 191$

$$\frac{s(4) - s(2)}{(4) - (2)} =$$ average velocity

$$\frac{(191) - (143)}{(4) - (2)} =$$

$$\frac{191 - 143}{4 - 2} =$$

$$\frac{48}{2} =$$

$$24 =$$

5. $\lim_{x \to 5} \frac{x^2 - 25}{x-5} =$

$\lim_{x \to 5} \frac{(x)^2 - (5)^2}{x-5} =$

$\lim_{x \to 5} \frac{(x+5)(x-5)}{(x-5)} =$

$\lim_{x \to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}} =$

$\lim_{x \to 5} (x+5) =$

$5+5=$

$10=$

formula

$a^2 - b^2 =$

$(a+b)(a-b) =$

⑥ $\lim_{t \to 36} \frac{t-36}{\sqrt{t}-6} =$

$\lim_{t \to 36} \left(\frac{t-36}{\sqrt{t}-6}\right)\left(\frac{\sqrt{t}+6}{\sqrt{t}+6}\right) =$ mult

$\lim_{t \to 36} \frac{(t-36)(\sqrt{t}+6)}{(\sqrt{t})^2 + \cancel{6\sqrt{t}} - \cancel{6\sqrt{t}} - 36}$

$\lim_{t \to 36} \frac{(t-36)(\sqrt{t}+6)}{(\sqrt{t})^2 - 36} =$

$\lim_{t \to 36} \frac{(t-36)(\sqrt{t}+6)}{(t-36)} =$

$\lim_{t \to 36} \frac{\cancel{(t-36)}(\sqrt{t}+6)}{\cancel{(t-36)}}$

$\lim_{t \to 36} (\sqrt{t}+6) =$

$\sqrt{36} + 6 =$

$6 + 6 =$

$12 =$

⑧ $\lim_{x \to -5} \dfrac{x^2+3x-10}{x+5} =$

$\lim_{x \to -5} \dfrac{(x-2)(x+5)}{(x+5)} =$

$\lim_{x \to -5} \dfrac{(x-2)\cancel{(x+5)}}{\cancel{(x+5)}} =$

$\lim_{x \to -5} (x-2) =$

$-5-2 =$

$-7 =$

(9) assume $\lim_{x \to 9} f(x) = 20$ and $\lim_{x \to 9} h(x) = 4$

find

$$\lim_{x \to 9} \frac{f(x)}{h(x)} =$$

$$\frac{\lim_{x \to 9} f(x)}{\lim_{x \to 9} h(x)} =$$

$$\frac{20}{4} =$$

$$\frac{\cancel{4}(5)}{\cancel{4}} =$$

$$5 =$$

(10) $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} =$

$\lim_{x \to 9} \left(\frac{\sqrt{x}-3}{x-9}\right)\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right) =$ Mult

$\lim_{x \to 9} \frac{(\sqrt{x})^2 + \cancel{3\sqrt{x}} - \cancel{3\sqrt{x}} - 9}{(x-9)(\sqrt{x}+3)} =$

$\lim_{x \to 9} \frac{(\sqrt{x})^2 - 9}{(x-9)(\sqrt{x}+3)} =$

$\lim_{x \to 9} \frac{(x - 9)}{(x-9)(\sqrt{x}+3)} =$

$\lim_{x \to 9} \frac{1\cancel{(x-9)}}{\cancel{(x-9)}(\sqrt{x}+3)} =$

$\lim_{x \to 9} \frac{1}{\sqrt{x}+3} =$

$\frac{1}{\sqrt{9}+3} =$

$\frac{1}{3+3} =$

$\frac{1}{6} =$

(12) find limit at infinity

$\lim_{x \to \infty} \frac{f(x)}{g(x)}$ if $f(x) \to 700000$ and $g(x) \to \infty$ as $x \to \infty$

$\lim_{x \to \infty} f(x) = 700{,}000$

$\lim_{x \to \infty} g(x) = \infty$

formula

$\lim_{x \to \infty} \frac{1}{x} = 0$

OR

$\lim_{x \to \infty} \frac{1}{x^n} = 0$

$\lim_{x \to \infty} \frac{f(x)}{g(x)} =$

$$\frac{\lim_{x \to \infty} f(x)}{\lim_{x \to \infty} g(x)} =$$

$$\frac{700{,}000}{\infty} =$$

$0 =$

(13) $\lim_{x \to \infty} \dfrac{2+6x+9x^4}{x^4} =$

$\lim_{x \to \infty} \left(\dfrac{2+6x+9x^4}{x^4}\right) \dfrac{\frac{1}{x^4}}{\frac{1}{x^4}} =$ mult

$\lim_{x \to \infty} \left(\dfrac{\frac{2}{1} + \frac{6x}{1} + \frac{9x^4}{1}}{\frac{x^4}{1}}\right) \dfrac{\frac{1}{x^4}}{\frac{1}{x^4}} =$

$\lim_{x \to \infty} \left(\dfrac{\frac{2}{x^4} + \frac{6x}{x^4} + \frac{9x^4}{x^4}}{\frac{x^4}{x^4}}\right) =$

$\lim_{x \to \infty} \dfrac{\frac{2}{x^4} + \frac{6}{x^3} + 9}{1} =$

$\dfrac{0+0+9}{1} =$

$\dfrac{9}{1} =$

$9 =$

(29) Find the average velocity of the function over the given interval

$y = \frac{3}{x-2}$ $\quad [4, 7]$ (a = 4, b = 7)

$\frac{f(b) - f(a)}{(b)-(a)} =$ average velocity

$\frac{f(7) - f(4)}{(7) - (4)} =$

$\frac{\left(\frac{3}{7-2}\right) - \left(\frac{3}{4-2}\right)}{(7)-(4)} =$

$\frac{\frac{3}{5} - \frac{3}{2}}{7-4} =$

$\frac{\frac{3}{5} - \frac{3}{2}}{3} =$

$\frac{\frac{3}{5}\left(\frac{2}{2}\right) - \frac{3}{2}\left(\frac{5}{5}\right)}{3} =$

$\frac{\frac{6}{10} - \frac{15}{10}}{3} =$

$\frac{\frac{6-15}{10}}{3} =$

$\frac{\frac{-9}{10}}{3} =$

$\frac{-\frac{9}{10}}{\frac{3}{1}} =$

$-\frac{9}{10} \cdot \frac{1}{3} =$

$\frac{-3\cancel{(3)}}{10} \cdot \frac{1}{\cancel{(3)}} =$

$-\frac{3}{10} =$

30. Find all vertical asymptotes of the given function

$$h(x) = \frac{x+11}{x^2+36x}$$

Let $x^2+36x = 0$

$x(x+36) = 0$

$x = 0$ or $x+36 = 0$

$x + \cancel{36} - \cancel{36} = 0 - 36$

$x = -36$

Vertical asymptotes

$x = 0$ or $x = -36$

(31) divide numerator and denominator by the highest power of x in the denominator. to find the limit at infinity.

formula

$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to\infty} \frac{1}{x^n} = 0$

$$\lim_{x\to\infty} \frac{\sqrt[3]{x}+6x-7}{3x+x^{2/3}+2} =$$

$$\lim_{x\to\infty} \frac{(\sqrt[3]{x}+6x-7)}{(3x+x^{2/3}+2)}\left(\frac{\frac{1}{x^{3/3}}}{\frac{1}{x^{3/3}}}\right) =$$

$$\lim_{x\to\infty} \left(\frac{x^{1/3}+6x^{3/3}-7}{3x^{3/3}+x^{2/3}+2}\right)\left(\frac{\frac{1}{x^{3/3}}}{\frac{1}{x^{3/3}}}\right) =$$

$$\lim_{x\to\infty} \frac{\frac{x^{1/3}}{x^{3/3}}+\frac{6\cancel{x^{3/3}}}{\cancel{x^{3/3}}}-\frac{7}{x^{3/3}}}{\frac{3\cancel{x^{3/3}}}{\cancel{x^{3/3}}}+\frac{x^{2/3}}{x^{3/3}}+\frac{2}{x^{3/3}}} =$$

$$\lim_{x\to\infty} \frac{\frac{1}{x^{\frac{3}{3}-\frac{1}{3}}}+6-\frac{7}{x^{3/3}}}{3+\frac{1}{x^{\frac{3}{3}-\frac{2}{3}}}-\frac{2}{x^{3/3}}} =$$

$$\lim_{x\to\infty} \frac{\frac{1}{x^{2/3}}+6-\frac{7}{x}}{3+\frac{1}{x^{1/3}}-\frac{2}{x}}$$

$$\frac{0+6-0}{3+0-0} =$$

$$\frac{6}{3} =$$

$$2 =$$

(32.) Use the graph to find $\delta > 0$ such that for all x $0<|x-x_0|<\delta$ and $|f(x)-L|<\epsilon$.

$f(x)=2x+3$, $\epsilon=0.2$, $x_0=1$, $L=5$

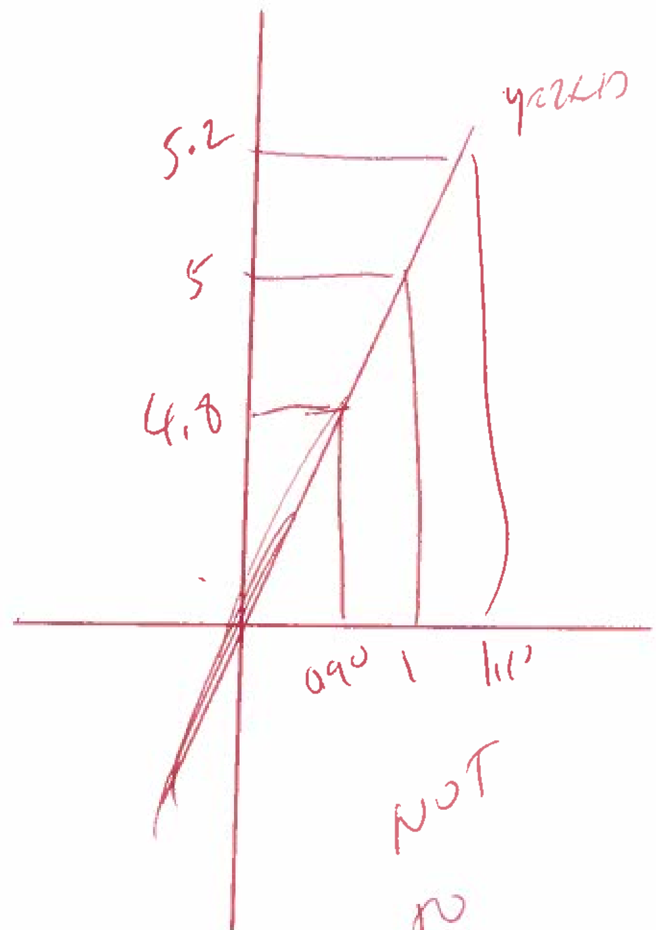
$$|f(x)-L|<\epsilon$$
$$|(2x+3)-(5)|<0.2$$
$$|2x+3-5|<0.2$$
$$|2x-2|<0.2$$
$$|2(x-1)|<0.2$$
$$|2|\cdot|x-1|<0.2$$
$$2|x-1|<0.2$$
$$\frac{2|x-1|}{2}<\frac{0.2}{2}$$
$$|x-1|<0.1$$

let $\delta=0.1$

$$|x-1|<0.1$$

OR

$$|x-1|<\delta=0.1$$



NOT to scale

15) Find an equation for the tangent to the curve at the given point.

$y = x^2 - 3$ $\quad$ $(-2, 1)$

$\quad\quad\quad\quad\quad\quad\quad\quad$ $x_1 \quad y_1$

$y' = 2x - 0$

$y' = 2x$

$y'(-2) = 2(-2)$

$y'(-2) = -4 = m = \text{slope}$

$y - y_1 = m(x - x_1)$ $\quad$ Point Slope formula

$y - (1) = -4(x - (-2))$

$y - 1 = -4(x + 2)$

$y - 1 = -4x - 8$

$y - 1 + 1 = -4x - 8 + 1$

$y = -4x - 7$

76 at time t, the position of a body moving along the s-axis is $s = t^3 - 12t^2 + 36t$ (m). Find the displacement of the body from $t=0$ to $t=3$.

$f(3) - f(0) =$

$((3)^3 - 12(3)^2 + 36(3)) - ((0)^3 - 12(0)^2 + 36(0)) =$

$((3)(3)(3) - 12(3)(3) + 36(3)) - ((0)(0)(0) - 12(0)(0) + 36(0)) =$

$(27 - \cancel{108} + \cancel{108}) - (0 - 0 + 0)$

$(27) - (0) =$

$27 - 0 =$

27 m

77) use implicit differentiation to find $\frac{dy}{dx}$

$$xy + x = 2$$

$$(xy)' + (x)' = (2)'$$

$$(x)'(y) + (x)(y)' + 1 = 0$$

$$(1)(y) + (x)(y') + 1 = 0$$

$$y + xy' + 1 = 0$$

$$xy' = -1 - y$$

$$\frac{\cancel{x}y'}{\cancel{x}} = \frac{-1-y}{x}$$

$$y' = \frac{-1-y}{x}$$

$$y' = \frac{-1(1+y)}{x}$$

$$y' = -\frac{1+y}{x}$$

78 find the derivative of the function

$y = \log_8 \sqrt{5x+7}$

$y = \log_8 (5x+7)^{1/2}$

$y = \frac{1}{2} \log_8 (5x+7)$

$y' = \frac{1}{2} \frac{(5x+7)'}{(5x+7)\ln(8)}$

$y' = \frac{1}{2} \frac{(5+0)}{(5x+7)\ln(8)}$

$y' = \frac{1}{2} \frac{5}{(5x+7)\ln(8)}$

$y' = \frac{1}{2} \frac{5}{\ln(8)(5x+7)}$

$y' = \frac{5}{2\ln(8)(5x+7)}$

formula

$y = \log_b \sqrt{Ax+B}$

$y = \log_b (Ax+B)^{1/2}$

$y = \frac{1}{2}\log_b (Ax+B)$

$y = \log_b f(x)$

$y' = \frac{f'(x)}{f(x)\ln(b)}$

(79) Boyle's law states that if the temperature of a gas remains constant, then $PV = c$, where P = pressure, V = volume, and c is a constant. Given ~~the~~ a quantity of gas at constant temperature, if V is <u>decreasing</u> at a rate of $11 \text{ in}^3/\text{sec}$, at what rate is P increasing when $P = 90 \text{ lb/in}^2$ and $V = 30 \text{ in}^3$? (do not round)

$$PV = c$$

$$(PV)' = (c)'$$

$$P'V + PV' = 0$$

$$\frac{dP}{dt}V + P\frac{dV}{dt} = 0$$

$$\frac{dP}{dt}(30) + (90)(-11) = 0$$

$$\frac{dP}{dt}(30) - 990 = 0$$

$$30\frac{dP}{dt} - 990 = 0$$

$$30\frac{dP}{dt} \cancel{-990} = 0 + 990$$

$$30\frac{dP}{dt} = 990$$

$V = 30 \text{ in}^3$

$P = 90 \text{ lb/in}^2$

$\frac{dV}{dt} = -11 \text{ in}^3/\text{sec}$

↑ decreasing

$$\frac{\cancel{30}\frac{dP}{dt}}{\cancel{30}} = \frac{990}{30}$$

$$\boxed{\frac{dP}{dt} = 33 \text{ lb/in}^2 \text{ per sec}}$$

Formula

$y = fg$

$y' = f'g + fg'$

(101) $\int (5x^9 - 3x^5)\,dx =$

$$\frac{5x^{9+1}}{9+1} - 3\frac{x^{5+1}}{5+1} + C =$$

$$\frac{5x^{10}}{10} - \frac{3x^6}{6} + C =$$

$$\frac{5x^{10}}{5(2)} - \frac{3x^6}{3(2)} + C =$$

$$\frac{\cancel{5}x^{10}}{\cancel{5}(2)} - \frac{\cancel{3}x^6}{\cancel{3}(2)} + C =$$

$$\frac{x^{10}}{2} - \frac{x^6}{2} + C =$$

formula

$$\int x^n\,dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

(102)

$$\int\left(\frac{6}{\sqrt{x}}+6\sqrt{x}\right)dx=$$

$$\int\left(\frac{6}{x^{1/2}}+6x^{1/2}\right)dx=$$

$$\int\left(6x^{-1/2}+6x^{1/2}\right)dx=$$

$$\frac{6x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+\frac{6x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=$$

$$\frac{6x^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}}+\frac{6x^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}}+C=$$

$$\frac{6x^{\frac{-1+2}{2}}}{\frac{-1+2}{2}}+\frac{6x^{\frac{1+2}{2}}}{\frac{1+2}{2}}+C=$$

$$\frac{6x^{1/2}}{\frac{1}{2}}+\frac{6x^{3/2}}{\frac{3}{2}}+C=$$

$$\frac{2}{1}(6)x^{1/2}+\frac{2}{3}(6)x^{3/2}+C=$$

$$12x^{1/2}+\frac{12}{3}x^{3/2}+C=$$

$$12x^{1/2}+4x^{3/2}+C=$$

$$12\sqrt{x}+4x^{3/2}+C$$

(103) $\int (6x+5)^2\, dx =$

$\frac{1}{6}\int (6x+5)^2 (6)\, dx =$

$\frac{1}{6}\frac{(6x+5)^{2+1}}{2+1} + C =$

$\frac{1}{6}\frac{(6x+5)^3}{3} + C =$

$\frac{1}{18}(6x+5)^3 + C =$

formula

$\int x^n\, dx =$

$\frac{x^{n+1}}{n+1} + C =$

$\int a\, dx =$

$ax + C =$

OR

$\int (6x+5)^2\, dx$

$\int (6x+5)(6x+5)\, dx$

$\int (36x^2 + 30x + 30x + 25)\, dx$

$\int (36x^2 + 60x + 25)\, dx$

$\frac{36x^{2+1}}{2+1} + \frac{60x^{1+1}}{1+1} + 25x + C =$

$\frac{36x^3}{3} + \frac{60x^2}{2} + 25x + C =$

$12x^3 + 30x^2 + 25x + C =$

(104) $\int 5m(10m^3-5m)\,dm =$

$\int (50m^4-25m^2)\,dm =$

$\frac{50m^{4+1}}{4+1} - \frac{25m^{2+1}}{2+1} + C =$

$\frac{50m^5}{5} - \frac{25m^3}{3} + C =$

$10m^5 - \frac{25m^3}{3} + C =$

Formula

$\int x^n\,dx =$

$\frac{x^{n+1}}{n+1} + C =$

105

$$\int (2x^{1/3} + 3x^{-2/3} + 6)\,d_x =$$

Formula

$$\int x^n d_x$$

$$\frac{x^{n+1}}{n+1} + C$$

$$\int a\, d_x$$

$$ax + C =$$

$$\frac{2x^{1/3+1}}{\frac{1}{3}+1} + \frac{3x^{-2/3+1}}{-\frac{2}{3}+1} + 6x + C =$$

$$\frac{2x^{\frac{1}{3}+\frac{3}{3}}}{\frac{1}{3}+\frac{3}{3}} + \frac{3x^{-2/3+\frac{3}{3}}}{-\frac{2}{3}+\frac{3}{3}} + 6x + C =$$

$$\frac{2x^{\frac{1+3}{3}}}{\frac{1+3}{3}} + \frac{3x^{\frac{-2+3}{3}}}{\frac{-2+3}{3}} + 6x + C =$$

$$\frac{2x^{4/3}}{\frac{4}{3}} + \frac{3x^{1/3}}{\frac{1}{3}} + 6x + C =$$

$$\frac{3}{4}(2)\,x^{4/3} + \frac{3}{1}(3x^{1/3}) + 6x + C =$$

$$\frac{3}{(2)\cancel{(2)}}\cancel{(2)}\,x^{4/3} + 9x^{1/3} + 6x + C =$$

$$\frac{3}{2}x^{4/3} + 9x^{1/3} + 6x + C =$$

106

$$\int 5 \sqrt[10]{x}\, dx =$$

$$\int 5 x^{1/10}\, dx =$$

$$\frac{5x^{\frac{1}{10}+1}}{\frac{1}{10}+1} + C =$$

$$\frac{5x^{\frac{1}{10}+\frac{10}{10}}}{\frac{1}{10}+\frac{10}{10}} + C =$$

$$\frac{5x^{\frac{1+10}{10}}}{\frac{1+10}{10}} + C =$$

$$\frac{5x^{\frac{11}{10}}}{\frac{11}{10}} + C =$$

$$\frac{10}{11}(5)\, x^{\frac{11}{10}} + C =$$

$$\frac{50}{11} x^{\frac{11}{10}} + C =$$

formula

$$\int x^n dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

$$\int a\, dx =$$

$$ax + C =$$

107

$$\int (7x+1)(2-x)\,dx =$$

$$\int (14x - 7x^2 + 2 - 1x)\,dx =$$

$$\int (-7x^2 + 13x + 2)\,dx =$$

$$\frac{-7x^{2+1}}{2+1} + \frac{13x^{1+1}}{1+1} + 2x + C =$$

$$\frac{-7x^3}{3} + \frac{13x^2}{2} + 2x + C =$$

formula

$$\int x^n\,dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

$$\int a\,dx =$$

$$ax + C =$$

(108) $\int \left(\frac{7x^8 - 25x^6}{x^2}\right) dx$

$$\int \left(\frac{7x^8}{x^2} - \frac{25x^6}{x^2}\right) dx =$$

$$\int \left(7x^{8-2} - 25x^{6-2}\right) dx =$$

$$\int \left(7x^6 - 25x^4\right) dx =$$

$$\frac{7x^{6+1}}{6+1} - \frac{25x^{4+1}}{4+1} + C =$$

$$\frac{7x^7}{7} - \frac{25x^5}{5} + C =$$

$$x^7 - 5x^5 + C =$$

Formula

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C =$$

$$\int a\,dx = ax + C =$$

(109) For the function f. find the antiderivative F that satisfies the given condition

$f(x) = 2x^3 + 9\sin(x)$, $\quad F(0) = 5$

$\int f(x)\,dx = \int (2x^3 + 9\sin(x))\,dx$

formula

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$\int \sin(x)\,dx = -\cos(x) + C$

$F(x) = \frac{2x^{3+1}}{3+1} - 9\cos(x) + C$

$F(x) = \frac{2x^4}{4} - 9\cos(x) + C$

$F(x) = \frac{\cancel{2}x^4}{(\cancel{2})(2)} - 9\cos(x) + C$

$F(x) = \frac{x^4}{2} - 9\cos(x) + C$

$F(0) = \frac{(0)^4}{2} - 9\cos(0) + C = 5$

$\frac{(0)^4}{2} - 9\cos(0) + C = 5$

$0 - 9(1) + C = 5$

$-9 + C = 5$

$\cancel{-9} + C \cancel{+9} = 5 + 9$

$C = 14$

$F(x) = \frac{x^4}{2} - 9\cos(x) + 14$

(110) for the function f. find the antiderivative F that satisfies the given condition.

$f(u) = 4e^u - 2, \quad F(0) = 7$

formula

$\int e^{f(x)} f'(x)\, dx$

$e^{f(x)} + c$

$\int x^n dx$

$\frac{x^{n+1}}{n+1} + c$

$\int a\, dx =$

$ax + c$

$\int f(u)\, du = \int (4e^u - 2)\, du$

$F(u) = 4e^u - 2u + c$

$F(0) = 4e^0 - 2(0) + c = 7$

$4e^0 - 2(0) + c = 7$

$4(1) - 0 + c = 7$

$4 + c = 7$

$\cancel{4} + c - \cancel{4} = 7 - 4$

$c = 3$

$F(u) = 4e^u - 2u + 3$

(111) given the velocity function of an object moving along a line. find the position function with the given initial position.

$V(t) = 9t^2 + 2t - 1 \quad , \quad S(0) = 0$

formula

$\int x^n dx = \frac{x^{n+1}}{n+1} + C =$

$\int a\,dx = ax + C =$

$\int V(t)\,dt = \int (9t^2 + 2t - 1)\,dt$

$S(t) = \frac{9t^{2+1}}{2+1} + \frac{2t^{1+1}}{1+1} - 1t + C$

$S(t) = \frac{9t^3}{3} + \frac{2t^2}{2} - 1t + C$

$S(t) = 3t^3 + t^2 - 1t + C$

$S(0) = 3(0)^3 + (0)^2 - 1(0) + C = 0$

$3(0)^3 + (0)^2 - 1(0) + C = 0$

$0 + 0 - 0 + C = 0$

$C = 0$

$S(t) = 3t^3 + t^2 - 1t$

(112) Find the absolute extreme value of the function on the interval

$F(x) = \sqrt[3]{x}$ $\qquad -27 \leq x \leq 8$

$F(x) = x^{1/3}$

$F'(x) = \frac{1}{3} x^{1/3 - 1}$

$F'(x) = \frac{1}{3} x^{1/3 - 3/3}$

$F'(x) = \frac{1}{3} x^{\frac{1-3}{3}}$

$F'(x) = \frac{1}{3} x^{-2/3}$

$F'(x) = \frac{1}{3} x$

$F'(x) = \frac{1}{3x^{2/3}}$ undefined if $x = 0$

Critical point

Check $x = 0$, $x = -27$, $x = 8$

$F(x) = \sqrt[3]{x}$

$F(0) = \sqrt[3]{0} = 0$ ✓

$F(-27) = \sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3$ ✓

$F(+8) = \sqrt[3]{8} = \sqrt[3]{(2)^3} = 2$ ✓

absolute maximum is 2 at $x = 8$

absolute minimum is -3 at $x = -27$

(11/3) Find the value or values of c that satisfy the equation $\frac{f(b)-f(a)}{b-a} = f'(c)$ in the conclusion of the Mean Value Theorem for the function and interval.

$f(x) = x^2+5x+2$ $\quad [-3, 2]$
$\qquad\qquad\qquad\qquad\quad a \quad b$

$f(x) = x^2+5x+2$
$f'(x)\ 2x+5+0$
$f'(x) = 2x+5$

$$\frac{f(b)-f(a)}{(b)-(a)} = f'(x)$$

$$\frac{f(2)-f(-3)}{(2)-(-3)} = 2x+5$$

$$\frac{((2)^2+5(2)+2)-((-3)^2+5(-3)+2)}{(2)-(-3)} = 2x+5$$

$$\frac{((2)(2)+5(2)+2)-((-3)(-3)+5(-3)+2)}{(2)-(-3)} = 2x+5$$

$$\frac{(4+10+2)-(9-15+2)}{2+3} = 2x+5$$

$$\frac{(16)-(-4)}{2+3} = 2x+5$$

$$\frac{16+4}{5} = 2x+5$$

$$\frac{20}{5} = 2x+5$$

$4 = 2x+5$

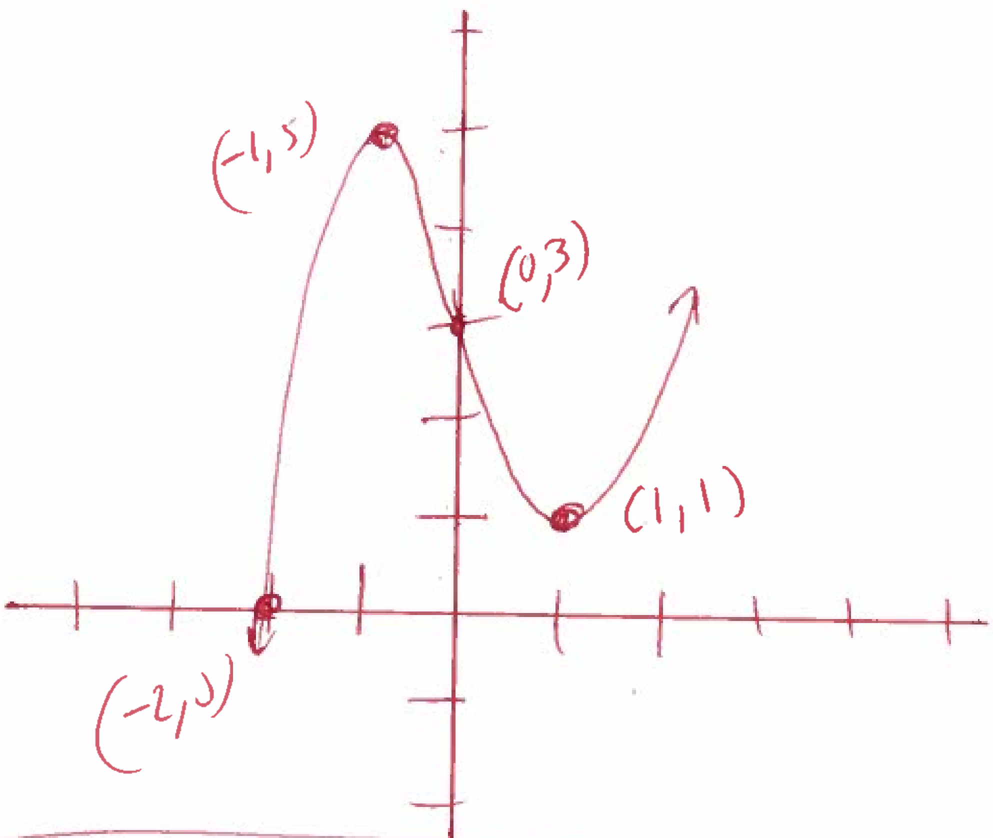
$4-5 = 2x+5-5$

$-1 = 2x$

$\frac{-1}{2} = \frac{\cancel{2}x}{\cancel{2}}$

$-\frac{1}{2} = x$

(114) use the graph of the function $f(x)$ to locate the local extrema and identify the intervals where the function is concave up and concave down.



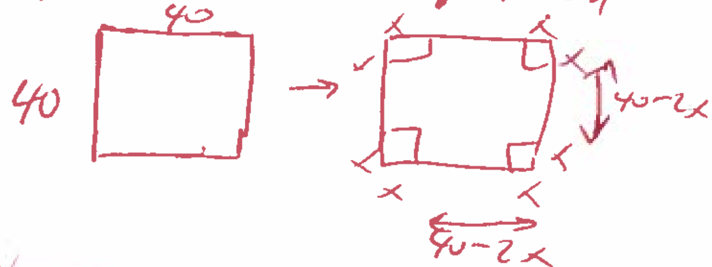
local minimum at $x=1$

local maximum at $x=-1$

Concave up on $(0,\infty)$

Concave down on $(-\infty,0)$

115 From a thin piece of cardboard 40 in by 40 in, square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth.



$V = LWH$

$V = (40-2x)(40-2x)(x)$

$V = (1600 - 80x - 80x + 4x^2)(x)$

$V = (1600 - 160x + 4x^2)(x)$

$V = 1600x - 160x^2 + 4x^3$

$V = 4x^3 - 160x^2 + 1600x$

$V' = 12x^2 - 160(2x) + 1600$

$V' = 12x^2 - 320x + 1600$

set $12x^2 - 320x + 1600 = 0$

$4(3x^2 - 80x + 400) = 0$

$4(3x-20)(x-20) = 0$

$4 = 0$ OR $3x - 20 = 0$ or $x - 20 = 0$

$3x - 20 + 20 = 0 + 20$ OR $x - 20 + 20 = 0 + 20$

$3x = 20$ OR $x = 20$

$\frac{3x}{3} = \frac{20}{3}$

$x = \frac{20}{3}$

$V'(x) = 12x^2 - 320x + 1600$

$V''(x) = 24x - 320 + 0$

$V''(x) = 24x - 320$

$V''(x) = 24x - 320$

$V''(\frac{20}{3}) = 24(\frac{20}{3}) - 320$

$V''(\frac{20}{3}) = 8(20) - 320$

$V''(\frac{20}{3}) = 160 - 320$

$V''(\frac{20}{3}) = -160 < 0$ concave down max

$V''(20) = 24(20) - 320$

$V''(20) = 480 - 320$

$V''(20) = 160 > 0$ concave up min

Max at $x = \frac{20}{3}$

$V = (40-2x)(40-2x)(x)$

$V = (40 - 2(\frac{20}{3}))(40 - 2(\frac{20}{3}))(\frac{20}{3})$

$V = (26.6666)(26.6666)(6.6666)$

$V = 4740.6696$

OR

$V = 4740.7$

(116) Solve the initial value problem

$$\frac{ds}{dt} = \cos(t) - \sin(t), \quad s\left(\frac{\pi}{2}\right) = 11$$

$$\int ds = \int (\cos(t) - \sin(t))\,dt$$

$$s = \sin(t) + \cos(t) + C$$

$$s(t) = \sin(t) + \cos(t) + C$$

$$s\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) + C = 11$$

$$\sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) + C = 11$$

$$(1) + 0 + C = 11$$

$$1 + 0 + C = 11$$

$$1 + C = 11$$

$$1 + C - 1 = 11 - 1$$

$$C = 10$$

$$s = \sin(t) + \cos(t) + 10$$

(127) Find the area of the region bounded by the graph of f and the x-axis

$f(x) = x^2 - 25 \qquad [1, 2] \qquad \int_a^b (f(x) - g(x))\,dx$

$\int_1^2 (0) - (x^2 - 25)\,dx$

$\int_1^2 (0 - x^2 + 25)\,dx =$

$\int_1^2 (-x^2 + 25)\,dx =$

$\frac{-x^{2+1}}{2+1} + 25x \Big|_1^2 =$

$\left(\frac{-x^3}{3} + 25x\right) \Big|_1^2 =$

$\left(-\frac{(2)^3}{3} + 25(2)\right) - \left(-\frac{(1)^3}{3} + 25(1)\right) =$

$\left(-\frac{8}{3} + 50\right) - \left(-\frac{1}{3} + 25\right) =$

$\left(-\frac{8}{3} + \frac{50}{1}\left(\frac{3}{3}\right)\right) - \left(-\frac{1}{3} + \frac{25}{1}\left(\frac{3}{3}\right)\right) =$

$\left(-\frac{8}{3} + \frac{150}{3}\right) - \left(-\frac{1}{3} + \frac{75}{3}\right) =$

$\left(\frac{-8+150}{3}\right) - \left(\frac{-1+75}{3}\right)$

$\left(\frac{142}{3}\right) - \left(\frac{74}{3}\right) =$

$\frac{142}{3} - \frac{74}{3} =$

$\frac{142-74}{3} =$

$\frac{68}{3} =$

(13)

$$\int_0^3 (x-2)^2\, dx =$$

$$\int_0^3 (x-2)^2 (1)\, dx =$$

$$\frac{(x-2)^{2+1}}{2+1} \Big/_0^3 =$$

$$\frac{(x-2)^3}{3} \Big/_0^3 =$$

$$\frac{(3-2)^3}{3} - \left(\frac{(0-2)^3}{3}\right) =$$

$$\left(\frac{(1)^3}{3}\right) - \left(\frac{(-2)^3}{3}\right) =$$

$$\left(\frac{1}{3}\right) - \left(-\frac{8}{3}\right) =$$

$$\frac{1}{3} + \frac{8}{3} =$$

$$\frac{1+8}{3} =$$

$$\frac{9}{3} =$$

$$3 =$$

131. $f(x) = x(x-1)$ find average value $[1, 4]$
$a = 1$, $b = 4$

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{(4)-(1)}\int_1^4 x(x-1)\,dx =$$

$$\frac{1}{4-1}\int_1^4 (x^2-x)\,dx =$$

$$\frac{1}{3}\int_1^4 (x^2-x)\,dx =$$

$$\frac{1}{3}\left(\frac{x^{2+1}}{2+1}-\frac{x^{1+1}}{1+1}\right)\Big|_1^4 =$$

$$\frac{1}{3}\left(\frac{x^3}{3}-\frac{x^2}{2}\right)\Big|_1^4 =$$

$$\frac{1}{3}\left(\frac{(4)^3}{3}-\frac{(4)^2}{2}\right)-\frac{1}{3}\left(\frac{(1)^3}{3}-\frac{(1)^2}{2}\right) =$$

$$\frac{1}{3}\left(\frac{64}{3}-\frac{16}{2}\right)-\frac{1}{3}\left(\frac{1}{3}-\frac{1}{2}\right) =$$

$$\frac{1}{3}\left(\frac{64}{3}\left(\frac{2}{2}\right)-\frac{16}{2}\left(\frac{3}{3}\right)\right)-\frac{1}{3}\left(\frac{1}{3}\left(\frac{2}{2}\right)-\frac{1}{2}\left(\frac{3}{3}\right)\right) =$$

$$\frac{1}{3}\left(\frac{128}{6}-\frac{48}{6}\right)-\frac{1}{3}\left(\frac{2}{6}-\frac{3}{6}\right) =$$

$$\frac{1}{3}\left(\frac{128-48}{6}\right)-\frac{1}{3}\left(\frac{2-3}{6}\right) =$$

$$\frac{1}{3}\left(\frac{80}{6}\right)-\frac{1}{3}\left(\frac{-1}{6}\right) =$$

$$\frac{1}{3}\left(\frac{80}{6}\right)+\frac{1}{3}\left(\frac{1}{6}\right) =$$

$$\frac{80}{18}+\frac{1}{18} =$$

$$\frac{81}{18} =$$

$$\frac{\cancel{9}(9)}{\cancel{9}(2)} =$$

$$\frac{9}{2}$$

132 $f(x) = x^3 - 5x^2 + 2$ find average value

$0 \le x \le 4$

$\frac{1}{b-a}\int f(x)\,dx$

$\frac{1}{(4)-(0)}\int_0^4 (x^3 - 5x^2 + 2)\,dx$

$\frac{1}{4-0}\int_0^4 (x^3 - 5x^2 + 2)\,dx$

$\frac{1}{4}\int_0^4 (x^3 - 5x^2 + 2)\,dx$

$\frac{1}{4}\left(\frac{x^{3+1}}{3+1} - \frac{5x^{2+1}}{2+1} + 2x\right)\Big|_0^4$

$\frac{1}{4}\left(\frac{x^4}{4} - \frac{5x^3}{3} + 2x\right)\Big|_0^4$

$\frac{1}{4}\left(\frac{(4)^4}{4} - \frac{5(4)^3}{3} + 2(4)\right) - \frac{1}{4}\left(\frac{(0)^4}{4} - \frac{5(0)^3}{3} + 2(0)\right) =$

$\frac{1}{4}\left(\frac{256}{4} - \frac{5(64)}{3} + 8\right) - \frac{1}{4}(0 + 0 + 0) =$

$\frac{1}{4}\left(\frac{256}{4} - \frac{320}{3} + \frac{8}{1}\right) - \frac{1}{4}(0) =$

$\frac{1}{4}\left(\frac{256}{4}\left(\frac{3}{3}\right) - \frac{320}{3}\left(\frac{4}{4}\right) + \frac{8}{1}\left(\frac{12}{12}\right)\right) - \frac{1}{4}(0) =$

$\frac{1}{4}\left(\frac{768}{12} - \frac{1280}{12} + \frac{96}{12}\right) - \frac{1}{4}(0) =$

$\frac{1}{4}\left(\frac{768 - 1280 + 96}{12}\right) - \frac{1}{4}(0) =$

$\frac{1}{4}\left(\frac{-416}{12}\right) =$

$\frac{-104}{12} =$

$\frac{\cancel{4}(-26)}{\cancel{4}(3)} =$

$-\frac{26}{3}$

(133.) find the points at which the function $f(x) = 9 - 4x$ equals its average value on the interval $[\underset{a}{0}, \underset{b}{6}]$.

$\frac{1}{b-a}\int_a^b f(x)\,dx$ average value

$\frac{1}{(6)-(0)}\int_0^6 (9-4x)\,dx =$

$\frac{1}{6-0}\int_0^6 (9-4x)\,dx =$

$\frac{1}{6}\int_0^6 (9-4x)\,dx =$

$\frac{1}{6}\left(9x - \frac{4x^{1+1}}{1+1}\right)\Big|_0^6 =$

$\frac{1}{6}\left(9x - \frac{4x^2}{2}\right)\Big|_0^6 =$

$\frac{1}{6}(9x - 2x^2)\Big|_0^6 =$

$\frac{1}{6}(9(6) - 2(6)^2) - \frac{1}{6}(9(0) - 2(0)^2) =$

$\frac{1}{6}(9(6) - 2(6)(6)) - \frac{1}{6}(9(0) - 2(0)(0)) =$

$\frac{1}{6}(54 - 72) - \frac{1}{6}(0 - 0) =$

$\frac{1}{6}(-18) - \frac{1}{6}(0) =$

$-\frac{18}{6} - 0 =$

$-3 - 0$

$-3 =$ average value

Let $9 - 4x = -3$

$9 - 4x - 9 = -3 - 9$

$-4x = -12$

$\frac{-4x}{-4} = \frac{-12}{-4}$

$x = 3$ ✓

ANSWER

134

$$\int 2x\,(x^2+17)^9\,dx =$$

$$\int (x^2+17)^9\,(2x)\,dx =$$

$$\frac{(x^2+17)^{9+1}}{9+1} + C =$$

$$\frac{(x^2+17)^{10}}{10} + C =$$

formula

$$\int (f(x))^N \cdot f'(x)\,dx =$$

$$\frac{(f(x))^{N+1}}{N+1} + C =$$

135

$$\int -12x \sin(6x^2+1)\,dx =$$

$$\int \sin(6x^2+1)(-12x)\,dx =$$

$$\int -\sin(6x^2+1)(12x)\,dx =$$

$$\cos(6x^2+1)+C =$$

formula

$$\int -\sin(f(x))\cdot f'(x)\,dx =$$

$$\cos(f(x))+C$$

136

$$\int (12x+5)\sqrt{6x^2+5x}\,dx =$$

$$\int \sqrt{6x^2+5x}\,(12x+5)\,dx =$$

$$\int (6x^2+5x)^{1/2}(12x+5)\,dx =$$

$$\frac{(6x^2+5x)^{1/2+1}}{\frac{1}{2}+1} + C =$$

$$\frac{(6x^2+5x)^{1/2+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C =$$

$$\frac{(6x^2+5x)^{\frac{1+2}{2}}}{\frac{1+2}{2}} + C =$$

$$\frac{(6x^2+5x)^{3/2}}{3/2} + C =$$

$$\frac{2}{3}(6x^2+5x)^{3/2} + C =$$

(137) $\int \frac{e^{5x}}{e^{5x}+2}\,dx =$

$\frac{1}{5}\int \frac{e^{5x}(5)}{e^{5x}+2}\,dx =$

$\frac{1}{5}\ln\left|e^{5x}+2\right|+C =$

Formula

$\int \frac{f'(x)}{f(x)}\,dx =$

$\ln\left|f(x)\right|+C =$

$y = e^{f(x)}$

$y' = e^{f(x)} \cdot f'(x)$

138

$$\int_0^{\pi/18} \cos(6x)\,dx =$$

$$\frac{1}{6}\int_0^{\pi/18} \cos(6x)(6)\,dx =$$

$$\frac{1}{6}\sin(6x)\Big|_0^{\frac{\pi}{18}} =$$

$$\frac{1}{6}\sin\left(6\left(\frac{\pi}{18}\right)\right) - \frac{1}{6}\sin(6(0)) =$$

$$\frac{1}{6}\sin\left(\frac{6\pi}{18}\right) - \frac{1}{6}\sin(0) =$$

$$\frac{1}{6}\sin\left(\frac{\cancel{6}\pi}{(\cancel{6})(3)}\right) - \frac{1}{6}\sin(0) =$$

$$\frac{1}{6}\sin\left(\frac{\pi}{3}\right) - \frac{1}{6}\sin(0) =$$

$$\frac{1}{6}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{6}(0) =$$

$$\frac{\sqrt{3}}{12} - 0 =$$

$$\frac{\sqrt{3}}{12} =$$

1139

$$\int_0^3 4e^{2x}\,dx =$$

$$4\int_0^3 e^{2x}\,dx =$$

$$4\left(\tfrac{1}{2}\right)\int_0^3 e^{2x}(2)\,dx =$$

$$\tfrac{4}{2}\int_0^3 e^{2x}(2)\,dx =$$

$$2e^{2x}\Big/_0^3 =$$

$$2e^{2(3)} - 2e^{2(0)} =$$

$$2e^{6} - 2e^{0} =$$

$$2e^{6} - 2(1) =$$

$$2e^{6} - 2 =$$

~~Formula~~

$$\int e^{f(x)}\cdot f'(x)\,dx =$$

$$e^{f(x)} + C =$$

(140) $\int_0^3 \frac{2x}{(x^2+2)^3}\,dx$

$\int_0^3 (x^2+2)^{-3}(2x)\,dx$

$\frac{(x^2+2)^{-3+1}}{-3+1}\Big/_0^3 =$

$\frac{(x^2+2)^{-2}}{-2}\Big/_0^3 =$

$\frac{1}{-2(x^2+2)^2}\Big/_0^3 =$

$\frac{1}{-2((3)^2+2)^2} - \frac{1}{-2((0)^2+2)^2} =$

$\frac{1}{-2(9+2)^2} - \frac{1}{-2(0+2)^2} =$

$\frac{1}{-2(11)^2} - \frac{1}{-2(2)^2} =$

$\frac{1}{-2(121)} - \frac{1}{-2(4)} =$

$\frac{1}{-242} - \frac{1}{-8} =$

$\frac{1}{-242} + \frac{1}{8} =$

$$\begin{array}{r|l} 2 & 242 \\ 11 & 121 \\ 11 & 11 \\ & 1 \end{array} \qquad \begin{array}{r|l} 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

$LCD = 2 \cdot 2 \cdot 2 \cdot 11 \cdot 11$

968

$\frac{1}{-242}\left(\frac{4}{4}\right) + \frac{1}{8}\left(\frac{121}{121}\right) =$

$\frac{4}{-968} + \frac{121}{968} =$

$\frac{-4+121}{968} =$

$\frac{117}{968}$

141

$$\int_0^3 \frac{p}{\sqrt{16+p^2}}\,dp =$$

$$\int_0^3 \frac{p}{(16+p^2)^{1/2}}\,dp =$$

$$\int_0^3 (16+p^2)^{-1/2}(p)\,dp =$$

$$\frac{1}{2}\int_0^3 (16+p^2)^{-1/2}(2p)\,dp =$$

$$\frac{1}{2}\,\frac{(16+p^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Big/_0^3 =$$

$$\frac{1}{2}\,\frac{(16+p^2)^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}}\Big/_0^3 =$$

$$\frac{1}{2}\,\frac{(16+p^2)^{\frac{-1+2}{2}}}{\frac{-1+2}{2}}\Big/_0^3 =$$

$$\frac{1}{2}\,\frac{(16+p^2)^{\frac{1}{2}}}{\frac{1}{2}}\Big/_0^3 =$$

$$\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{1}(16+p^2)^{\frac{1}{2}}\Big/_0^3 =$$

$$(16+p^2)^{\frac{1}{2}}\Big/_0^3 =$$

$$\sqrt{16+p^2}\Big/_0^3 =$$

$$\sqrt{16+(3)^2} - \sqrt{16+(0)^2} =$$

$$\sqrt{16+9} - \sqrt{16+0} =$$

$$\sqrt{25} - \sqrt{16} =$$

$$5 - 4 =$$

$$1 =$$

(142) use a finite approximation to estimate the area under the graph of the given function on the stated interval as instructed

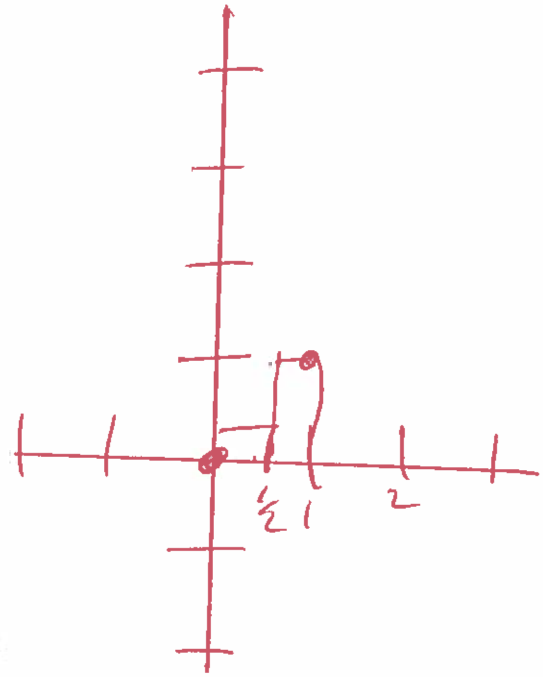
$f(x) = x^2$ between $x = 0$ and $x = 1$ using a right sum with two rectangles of equal width.

$f(x) = x^2$

$f(0) = (0)^2 = (0)(0) = 0$

$f(1) = (1)^2 = (1)(1) = 1$

$f(\frac{1}{2}) = (\frac{1}{2})^2 = (\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$



area 1 $= (\frac{1}{2})(\frac{1}{4}) = \frac{1}{8}$

area 2 $= (\frac{1}{2})(1) = \frac{1}{2}$

area 1 + area 2

$(\frac{1}{8}) + (\frac{1}{2}) =$

$\frac{1}{8} + \frac{1}{2}(\frac{4}{4}) =$

$\frac{1}{8} + \frac{4}{8} =$

$\frac{1+4}{8} =$

$\frac{5}{8} =$ dwell

$.625 =$

(143) $\int_{10}^{11} f(x)\,dx = 6$ find $\int_{10}^{11} 4f(u)\,du$ and $\int_{10}^{11} -f(u)\,du$

$\int_{10}^{11} 4f(u)\,du =$

$4\int_{10}^{11} f(u)\,du =$

$4(6) =$

$24 =$

$\int_{10}^{11} -f(u)\,du =$

$-\int_{10}^{11} f(u)\,du =$

$-(6) =$

$-6 =$

answers

24, −6

144. find the derivative

$$\frac{d}{dx}\int_{1}^{\sqrt{x}} 18t^5\,dt$$

(145) evaluate the integral using the given substitution

$$\int x \cos(5x^2)\,dx =$$

$u = 5x^2$
$du = 10x$

$$\int \cos(5x^2)(x)\,dx =$$

$$\frac{1}{10}\int \cos(5x^2)\,(10x)\,dx =$$

$$\frac{1}{10}\sin(5x^2) + C =$$

formula

$$\int \cos(f(x)) \cdot f'(x)\,dx =$$

$$\sin(f(x)) + C =$$

(147.) If the general solution of a differential equation is $y(t) = Ce^{-3t} + 3$, what is the solution that satisfies the initial condition $y(0) = 9$?

$$y(t) = Ce^{-3t} + 3$$

$$y(0) = Ce^{-3(0)} + 3 = 9$$

$$Ce^{-3(0)} + 3 = 9$$

$$Ce^{0} + 3 = 9$$

$$C(1) + 3 = 9$$

$$C + 3 = 9$$

$$C + \cancel{3} - \cancel{3} = 9 - 3$$

$$C = 6$$

$$\boxed{y(t) = 6e^{-3t} + 3}$$ ✓