calmath24131 SONEXTSTEP

09-30-18
09-31-18
09-31-18
10-10-18
~~10-11-18~~
~~10-18-18~~
10-24-18

1) $f(x) = 3x^2 - 4x + 1$

$$\frac{f(x+h) - f(x)}{h} =$$

$$\frac{(3(x+h)^2 - 4(x+h) + 1) - (3x^2 - 4x + 1)}{h} =$$

$$\frac{3(x+h)(x+h) - 4x - 4h + 1 - 3x^2 + 4x - 1}{h} =$$

$$\frac{3(x^2 + xh + xh + h^2) - 4x - 4h + 1 - 3x^2 + 4x - 1}{h} =$$

$$\frac{3(x^2 + 2xh + h^2) - 4x - 4h + 1 - 3x^2 + 4x - 1}{h} =$$

$$\frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{4x} - 4h + \cancel{1} - \cancel{3x^2} + \cancel{4x} - \cancel{1}}{h} =$$

$$\frac{6xh + 3h^2 - 4h}{h} =$$

$$\frac{\cancel{h}(6x + 3h - 4)}{\cancel{h}} =$$

$$6x + 3h - 4 =$$

② $f(x) = \frac{21}{x}$

$$\frac{f(x+h) - f(x)}{h} =$$

$$\frac{\frac{21}{x+h} - \frac{21}{x}}{h} =$$

$$\frac{\left(\frac{21}{x+h} - \frac{21}{x}\right)\frac{x(x+h)}{1}}{(h)\ x(x+h)} =$$ mult

$$\frac{\frac{21x\cancel{(x+h)}}{\cancel{(x+h)}} - \frac{21\cancel{x}(x+h)}{\cancel{x}}}{h\,(x)(x+h)} =$$

$$\frac{21x - 21(x+h)}{h(x)(x+h)} =$$

$$\frac{21x - 21x - 21h}{h(x)(x+h)} =$$

$$\frac{-21\cancel{h}}{\cancel{h}\cdot(x)(x+h)} =$$

$$\frac{-21}{x(x+h)} =$$

③ $f(x) = 2 - 5x - x^2$

$$\frac{f(x) - f(a)}{x - a} =$$

$$\frac{(2 - 5x - x^2) - (2 - 5(a) - (a)^2)}{x - a} =$$

$$\frac{\cancel{2} - 5x - x^2 \cancel{-2} + 5a + a^2}{x - a} =$$

$$\frac{-5x - x^2 + 5a + a^2}{x - a} =$$

$$\frac{-5x + 5a - x^2 + a^2}{x - a} =$$

$$\frac{-5(x - a) - (x^2 - a^2)}{(x - a)} =$$

$$\frac{-5(x - a) - ((x + a)(x - a))}{(x - a)} =$$

$$\frac{(x - a)(-5 - (x + a))}{x - a} =$$

$$\frac{\cancel{(x - a)}(-5 - x - a)}{\cancel{(x - a)}} =$$

$$-5 - x - a =$$

$$-a - x - 5 =$$

(4) the function $s(t)$ represents the position of an object at time t moving along a line. Suppose $s(3)=155$ and $s(5)=205$. find the average velocity of the object over the interval of time $[3,5]$.

The average velocity over the interval $[3,5]$ is $V_{av} = \boxed{}$

$$\frac{s(5)-s(3)}{(5)-(3)} =$$

$$\frac{(205)-(155)}{(5)-(3)}$$

$$\frac{205-155}{5-3} =$$

$$\frac{50}{2} =$$

$$25 =$$

⑤ What is the slope of the secant line between the points $(a, f(a))$ and $(b, f(b))$ on the graph of f?

$$m_{sec} = \frac{f(b) - f(a)}{b - a}$$

⑥ What is the slope of the line tangent to the graph of f at $(a, f(a))$?

$$m_{tan} = \lim_{t \to a} \frac{f(t) - f(a)}{t - a}$$

7. The position of an object moving along a line is given by the function $s(t) = -20t^2 + 160t$. Find the average velocity of the object over the following intervals.

(a) [1, 4] (b) [1, 3]
(c) [1, 2] (d) [1, 1 + h] where h > 0 is any real number.

(a) The average velocity of the object over the interval [1, 4] is 60 $= \frac{s(4)-s(1)}{4-1}$

(b) The average velocity of the object over the interval [1, 3] is 80 $= \frac{s(3)-s(1)}{3-1}$

(c) The average velocity of the object over the interval [1, 2] is 100. $= \frac{s(2)-s(1)}{2-1}$

(d) The average velocity of the object over the interval [1, 1 + h] is $-20h+120$

$\frac{s(1+h)-s(1)}{(1+h)-1} = \frac{(-20(1+h)^2+160(1+h))-(-20(1)^2+160(1))}{1+h-1} = \frac{(-20(1+h)(1+h)+160(1+h))-(-20(1)(1)+160(1))}{h} = \frac{(-20(1+2h+h^2)+160(1+h))-(-20+160)}{h} = \frac{-20-40h-20h^2+160+160h-140}{h} = \frac{-20h^2+120h}{h} = \frac{h(-20h+120)}{h} = -20h+120$

Answers 60

80

100

$-20h+120$

8. For the position function $s(t) = -16t^2 + 102t$, complete the following table with the appropriate average velocities. Then make a conjecture about the value of the instantaneous velocity at t = 1.

Time Interval	[1, 2]	[1, 1.5]	[1, 1.1]	[1, 1.01]	[1, 1.001]
Average Velocity	___	___	___	___	___

Complete the following table.

Time Interval	[1, 2]	[1, 1.5]	[1, 1.1]	[1, 1.01]	[1, 1.001]
Average Velocity	54	62	68.4	69.84	69.984

(Type exact answers. Type integers or decimals.)

The value of the instantaneous velocity at t = 1 is 70.
(Round to the nearest integer as needed.)

$54 = \frac{s(2)-s(1)}{2-1}$

$62 = \frac{s(1.5)-s(1)}{1.5-1}$

$68.4 = \frac{s(1.1)-s(1)}{1.1-1}$

$69.84 = \frac{s(1.01)-s(1)}{1.01-1}$

$69.984 = \frac{s(1.001)-s(1)}{1.001-1}$

Answers 54

62

68.4

69.84

69.984

70

9. For the function $f(x) = 17x^3 - x$, make a table of slopes of secant lines and make a conjecture about the slope of the tangent line at $x = 1$.

Complete the table.

(Round the final answer to three decimal places as needed. Round all intermediate values to four decimal places as needed.)

Interval	Slope of secant line
[1, 2]	118,000
[1, 1.5]	79,750
[1, 1.1]	55,270
[1, 1.01]	50,510
[1, 1.001]	50,100

$118000 = \frac{f(2) - f(1)}{2-1}$

$79,750 = \frac{f(1.5) - f(1)}{1.5-1}$

$55,270 = \frac{f(1.1) - f(1)}{1.1-1}$

$50,510 = \frac{f(1.01) - f(1)}{1.01-1}$

$50100 = \frac{f(1.001) - f(1)}{1.001-1}$

An accurate conjecture for the slope of the tangent line at $x = 1$ is 50.
(Round to the nearest integer as needed.)

Answers 118.000

79.750

55.270

50.510

50.100

50

10.

Let $f(x) = \frac{x^2-16}{x-4}$. **(a)** Calculate f(x) for each value of x in the following table. **(b)** Make a conjecture about the value of $\lim\limits_{x \to 4} \frac{x^2-16}{x-4}$.

(a) Calculate f(x) for each value of x in the following table.

x	3.9	3.99	3.999	3.9999
$f(x) = \frac{x^2-16}{x-4}$	7.9	7.99	7.999	7.9999
x	4.1	4.01	4.001	4.0001
$f(x) = \frac{x^2-16}{x-4}$	8.1	8.01	8.001	8.001

(Type an integer or decimal rounded to four decimal places as needed.)

(b) Make a conjecture about the value of $\lim\limits_{x \to 4} \frac{x^2-16}{x-4}$.

$\lim\limits_{x \to 4} \frac{x^2-16}{x-4} =$ 8 (Type an integer or a decimal.)

formula
a^2-b^2
$(a+b)(a-b)$

$\lim\limits_{x \to 4} \frac{x^2-16}{x-4} =$

$\lim\limits_{x \to 4} \frac{(x)^2-(4)^2}{x-4} =$

$\lim\limits_{x \to 4} \frac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}} =$

$\lim\limits_{x \to 4} (x+4) =$

$4+4 =$

$8 =$

Answers 7.9

7.99

7.999

7.9999

8.1

8.01

8.001

8.0001

8

$f(x) = \frac{x^2-16}{x-4}$

$f(3.9) = \frac{(3.9)^2-16}{3.9-4} = 7.9$

$f(3.99) = \frac{(3.99)^2-16}{3.99-4} = 7.99$

$f(3.999) = \frac{(3.999)^2-16}{3.999-4} = 7.999$

$f(3.9999) = \frac{(3.9999)^2-16}{3.9999-4} = 7.9999$

$f(4.1) = \frac{(4.1)^2-16}{4.1-4} = 8.1$

$f(4.01) = \frac{(4.01)^2-16}{4.01-4} = 8.01$

$f(4.001) = \frac{(4.001)^2-16}{4.001-4} = 8.001$

$f(4.0001) = \frac{(4.0001)^2-16}{4.0001-4} = 8.001$

11. Let $g(t) = \frac{t-49}{\sqrt{t}-7}$.

a. Make two tables, one showing the values of g for t = 48.9, 48.99, and 48.999 and one showing values of g for t = 49.1, 49.01, and 49.001.

b. Make a conjecture about the value of $\lim_{t\to 49} \frac{t-49}{\sqrt{t}-7}$.

a. Make a table showing the values of g for t = 48.9, 48.99, and 48.999.

t	48.9	48.99	48.999
g(t)	13.9929	13.9993	13.9999

(Round to four decimal places.)

Make a table showing the values of g for t = 49.1, 49.01, and 49.001.

t	49.1	49.01	49.001
g(t)	14.0071	14.0007	14.0001

(Round to four decimal places.)

b. Make a conjecture about the value of $\lim_{t\to 49} \frac{t-49}{\sqrt{t}-7}$. Select the correct choice below and ~~fill in any answer boxes in~~ your choice.

A. $\lim_{t\to 49} \frac{t-49}{\sqrt{t}-7} =$ 14 (Simplify your answer.)

B. The limit does not exist.

Answers 13.9929

13.9993

13.9999

14.0071

14.0007

14.0001

A. $\lim_{t\to 49} \frac{t-49}{\sqrt{t}-7} =$ 14 (Simplify your answer.)

$g(48.9) = \frac{48.9-49}{\sqrt{48.9}-7} = 13.9929$

$g(48.99) = \frac{48.99-49}{\sqrt{48.99}-7} = 13.9993$

$g(13.9999) = \frac{48.999-49}{\sqrt{48.999}-7} = 13.9999$

$g(49.1) = \frac{49.1-49}{\sqrt{49.1}-7} = 14.0071$

$g(49.01) = \frac{49.01-49}{\sqrt{49.1}-7} = 14.0007$

$g(49.001) = \frac{49.001-49}{\sqrt{49.001}-7} = 14.0001$

$\lim_{t\to 49} \frac{t-49}{\sqrt{t}-7} =$

mult

$\lim_{t\to 49} \left(\frac{t-49}{\sqrt{t}-7}\right)\left(\frac{\sqrt{t}+7}{\sqrt{t}+7}\right) =$

$\lim_{t\to 49} \frac{(t-49)(\sqrt{t}+7)}{(\sqrt{t})^2+\cancel{7\sqrt{t}}-\cancel{7\sqrt{t}}-49} =$

$\lim_{t\to 49} \frac{\cancel{(t-49)}(\sqrt{t}+7)}{\cancel{(t-49)}} =$

$\lim_{t\to 49} (\sqrt{t}+7) =$

$\sqrt{49}+7 =$

$7+7 =$

14

12. If $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = M$, where L and M are finite real numbers, then what must be true about L and M in order for $\lim_{x \to a} f(x)$ to exist?

Choose the correct answer below.

- A. $L < M$
- B. $L = M$
- C. $L > M$
- D. $L \neq M$

Answer: B. $L = M$

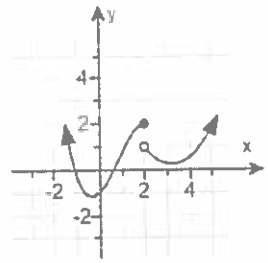
13. Use the graph to find the following limits and function value.

a. $\lim_{x \to 2^-} f(x)$

b. $\lim_{x \to 2^+} f(x)$

c. $\lim_{x \to 2} f(x)$

d. f(2)



a. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 2^-} f(x) =$ 2 (Type an integer.)

B. The limit does not exist.

b. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 2^+} f(x) =$ 1 (Type an integer.)

B. The limit does not exist.

c. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 2} f(x) =$ ______ (Type an integer.)

B. The limit does not exist.

d. Find the function value. Select the correct choice below and fill in any answer boxes in your choice.

A. f(2) = 2 (Type an integer.)

B. The answer is undefined.

Answers A. $\lim_{x \to 2^-} f(x) =$ 2 (Type an integer.)

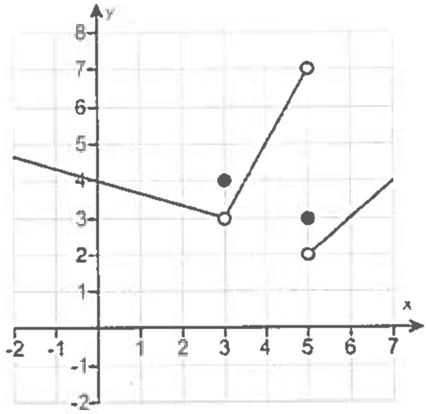
A. $\lim_{x \to 2^+} f(x) =$ 1 (Type an integer.)

B. The limit does not exist.

A. f(2) = 2 (Type an integer.)

14.

Use the graph of f to complete parts (a) through (l). If a limit does not exist, explain why.



(a) Find f(3). Select the correct choice below, and fill in the answer box if necessary.

A. f(3) = 4
(Type an integer or a fraction.)

B. The value of f(3) is undefined.

(b) Find $\lim_{x \to 3^-} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

A. $\lim_{x \to 3^-} f(x) =$ 3
(Type an integer or a fraction.)

B. The limit does not exist because f(x) is not defined for all $x < 3$.

(c) Find $\lim_{x \to 3^+} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

A. $\lim_{x \to 3^+} f(x) =$ 3
(Type an integer or a fraction.)

B. The limit does not exist because f(x) is not defined for all $x > 3$.

(d) Find $\lim_{x \to 3} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

A. $\lim_{x \to 3} f(x) =$ 3
(Type an integer or a fraction.)

B. The limit does not exist because $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$.

(e) Find f(5). Select the correct choice below, and fill in the answer box if necessary.

A. f(5) = 3
(Type an integer or a fraction.)

B. The value of f(5) is undefined.

(f) Find $\lim_{x \to 5^-} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

A. $\lim_{x \to 5^-} f(x) =$ 7
(Type an integer or a fraction.)

B. The limit does not exist because f(x) is not defined for all $x < 5$.

Answers A. $f(3) =$ **4** (Type an integer or a fraction.)

A. $\lim_{x \to 3^-} f(x) =$ **3** (Type an integer or a fraction.)

A. $\lim_{x \to 3^+} f(x) =$ **3** (Type an integer or a fraction.)

A. $\lim_{x \to 3} f(x) =$ **3** (Type an integer or a fraction.)

A. $f(5) =$ **3** (Type an integer or a fraction.)

A. $\lim_{x \to 5^-} f(x) =$ **7** (Type an integer or a fraction.)

A. $\lim_{x \to 5^+} f(x) =$ **2** (Type an integer or a fraction.)

B. The limit does not exist because $\lim_{x \to 5^-} f(x) \neq \lim_{x \to 5^+} f(x)$.

A. $f(4) =$ **5** (Type an integer or a fraction.)

A. $\lim_{x \to 4^-} f(x) =$ **5** (Type an integer or a fraction.)

A. $\lim_{x \to 4^+} f(x) =$ **5** (Type an integer or a fraction.)

A. $\lim_{x \to 4} f(x) =$ **5** (Type an integer or a fraction.)

15. Explain why $\lim_{x \to 3} \frac{x^2 - 7x + 12}{x - 3} = \lim_{x \to 3} (x - 4)$, and then evaluate $\lim_{x \to 3} \frac{x^2 - 7x + 12}{x - 3}$.

Choose the correct answer below.

- A. The numerator of the expression $\frac{x^2 - 7x + 12}{x - 3}$ simplifies to $x - 4$ for all x, so the limits are equal.
- B. The limits $\lim_{x \to 3} \frac{x^2 - 7x + 12}{x - 3}$ and $\lim_{x \to 3} (x - 4)$ equal the same number when evaluated using direct substitution.
- C. Since $\frac{x^2 - 7x + 12}{x - 3} = x - 4$ whenever $x \neq 3$, it follows that the two expressions evaluate to the same number as x approaches 3.
- D. Since each limit approaches 3, it follows that the limits are equal.

Now evaluate the limit.

$\lim_{x \to 3} \frac{x^2 - 7x + 12}{x - 3} =$ $\boxed{-1}$ (Simplify your answer.)

$\lim_{x \to 3} \frac{x^2 - 7x + 12}{x - 3} =$

$\lim_{x \to 3} \frac{(x-3)(x-4)}{(x-3)} =$

$\lim_{x \to 3} (x-4) =$

$3 - 4 =$

$-1 =$

Answers C.

Since $\frac{x^2 - 7x + 12}{x - 3} = x - 4$ whenever $x \neq 3$, it follows that the two expressions evaluate to the same number as x approaches 3.

-1

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16. Find the following limit or state that it does not exist.

$$\lim_{x \to -1} \frac{x^2 + 3x + 2}{x + 1}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $\lim_{x \to -1} \frac{x^2 + 3x + 2}{x + 1} =$ ______ (Type an exact answer.)
- B. The limit does not exist.

Answer: A. $\lim_{x \to -1} \frac{x^2 + 3x + 2}{x + 1} =$ $\boxed{1}$ (Type an exact answer.)

$\lim_{x \to -1} \frac{x^2 + 3x + 2}{x + 1} =$

$\lim_{x \to -1} \frac{(x+1)(x+2)}{(x+1)} =$

$\lim_{x \to -1} (x+2) =$

$-1 + 2 =$

$1 =$

(17) $\lim_{x \to 289} \frac{\sqrt{x} - 17}{x - 289} =$

$$\lim_{x \to 289} \left(\frac{\sqrt{x} - 17}{x - 289}\right)\left(\frac{\sqrt{x} + 17}{\sqrt{x} + 17}\right) \stackrel{\text{Mult}}{=}$$

$$\lim_{x \to 289} \frac{(\sqrt{x})^2 + \cancel{17\sqrt{x}} - \cancel{17\sqrt{x}} - 289}{(x - 289)(\sqrt{x} + 17)} =$$

$$\lim_{x \to 289} \frac{(x - 289)}{(x - 289)(\sqrt{x} + 17)} =$$

$$\lim_{x \to 289} \frac{1\cancel{(x - 289)}}{\cancel{(x - 289)}(\sqrt{x} + 17)} =$$

$$\lim_{x \to 289} \frac{1}{\sqrt{x} + 17} =$$

$$\frac{1}{\sqrt{289} + 17} =$$

$$\frac{1}{17 + 17} =$$

$$\frac{1}{34} =$$

(18) determine the limit

$$\lim_{x \to -\infty} 6x^{17} =$$

$-\infty =$

(19) determine the limit

$$\lim_{x \to \infty} x^{-7} =$$

$$\lim_{x \to \infty} \frac{1}{x^7} =$$

$$0 =$$

formula

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to \infty} \frac{1}{x^n} = 0$$

20 determine the limit at infinity

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} =$$

formula

if $f(x) \to 100{,}000$
and $g(x) \to \infty$
as $x \to \infty$

$$\frac{\lim_{x \to \infty} f(x)}{\lim_{x \to \infty} g(x)} =$$

then

$\lim_{x \to \infty} f(x) = 100{,}000$

$\lim_{x \to \infty} g(x) = \infty$

$$\frac{100{,}000}{\infty} =$$

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$0 =$$

$$\lim_{x \to \infty} \frac{1}{x^n} = 0$$

(21.) determine the limit at infinity

$$\lim_{x\to\infty} \frac{4+9x+9x^4}{x^4}$$

$$\lim_{x\to\infty} \left(\frac{4+9x+9x^4}{x^4}\right) \frac{\frac{1}{x^4}}{\frac{1}{x^4}} = \text{mult}$$

$$\lim_{x\to\infty} \left(\frac{\frac{4}{1}+\frac{9x}{1}+\frac{9x^4}{1}}{\frac{x^4}{1}}\right) \frac{\frac{1}{x^4}}{\frac{1}{x^4}} =$$

$$\lim_{x\to\infty} \frac{\frac{4}{x^4}+\frac{9x}{x^4}+\frac{9\cancel{x^4}}{\cancel{x^4}}}{\frac{\cancel{x^4}}{\cancel{x^4}}} =$$

$$\lim_{x\to\infty} \frac{\frac{4}{x^4}+\frac{9}{x^3}+9}{1} =$$

$$\frac{0+0+9}{1} =$$

$$\frac{9}{1} =$$

9

formula

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

(22) $\lim\limits_{x\to\infty} \dfrac{\sin(10x)}{3x}$

Squeeze theorem

$g(x) \le f(x) \le h(x)$, for all x in some open interval containing c, except possibly at $x = c$.

If $\lim\limits_{x\to c} g(x) = \lim\limits_{x\to c} h(x) = L$

then $\lim\limits_{x\to c} f(x) = L$

$$-1 \le \sin(10x) \le 1$$

$$\frac{-1}{3x} \le \frac{\sin(10x)}{3x} \le \frac{1}{3x}$$

$$\lim_{x\to\infty}\left(\frac{-1}{3x}\right) \le \lim_{x\to\infty} \frac{\sin(10x)}{3x} \le \lim_{x\to\infty}\left(\frac{1}{3x}\right)$$

$$0 \le \lim_{x\to\infty} \frac{\sin(10x)}{3x} \le 0$$

$$\lim_{x\to\infty} \frac{\sin(10x)}{3x} = 0$$

23. $\lim_{x \to \infty} (3x^5 - 5x^4 + 1) =$

$\infty =$

(24) determine the limit

formula

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to \infty} \frac{1}{x^n} = 0$

$$\lim_{w \to \infty} \frac{20w^2+3w+3}{\sqrt{25w^4+5w^3}} =$$

$$\lim_{w \to \infty} \left(\frac{20w^2+3w+3}{\sqrt{25w^4+5w^3}} \right) \frac{\sqrt{\frac{1}{w^4}}}{\sqrt{\frac{1}{w^4}}} =$$ mult

$$\lim_{w \to \infty} \frac{20w^2\sqrt{\frac{1}{w^4}}+3w\sqrt{\frac{1}{w^4}}+3\sqrt{\frac{1}{w^4}}}{\sqrt{(25w^4+5w^3)\left(\frac{1}{w^4}\right)}} =$$

$$\lim_{w \to \infty} \frac{20w^2\left(\frac{1}{w^2}\right)+3w\left(\frac{1}{w^2}\right)+3\left(\frac{1}{w^2}\right)}{\sqrt{25w^4\left(\frac{1}{w^4}\right)+5w^3\left(\frac{1}{w^4}\right)}} =$$

$$\lim_{w \to \infty} \frac{\cancel{\frac{20w^2}{w^2}}+\frac{3w}{w^2}+\frac{3}{w^2}}{\sqrt{\cancel{\frac{25w^4}{w^4}}+\frac{5w^3}{w^4}}} =$$

$$\lim_{w \to \infty} \frac{20+\frac{3}{w}+\frac{3}{w^2}}{\sqrt{25+\frac{5}{w}}} =$$

$$\frac{20+0+0}{\sqrt{25+0}} =$$

$$\frac{20}{\sqrt{25}} =$$

$$\frac{20}{5} =$$

$$4 =$$

(25) determine the limit

formula
$$\lim_{x\to\infty} \frac{1}{x} = 0$$
$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to-\infty} \frac{\sqrt{49x^2+x}}{x} =$$

$$\lim_{x\to-\infty} \left(\frac{\sqrt{49x^2+x}}{x}\right)\frac{\sqrt{\frac{1}{x^2}}}{\sqrt{\frac{1}{x^2}}} =$$

$$\lim_{x\to-\infty} \left(\frac{\sqrt{(49x^2+x)\left(\frac{1}{x^2}\right)}}{(x)\left(\sqrt{\frac{1}{x^2}}\right)}\right) =$$

$$\lim_{x\to-\infty} \left(\frac{\sqrt{(49x^2)\left(\frac{1}{x^2}\right) + x\left(\frac{1}{x^2}\right)}}{(x)\left(\frac{1}{x}\right)}\right) =$$

$$\lim_{x\to-\infty} \frac{\sqrt{\frac{49x^2}{x^2} + \frac{x}{x^2}}}{\frac{x}{x}} =$$

$$\lim_{x\to-\infty} \frac{\sqrt{49+\frac{1}{x}}}{1} =$$

$$-\frac{\sqrt{49+0}}{1} = \quad -\frac{\sqrt{49}}{1} = \quad -\frac{7}{1} = \quad -7 =$$

(26)

$$\lim_{x\to\infty} \frac{2x}{10x+4} =$$

$$\lim_{x\to\infty} \left(\frac{2x}{10x+4}\right)\left(\frac{\frac{1}{x}}{\frac{1}{x}}\right) = \text{mult}$$

formula

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} \frac{\frac{2x}{x}}{\frac{10x}{x} + \frac{4}{x}} =$$

$$\lim_{x\to\infty} \frac{2}{10 + \frac{4}{x}} =$$

$$\frac{2}{10+0} =$$

$$\frac{2}{10} =$$

$$\frac{\cancel{2}(1)}{\cancel{2}(5)}$$

$$\frac{1}{5} =$$

The function has one horizontal asymptote

$$y = \frac{1}{5}$$

(27)

$$\lim_{x\to\infty} \frac{9x^2-8x+7}{3x^2+1} =$$

$$\lim_{x\to\infty} \left(\frac{9x^2-8x+7}{3x^2+1}\right) \frac{\frac{1}{x^2}}{\frac{1}{x^2}} =$$ ~~Mult~~

$$\lim_{x\to\infty} \frac{\frac{9x^2}{x^2} - \frac{8x}{x^2} + \frac{7}{x^2}}{\frac{3x^2}{x^2} + \frac{1}{x^2}} =$$

formula

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} \frac{9 - \frac{8}{x} + \frac{7}{x^2}}{3 + \frac{1}{x^2}} =$$

$$\frac{9-0+0}{3+0} =$$

$$\frac{9}{3} =$$

3

The horizontal asymptote is $y=3$

28

$$\lim_{x\to\infty} \frac{10x^5-7}{2x^5-6x^4} =$$

Mult

$$\lim_{x\to\infty} \left(\frac{10x^5-7}{2x^5-6x^4}\right)\left(\frac{\frac{1}{x^5}}{\frac{1}{x^5}}\right) =$$

formula

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} \left(\frac{\frac{10\cancel{x^5}}{\cancel{x^5}} - \frac{7}{x^5}}{\frac{2\cancel{x^5}}{\cancel{x^5}} - \frac{6x^4}{x^5}}\right) =$$

$$\lim_{x\to\infty} \left(\frac{10 - \frac{7}{x^5}}{2 - \frac{6}{x}}\right) =$$

$$\frac{10-0}{2-0} =$$

$$\frac{10}{2} =$$

$$5 =$$

the function has a horizontal asymptote at $y=5$

(29)

$$\lim_{x\to\infty} \frac{3x^5-6}{x^6+4x^4} =$$

formula

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} \left(\frac{3x^5-6}{x^6+4x^4}\right)\left(\frac{\frac{1}{x^6}}{\frac{1}{x^6}}\right) = \text{Mult}$$

$$\lim_{x\to\infty} \left(\frac{\frac{3x^5}{x^6} - \frac{6}{x^6}}{\frac{x^6}{x^6} + \frac{4x^4}{x^6}}\right) =$$

$$\lim_{x\to\infty} \left(\frac{\frac{3}{x} - \frac{6}{x^6}}{1 + \frac{4}{x^2}}\right) =$$

$$\frac{0-0}{1+0} =$$

$$\frac{0}{1} =$$

$$0 =$$

the function has ~~one~~ horizontal asymptote $y=0$

(30) $\lim_{x\to\infty} \frac{9x^3+3}{1-2x^3} =$

Formula
$\lim_{x\to\infty} \frac{1}{x} = 0$
$\lim_{x\to\infty} \frac{1}{x^n} = 0$

$\lim_{x\to\infty} \left(\frac{9x^3+3}{1-2x^3}\right)\left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right) =$ Mult

$\lim_{x\to\infty} \left(\frac{\frac{9\cancel{x^3}}{\cancel{x^3}} + \frac{3}{x^3}}{\frac{1}{x^3} - \frac{2\cancel{x^3}}{\cancel{x^3}}}\right) =$

$\lim_{x\to\infty} \left(\frac{9 + \frac{3}{x^3}}{\frac{1}{x^3} - 2}\right) =$

$\frac{9+0}{0-2} =$

$\frac{9}{-2} =$

the function has one horizontal asymptote $y = -9/2$

31 find all asymptotes

formula

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} \frac{4x^2+8}{2x^2+x-10} =$$

$$\lim_{x\to\infty} \left(\frac{4x^2+8}{2x^2+x-10}\right) \frac{\frac{1}{x^2}}{\frac{1}{x^2}} =$$ Mult

$$\lim_{x\to\infty} \left(\frac{\frac{4x^2}{x^2}+\frac{8}{x^2}}{\frac{2x^2}{x^2}+\frac{x}{x^2}-\frac{10}{x^2}}\right) =$$

$$\lim_{x\to\infty} \left(\frac{4+\frac{8}{x^2}}{2+\frac{1}{x}-\frac{10}{x^2}}\right) =$$

$$\frac{4+0}{2+0-0} =$$

$$\frac{4}{2} =$$

$$2 =$$

find vertical asymptotes

set $2x^2+x-10=0$

$(2x+5)(x-2)=0$

$2x+5=0$ OR $x-2=0$

$2x+5-5=0-5$ OR $x-2+2=0+2$

$2x=-5$ OR $x=2$

$\frac{2x}{2}=\frac{-5}{2}$

$x=-5/2$

Vertical asymptotes

$x=-5/2$

$x=2$

The function has one horizontal asymptote

$y=2$

32. Complete the following sentences.

(a) A function is continuous from the left at a if ______.
(b) A function is continuous from the right at a if ______.

(a) A function is continuous from the left at a if (1) ______.

(b) A function is continuous from the right at a if (2) ______.

(1)
- ○ $\lim_{x \to a^+} f(x) = f(a)$
- ○ $\lim_{x \to a^-} f(x) = f(a)$
- ○ $\lim_{x \to a^-} f(x) = -f(a)$
- ○ $\lim_{x \to a^+} f(x) = -f(a)$

(2)
- ○ $\lim_{x \to a^-} f(x) = f(a)$
- ○ $\lim_{x \to a^-} f(x) = -f(a)$
- ○ $\lim_{x \to a^+} f(x) = -f(a)$
- ○ $\lim_{x \to a^+} f(x) = f(a)$

Answers (1) $\lim_{x \to a^-} f(x) = f(a)$

(2) $\lim_{x \to a^+} f(x) = f(a)$

---

33. Determine whether the following function is continuous at a. Use the continuity checklist to justify your answer.

$$f(x) = \frac{5x^2 + 18x + 9}{x^2 - 2x},\ a = 2$$

Select all that apply.

- ☐ **A.** The function is continuous at a = 2.
- ☐ **B.** The function is not continuous at a = 2 because f(2) is undefined.
- ☐ **C.** The function is not continuous at a = 2 because $\lim_{x \to 2} f(x)$ does not exist.
- ☐ **D.** The function is not continuous at a = 2 because $\lim_{x \to 2} f(x) \neq f(2)$.

Answer: B. The function is not continuous at a = 2 because f(2) is undefined. , C. The function is not continuous at a = 2 because $\lim_{x \to 2} f(x)$ does not exist. , D. The function is not continuous at a = 2 because $\lim_{x \to 2} f(x) \neq f(2)$.

(34) determine whether the function is continuous at a. Use the continuity checklist to justify your answer.

$$f(x) \begin{cases} \dfrac{x^2-64}{x-8} & \text{if } x \neq 8 \\ 7 & \text{if } x = 8 \end{cases}$$

$a = 8$

the function is not continuous at $a=8$ because $\lim_{x \to \infty} f(x) \neq f(8)$

(35) determine the intervals on which the function is continuous

$$f(x) = \frac{x^2-3x+2}{x^2-1}$$

formula $a^2-b^2=(a+b)(a-b)$

set $x^2-1=0$ ← undefined here

$$(x)^2-(1)^2=0$$

$$(x+1)(x-1)=0$$

$x+1=0$ or $x-1=0$

$x+1-1=0-1$ or $x-1+1=0+1$

$x=-1$ or $x=1$

$$\boxed{(-\infty,-1)\cup(-1,1)\cup(1,\infty)}$$

continuous

(36) find the limit

$$\lim_{x \to 7} \sqrt{x^2+15} =$$

subs $x=7$

since x^2+15 is continuous for all x and the square root function is continuous for $x \geq 0$

$$\sqrt{(7)^2+15} =$$

$$\sqrt{(7)(7)+15} =$$

$$\sqrt{49+15} =$$

$$\sqrt{64} =$$

$$8 =$$

(37) Suppose X lies in the interval $(5, 9)$ with $X \neq 7$. find the smallest positive value of δ such that the inequality $0 < |X-7| < \delta$ is true for all possible value of X.

$$0 < |X-7| < \delta$$

$$-\delta < X-7 < \delta$$

$$-\delta + 7 < X - 7 + 7 < \delta + 7$$

$$-\delta + 7 < X < \delta + 7$$

$-\delta + 7 = 5$ OR $\delta + 7 = 9$

$-\delta + 7 - 7 = 5 - 7$ OR $\delta + \cancel{7} - \cancel{7} = 9 - 7$

$-\delta = -2$ OR $\delta = 2$

$\dfrac{-\delta}{-1} = \dfrac{-2}{-1}$

$\delta = 2$

the smallest positive value of $\delta = 2$

(40) Use the precise definition of a limit to prove the limit. Specify a relationship between e and δ that guarantee the limit exists

$$\lim_{x \to 0} (4x-8) = -8$$

$$|(4x-8)-(-8)| < e$$

$$|4x-8+8| < e$$

$$|4x| < e$$

$$|4||x| < e$$

$$4|x| < e$$

$$\frac{\cancel{4}|x|}{\cancel{4}} < \frac{e}{4}$$

$$|x| < \frac{e}{4}$$

$$|x-0| < \frac{e}{4}$$

let $e > 0$ and $\delta = \frac{e}{4}$

let $\delta = \frac{e}{4}$ then

$|(4x-8)-(-8)| < e$ whenever $0 < |x-0| < \delta = \frac{e}{4}$

41 Find the value of the derivate of the function at the given point.

$f(x) = 4x^2 - 3x$ $\quad (-1, 7)$ point
$\quad x_1 \; y_1$

$f'(x) = 4(2x) - 3$

$f'(x) = 8x - 3$

$f'(-1) = 8(-1) - 3$

$f'(-1) = -8 - 3$

$f'(-1) = -11$

(42) use the definition $m_{tan} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$
to find the line tangent to the graph of f at P_0
determine the equation of the tangent line at P_0

$f(x) = x^2 - 5$ $\quad (4, 11)$ point
$\quad\quad\quad\quad\quad\quad x_1 \; y_1$

$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

$f'(a) = \lim_{h \to 0} \frac{((a+h)^2 - 5) - ((a)^2 - 5)}{h} =$

$f'(a) = \lim_{h \to 0} \frac{((a+h)(a+h) - 5) - (a^2 - 5)}{h} =$

$f'(a) = \lim_{h \to 0} \frac{(a^2 + ah + ah + h^2 - 5) - (a^2 - 5)}{h} =$

$f'(a) = \lim_{h \to 0} \frac{\cancel{a^2} + 2ah + h^2 \cancel{-5} \cancel{-a^2} \cancel{+5}}{h} =$

$f'(a) = \lim_{h \to 0} \frac{2ah + h^2}{h}$

$f'(a) = \lim_{h \to 0} \frac{\cancel{h}(2a+h)}{\cancel{h}} =$

$\lim_{h \to 0} (2a+h) =$

$2a + 0 =$

$f'(a) = 2a$

$f(x) = x^2 - 5$

$f'(a) = 2a$

$f'(4) = 2(4)$

$f'(4) = 8 = m$

slope

$y - y_1 = m(x - x_1)$

$y - (11) = 8(x - (4))$

$y - 11 = 8(x - 4)$

$y - 11 = 8x - 32$

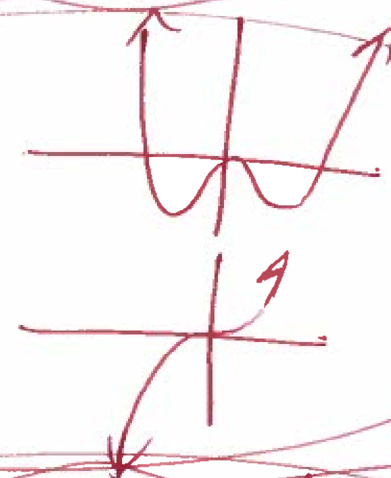
$y - \cancel{11} \cancel{+11} = 8x - 32 + 11$

$y = 8x - 21$

(43) Match the graph of the function with its derivative

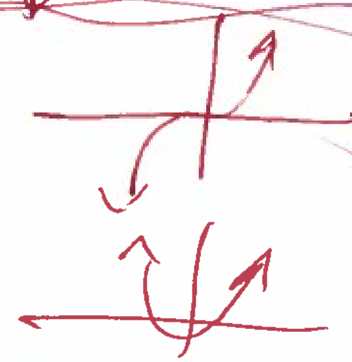
Examples $y=x^4$

$y'=4x^3$



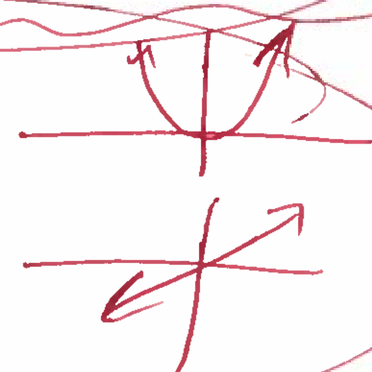
$y=x^3$

$y'=3x^2$



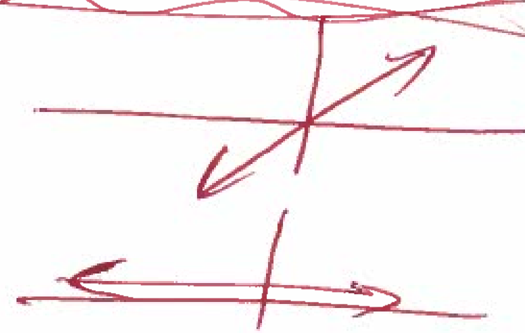
$y=x^2$

$y'=2x$



$y=x$

$y'=0$



(44) a line perpendicular to another line or to a tangent line is called a **normal line**. Find an equation of the line perpendicular to the line that is tangent to the following curve at the given point P.

$y = 5x + 11$ $\qquad (-2, 1)$ Point P

$\qquad\qquad\qquad\qquad x_1 \quad y_1$

$m = 5$

$m_{per} = -\frac{1}{5}$ Slope perpendicular

Example
m = Slope
$-\frac{1}{m}$ = perpendicular slope

$y - y_1 = m(x - x_1)$

$y - (1) = -\frac{1}{5}(x - (-2))$

$y - 1 = -\frac{1}{5}(x + 2)$

$y - 1 = -\frac{1}{5}x + \left(-\frac{2}{5}\right)$

$y - 1 = -\frac{1}{5}x - \frac{2}{5}$

$y - \cancel{1} + \cancel{1} = -\frac{1}{5}x - \frac{2}{5} + 1$

$y = -\frac{1}{5}x - \frac{2}{5} + \frac{5}{5}$

$y = -\frac{1}{5}x + \frac{-2+5}{5}$

$y = -\frac{1}{5}x + \frac{3}{5}$

(45) $y = \frac{x-3}{6x-5}$

$$y' = \frac{(x-3)'(6x-5) - (x-3)(6x-5)'}{(6x-5)^2}$$

$$y' = \frac{(1-0)(6x-5) - (x-3)(6-0)}{(6x-5)^2}$$

$$y' = \frac{(1)(6x-5) - (x-3)(6)}{(6x-5)^2}$$

$$y' = \frac{(6x-5) - (6x-18)}{(6x-5)^2}$$

$$y' = \frac{\cancel{6x} - 5 - \cancel{6x} + 18}{(6x-5)^2}$$

$$y' = \frac{13}{(6x-5)^2}$$

formula

$y = \frac{f}{g}$

$y' = \frac{f'g - fg'}{(g)^2}$

$y = ax$

$y' = a$

(46) use Quotient Rule to find $g'(1)$

formula

$$y = \frac{f}{g}$$

$$y' = \frac{f'g - fg'}{(g)^2}$$

$$g(x) = \frac{3x^2}{4x+1}$$

$$g'(x) = \frac{(3x^2)'(4x+1) - (3x^2)(4x+1)'}{(4x+1)^2}$$

$$g'(x) = \frac{(6x)(4x+1) - (3x^2)(4+0)}{(4x+1)^2}$$

$$g'(x) = \frac{(6x)(4x+1) - (3x^2)(4)}{(4x+1)^2}$$

$$g'(x) = \frac{24x^2 + 6x - 12x^2}{(4x+1)^2}$$

$$g'(x) = \frac{12x^2 + 6x}{(4x+1)^2}$$

$$g'(1) = \frac{12(1)^2 + 6(1)}{(4(1)+1)^2}$$

$$g'(1) = \frac{12(1)(1) + 6(1)}{(4+1)^2}$$

$$g'(1) = \frac{12+6}{(5)^2}$$

$$g'(1) = \frac{18}{25}$$

(47) $f(x) = (x-5)(3x+3)$

Use Product Rule

$f'(x) = (x-5)'(3x+3) + (x-5)(3x+3)'$

$f'(x) = (1-0)(3x+3) + (x-5)(3+0)$

$f'(x) = (1)(3x+3) + (x-5)(3)$

$f'(x) = 3x+3 + 3x-15$

$f'(x) = 6x-12$

formula

$y = f \cdot g$

$y' = f'g + fg'$

OR Expand first

$f(x) = (x-5)(3x+3)$

$f(x) = 3x^2 + 3x - 15x - 15$

$f(x) = 3x^2 - 12x - 15$

$f'(x) = 3(2x) - 12 - 0$

$f'(x) = 6x - 12$

$y = x^n$

$y' = nx^{n-1}$

(48) $h(w) = \frac{3w^3 - w}{w}$

$$h'(w) = \frac{(3w^3-w)'(w) - (3w^3-w)(w)'}{(w)^2}$$

$$h'(w) = \frac{(9w^2-1)(w) - (3w^3-w)(1)}{(w)^2}$$

$$h'(w) = \frac{(9w^2-1)(w) - (3w^3-w)}{(w)^2}$$

$$h'(w) = \frac{9w^3 - \cancel{w} - 3w^3 + \cancel{w}}{(w)^2}$$

$$h'(w) = \frac{6w^3}{(w)^2}$$

$$h'(w) = \frac{6w^3}{w^2}$$

$$h'(w) = 6w^{3-2}$$

$$h'(w) = 6w$$

Formula

$y = \frac{f}{g}$

$y' = \frac{f'g - fg'}{(g)^2}$

$y = x^n$

$y' = nx^{n-1}$

$y = a$

$y' = 0$

OR

$$h(w) = \frac{3w^3 - w}{w}$$

$$h(w) = \frac{3w^3}{w^1} - \frac{w^1}{w^1}$$

$$h(w) = 3w^{3-1} - 1$$

$$h(w) = 3w^2 - 1$$

$$h'(w) = 6w^{2-1}$$

$$h'(w) = 6w$$

49 $\lim\limits_{x \to 0} \dfrac{\sin(7x)}{\sin(2x)} =$

formula

$\lim\limits_{x \to 0} \dfrac{\sin(x)}{x} = 1$

$\lim\limits_{x \to 0} \left(\dfrac{\sin(7x)}{\sin(2x)}\right)\left(\dfrac{\frac{1}{x}}{\frac{1}{x}}\right)$ mult

$\lim\limits_{x \to 0} \dfrac{\sin(mx)}{(mx)} = 1$

$\lim\limits_{x \to 0} \dfrac{\frac{\sin(7x)}{x}}{\frac{\sin(2x)}{x}} =$

$\lim\limits_{x \to 0} \dfrac{\left(\frac{\sin(7x)}{x}\right)\frac{7}{7}}{\left(\frac{\sin(2x)}{x}\right)\frac{2}{2}} =$ mult

$\lim\limits_{x \to 0} \dfrac{\frac{7\sin(7x)}{7x}}{\frac{2\sin(2x)}{2x}} =$

$\dfrac{\lim\limits_{x \to 0} \frac{7\sin(7x)}{7x}}{\lim\limits_{x \to 0} \frac{2\sin(2x)}{2x}} =$

$\dfrac{7\lim\limits_{x \to 0} \frac{\sin(7x)}{(7x)}}{2\lim\limits_{x \to 0} \frac{\sin(2x)}{2x}} =$

$\dfrac{7(1)}{2(1)} =$

$\dfrac{7}{2} =$

50 $\lim_{x\to 0} \dfrac{\sin(2x)}{\tan(x)} =$

formula

$\lim_{x\to 0} \dfrac{\sin(x)}{x} = 1$

$\lim_{x\to 0} \dfrac{\sin(mx)}{(mx)} = 1$

$\lim_{x\to 0} \dfrac{\sin(2x)}{\dfrac{\sin(x)}{\cos(x)}} =$

$\lim_{x\to 0} \dfrac{(\sin(2x))}{\left(\dfrac{\sin(x)}{\cos x}\right)} \dfrac{\frac{1}{x}}{\frac{1}{x}} =$ mult

$\lim_{x\to 0} \dfrac{\dfrac{\sin(2x)}{x}}{\dfrac{\sin(x)}{\cos(x)\; x}} =$

$\lim_{x\to 0} \dfrac{\dfrac{\sin(2x)}{x}}{\dfrac{1}{\cos(x)} \cdot \dfrac{\sin x}{x}} =$ rewrite

$\lim_{x\to 0} \dfrac{\dfrac{\sin(2x)}{x} \cdot \dfrac{2}{2}}{\dfrac{1}{\cos(x)} \cdot \dfrac{\sin(x)}{x}} =$

$\lim_{x\to 0} \dfrac{\dfrac{2\sin(2x)}{2x}}{\dfrac{1}{\cos(x)} \cdot \dfrac{\sin(x)}{x}} =$

$\dfrac{\lim_{x\to 0} \dfrac{2\sin(2x)}{2x}}{\lim_{x\to 0} \dfrac{1}{\cos(x)} \cdot \lim_{x\to 0} \dfrac{\sin(x)}{x}} =$

$\dfrac{2 \lim_{x\to 0} \dfrac{\sin(2x)}{(2x)}}{\lim_{x\to 0} \dfrac{1}{\cos(x)} \cdot \lim_{x\to 0} \dfrac{\sin(x)}{x}} =$

$\dfrac{2(1)}{\dfrac{1}{\cos(0)} \cdot 1}$

$\dfrac{2(1)}{\frac{1}{1} \cdot 1} = \dfrac{2}{1} = 2$ ✓

51

$$\lim_{\theta \to 0} \frac{\cos^2(\theta) - 1}{\theta}$$

formula

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\lim_{\theta \to 0} \frac{\cos^2(\theta) - (1)^2}{\theta} =$$

$$\lim_{\theta \to 0} \frac{(\cos(\theta)+1)(\cos(\theta)-1)}{\theta} =$$

$$\lim_{\theta \to 0} (\cos(\theta)+1) \cdot \lim_{\theta \to 0} \left(\frac{\cos(\theta)-1}{\theta}\right) =$$

$$(\cos(0)+1) \cdot (0) =$$

$$(1+1) \cdot (0) =$$

$$(2) \cdot (0) =$$

$$0 =$$

(52) $y = 6\sin(x) + 8\cos(x)$

$y' = 6\cos(x)\,(x)' + (-8\sin(x)\,(x)')$

$y' = 6\cos(x)\,(1) + (-8\sin(x)\,(1))$

$y' = 6\cos(x) - 8\sin(x)$

OR

$\frac{dy}{dx} = 6\cos(x) - 8\sin(x)$

formula

$y = \sin(f(x))$

$y' = \cos(f(x)) \cdot f'(x)$

$y = \cos(f(x))$

$y' = -\sin(f(x)) \cdot f'(x)$

53

$$y = e^{-x}\sin(x)$$

$$y = fg$$
$$y' = f'g + fg'$$

$$y = e^{f(x)}$$
$$y' = e^{f(x)} f'(x)$$

$$y' = (e^{-x})'(\sin(x)) + (e^{-x})(\sin(x))'$$

$$y = \sin f(x)$$
$$y' = \cos f(x)\, f'(x)$$

$$y' = (e^{-x}(-x)')(\sin(x)) + (e^{-x})(\cos(x)(x)')$$

$$y' = (e^{-x}(-1))(\sin(x)) + (e^{-x})(\cos(x)(1))$$

$$y' = (-e^{-x})(\sin(x)) + (e^{-x})(\cos(x))$$

$$y' = -e^{-x}\sin(x) + e^{-x}\cos(x)$$

$$y' = e^{-x}\cos(x) - e^{-x}\sin(x)$$ rewrite

$$y' = e^{-x}(\cos(x) - \sin(x))$$

OR

$$\frac{dy}{dx} = e^{-x}(\cos(x) - \sin(x))$$

54 Find the equation of the line tangent to the curve at the given point

$y = 15x^2 + 8\sin(x)$ $\quad (0, 0)$ point; x_1, y_1

$y' = 30x + 8\cos(x)(x)'$

$y' = 30x + 8\cos x(x) \cdot (1)$

$y' = 30x + 8\cos(x)$

$y'(0) = 30(0) + 8\cos(0)$

$y'(0) = 0 + 8(1)$

$y'(0) = 0 + 8$

$y'(0) = 8 = m = \text{slope}$

$y - y_1 = m(x - x_1)$

$y - (0) = 8(x - (0))$

$y - 0 = 8(x - 0)$

$y = 8x$

(55) $h(x) = f(g(x))$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

x	1	2	3	4
$f(x)$	1	2	4	3
$f'(x)$	−9	−8	−4	−6
$g(x)$	3	1	2	4
$g'(x)$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{5}{7}$	$\frac{6}{7}$

$p(x) = g(f(x))$

$$p'(x) = g'(f(x)) \cdot f'(x)$$

$h'(1) = f'(g(1)) \cdot g'(1)$

$h'(1) = f'(3) \cdot \left(\frac{2}{7}\right)$

$h'(1) = -4 \cdot \left(\frac{2}{7}\right)$

$$h'(1) = \frac{-8}{7}$$

$p'(3) = g'(f(3))\, f'(3)$

$p'(3) = g'(4) \cdot (-4)$

$p'(3) = \left(\frac{6}{7}\right)(-4)$

$$p'(3) = \frac{-24}{7}$$

(56) $y=(5x-8)^3$

$y' = 3(5x-8)^{3-1} \cdot (5x-8)'$

$y' = 3(5x-8)^2 (5-0)$

$y' = 3(5x-8)^2 (5)$

$y' = 15(5x-8)^2$ OR

$\frac{dy}{dx} = 15(5x-8)^2$

formula chain

$y=(f(x))^n$

$y' = n(f(x))^{n-1} \cdot f'(x)$

$y = x^n$

$y' = nx^{n-1}$

$y = ax$

$y' = a$

(57) $y = 2(5x^4+2)^{-9}$

$y' = 2(-9)(5x^4+2)^{-9-1} \cdot (5x^4+2)'$

$y' = -18(5x^4+2)^{-10}(20x^3+0)$

$y' = -18(5x^4+2)^{-10}(20x^3)$

$y' = -360x^3(5x^4+2)^{-10}$

$y' = \dfrac{-360x^3}{(5x^4+2)^{10}}$

OR

$\dfrac{dy}{dx} = \dfrac{-360x^3}{(5x^4+2)^{10}}$

formula chain

$y = (f(x))^N$

$y' = N(f(x))^{N-1} \cdot f'(x)$

$y = x^n$

$y' = nx^{n-1}$

$y = ax$

$y' = a$

58 $y = \cos(9t-16)$

$y' = -\sin(9t-16) \cdot (9t-16)'$

$y' = -\sin(9t-16)(9-0)$

$y' = -\sin(9t-16)(9)$

$y' = -9\sin(9t-16)$

OR

$\frac{dy}{dt} = -9\sin(9t-16)$

~~formula~~

$y = \cos(f(x))$

$y' = -\sin(f(x)) \cdot f'(x)$

(59) $y = \tan(e^x)$

$y = \sec^2(e^x) \cdot (e^x)'$

$y = \sec^2(e^x) \cdot (e^x (x)')$

$y' = \sec^2(e^x) \cdot (e^x(1))$

$y' = \sec^2(e^x)\,(e^x)$

$y' = e^x \sec^2(e^x)$

OR

$$\frac{dy}{dx} = e^x \sec^2(e^x)$$

formula

$y = \tan(f(x))$

$y' = \sec^2(f(x)) \cdot f'(x)$

$y = e^{f(x)}$

$y' = e^{f(x)} \cdot f'(x)$

60

$$y = (\csc(x) + \cot(x))^{11}$$

$$y' = 11(\csc(x) + \cot(x))^{11-1} \cdot (\csc(x) + \cot(x))'$$

$$y' = 11(\csc(x) + \cot(x))^{10} \cdot (-\csc(x)\cot(x) - \csc^2(x))$$

$$y' = 11(\csc(x) + \cot(x))^{10} \cdot (-\csc(x))(\cot(x) + \csc(x))$$

$$y' = -11\csc(x)(\csc(x) + \cot(x))^{10}(\cot(x) + \csc(x))$$

$$y' = -11\csc(x)(\csc(x) + \cot(x))^{10} \cdot (\csc(x) + \cot(x))$$

$$y' = -11\csc(x)(\csc(x) + \cot(x))^{10} \cdot (\csc(x) + \cot(x))^1$$

$$y' = -11\csc(x)(\csc(x) + \cot(x))^{10+1}$$

$$y' = -11\csc(x)(\csc(x) + \cot(x))^{11}$$

OR

$$\frac{dy}{dx} = -11\csc(x)(\csc(x) + \cot(x))^{11}$$

61. $y = \cos(3\sin(x))$

$y' = -\sin(3\sin(x)) \cdot (3\sin(x))'$

$y' = -\sin(3\sin(x)) \cdot (3\cos(x)(x)')$

$y' = -\sin(3\sin(x)) \cdot (3\cos(x)(1))$

$y' = -\sin(3\sin(x)) \cdot (3\cos(x))$

move

$y' = -3\sin(3\sin(x)) \cdot \cos(x)$

OR

$\frac{dy}{dx} = -3\sin(3\sin(x)) \cdot \cos(x)$

62.

For some equations, such as $x^2 + y^2 = 1$ or $x - y^2 = 1$, it is possible to solve for y and then calculate $\frac{dy}{dx}$. Even in these cases, explain why implicit differentiation is usually a more efficient method for calculating the derivative.

Choose the correct answer below.

- A. Because implicit differentiation gives a single unified derivative.
- B. Because it produces $\frac{dy}{dx}$ in terms of x only.
- C. Because implicit differentiation gives two or more derivatives.
- D. Because it produces $\frac{dy}{dx}$ in terms of y only.

Answer: A. Because implicit differentiation gives a single unified derivative.

63

$$16x = y^2$$

$$16(1) = 2yy'$$

$$16 = 2yy'$$

$$\frac{16}{2y} = \frac{\cancel{2y}y'}{\cancel{2y}}$$

$$\frac{\cancel{2}(8)}{\cancel{2}(y)} = y'$$

$$\frac{8}{y} = y'$$

OR

$$\frac{8}{y} = \frac{dy}{dx}$$

(64)

$$\cos(y) + 9 = x$$

$$-\sin(y) \cdot y' + 0 = 1$$

$$-\sin(y)\, y' = 1$$

$$\frac{\cancel{-\sin(y)}(y')}{\cancel{-\sin(y)}} = \frac{1}{-\sin(y)}$$

$$y' = \frac{1}{-\sin(y)}$$

$$y' = -\frac{1}{\sin(y)}$$

formula

$$y = \cos(f(x))$$

$$y' = -\sin(f(x))\, f'(x)$$

$$\frac{1}{\sin(x)} = \csc(x)$$

$$y' = -\csc(y)$$

OR

$$\frac{dy}{dx} = -\csc(y)$$

65 $x = y^3$

$(1) = 3y^2 y'$

$1 = 3y^2 y'$

$\frac{1}{3y^2} = \frac{\cancel{3y^2} y'}{\cancel{3y^2}}$

$\frac{1}{3y^2} = y'$

$y' = \frac{1}{3y^2}$ OR $\boxed{\frac{dy}{dx} = \frac{1}{3y^2}}$ ✓✓

$y' = \frac{1}{3} y^{-2}$

$y'' = \frac{1}{3}(-2y^{-2-1} \cdot y')$

$y'' = -\frac{2}{3} y^{-3} \cdot y'$

$y'' = -\frac{2}{3} y^{-3} \cdot \left(\frac{1}{3y^2}\right)$ subst

$y'' = -\frac{2}{3} \left(\frac{1}{y^3}\right)\left(\frac{1}{3y^2}\right)$

$y'' = \frac{-2}{3 \cdot y^3 \cdot 3y^2}$

$y'' = \frac{-2}{9y^{3+2}}$

$y'' = \frac{-2}{9y^5}$ ✓

OR

$\boxed{\frac{d^2y}{dx^2} = \frac{-2}{9y^5}}$ ✓

(66) $x^5+y^5=244$ $\quad (\underset{x}{3},\underset{y}{1})$ point

$5x^4+5y^4\cdot y'=0$

$5x^4+5y^4y'=0$

$5y^4y'=-5x^4$

$$\frac{\cancel{5y^4}y'}{\cancel{5y^4}}=\frac{-5x^4}{5y^4}$$

$$y'=\frac{-1x^4}{y^4} \quad \text{OR} \quad \frac{dy}{dx}=\frac{-1x^4}{y^4}$$

$$y'(\underset{x}{3},\underset{y}{1})=\frac{-1(3)^4}{(1)^4}$$

$$y'(3,1)=\frac{-1(3)(3)(3)(3)}{(1)(1)(1)(1)}$$

$$y'(3,1)=\frac{-81}{1}$$

$y'(3,1)=-81$

the slope of $x^5+y^5=244$ at point $(3,1)$

67 $\sin(y) + \sin(x) = 5y$

$$\cos(y)\,y' + \cos(x) = 5y'$$

$$\cancel{\cos(y)\,y'} + \cos(x) - \cancel{\cos(y)\,y'} = 5y' - \cos(y)\,y'$$

$$\cos(x) = 5y' - \cos(y)\,y'$$

$$\cos(x) = y'(5 - \cos(y))$$

$$\frac{\cos(x)}{(5-\cos(y))} = \frac{y'\cancel{(5-\cos(y))}}{\cancel{(5-\cos(y))}}$$

$$\frac{\cos(x)}{5-\cos(y)} = y'$$

$$y' = \frac{\cos(x)}{5-\cos(y)}$$

OR

$$\frac{dy}{dx} = \frac{\cos(x)}{5-\cos(y)}$$

(68) $4\sin(xy) = 9x + 5y$

formula
$y = \sin(f(x))$
$y' = \cos(f(x))\, f'(x)$

$4\cos(xy) \cdot (xy)' = 9 + 5y'$

$4\cos(xy) \cdot ((x)'(y) + (x)(y)') = 9 + 5y'$

$4\cos(xy) \cdot ((1)(y) + (x)(y')) = 9 + 5y'$

$4\cos(xy) \cdot (y + xy') = 9 + 5y'$

$4\cos(xy) \cdot y + 4\cos(xy)(xy') = 9 + 5y'$

$\cancel{4\cos(xy)y} + 4\cos(xy)(xy') \cancel{- 4\cos(xy)y} = 9 + 5y' - 4\cos(xy)y$

$4\cos(xy)(xy') = 9 + 5y' - 4\cos(xy)y$

$4\cos(xy)(xy') - 5y' = 9 + \cancel{5y'} - 4\cos(xy)y \cancel{- 5y'}$

$4\cos(xy)(xy') - 5y' = 9 - 4\cos(xy)y$

$y'(4\cos(xy)(x) - 5) = 9 - 4\cos(xy)y$

$y'(4x\cos(xy) - 5) = 9 - 4\cos(xy)y$

$\cancel{\dfrac{y'(4x\cos(xy) - 5)}{(4x\cos(xy) - 5)}} = \dfrac{9 - 4\cos(xy)y}{(4x\cos(xy) - 5)}$

$y' = \dfrac{9 - 4y\cos(xy)}{4x\cos(xy) - 5} = \dfrac{dy}{dx}$

(69) $e^{4xy} = 5y$

Formula
$y = e^{f(x)}$
$y' = e^{f(x)} \cdot f'(x)$

$e^{4xy}(4xy)' = 5y'$

$e^{4xy} \cdot ((4x)'(y) + (4x)(y)') = 5y'$

$e^{4xy} \cdot ((4)(y) + (4x)(y')) = 5y'$

$e^{4xy} \cdot (4y + 4xy') = 5y'$

$e^{4xy} \cdot 4y + e^{4xy} \cdot 4xy' = 5y'$

$4ye^{4xy} + 4xy'e^{4xy} = 5y'$

$4ye^{4xy} = 5y' - 4xy'e^{4xy}$

$4ye^{4xy} = y'(5 - 4xe^{4xy})$

$$\frac{4ye^{4xy}}{(5 - 4xe^{4xy})} = \frac{y'\cancel{(5 - 4xe^{4xy})}}{\cancel{(5 - 4xe^{4xy})}}$$

$$\frac{4ye^{4xy}}{5 - 4xe^{4xy}} = y'$$

OR

$$\frac{dy}{dx} = \frac{4ye^{4xy}}{5 - 4xe^{4xy}}$$

(70) $6x^5+7y^5=13xy$

$6(5x^{5-1})+7(5y^{5-1}\cdot y')=(13x)'(y)+(13x)(y)'$

$6(5x^4)+7(5y^4\cdot y')=(13)(y)+(13x)(y')$

$30x^4+35y^4\cdot y'=13y+13xy'$

$30x^4=13y+13xy'-35y^4y'$

$30x^4-13y=13xy'-35y^4y'$

$30x^4-13y=y'(13x-35y^4)$

$$\frac{30x^4-13y}{(13x-35y^4)}=\frac{y'\cancel{(13x-35y^4)}}{\cancel{(13x-35y^4)}}$$

$$\frac{30x^4-13y}{13x-35y^4}=y'$$

OR

$$\frac{30x^4-13y}{13x-35y^4}=\frac{dy}{dx}$$

(71) State the rule of differentiation for the logarithmic function $f(x) = \log_b(x)$. How does it differ from the derivative formula for $\ln(x)$

If $b > 0$ and $b \neq 1$ then

$$\frac{d}{dx}(\log_b(x)) = \frac{1}{x\ln(b)} \text{ for } x > 0$$

and $$\frac{d}{dx}(\log_b|x|) = \frac{1}{x\ln(b)} \text{ for } x \neq 0.$$

$$\frac{d}{dx}\log_b(x) = \left[\frac{d}{dx}\ln(x)\right] \div \ln(b)$$

or

$$\frac{d}{dx}\log_b(x) = \left[\frac{d}{dx}\ln(x)\right] \cdot \frac{1}{\ln(b)}$$

(72) $y = \ln\sqrt{x^2+16}$

$y = \ln(x^2+16)^{1/2}$

$y = \frac{1}{2}\ln(x^2+16)$

$y' = \frac{1}{2}\cdot\frac{(x^2+16)'}{(x^2+16)}$

$y' = \frac{1}{2}\cdot\frac{(2x+0)}{(x^2+16)}$

$y' = \frac{1}{2}\cdot\frac{(2x)}{(x^2+16)}$

$y' = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}x}{x^2+16}$

$y' = \frac{x}{x^2+16}$ OR

$\frac{dy}{dx} = \frac{x}{x^2+16}$

Formula

$y = \ln f(x)$

$y' = \frac{f'(x)}{f(x)}$

$\ln\sqrt{f(x)} =$

$\ln(f(x))^{1/2} =$

$\frac{1}{2}\ln(f(x)) =$

73. express the function $f(x) = g(x)^{h(x)}$ in terms of the natural logarithmic and natural exponential functions (base e).

$$f(x) = g(x)^{h(x)}$$

$$f(x) = e^{\ln(g(x))^{h(x)}}$$

$$f(x) = e^{h(x)\ln(g(x))}$$

formula

$$e^{\ln(m x)} = m(x)$$

$$e^{\ln(m(x)^{n(x)})} = m(x)^{n(x)}$$

(74) $y = \ln(6x^2+5)$

$$y' = \frac{(6x^2+5)'}{(6x^2+5)}$$

$$y' = \frac{12x+0}{(6x^2+5)}$$

$$y' = \frac{12x}{6x^2+5}$$

OR

$$\frac{dy}{dx} = \frac{12x}{6x^2+5}$$

formula

$$y = \ln(fx)$$

$$y' = \frac{f'(x)}{f(x)}$$

75 $y = 4x^{5\pi}$

$y' = 4(5\pi x^{5\pi - 1})$

$y' = 20\pi x^{5\pi - 1}$

or

$\frac{dy}{dx} = 20\pi x^{5\pi - 1}$

Formula

$y = x^n$

$y' = nx^{n-1}$

76

$y = 7^x$

$y' = 7^x \ln(7)$

or

$\frac{dy}{dx} = 7^x \ln(7)$

formula

$y = a^x$

$y' = a^x \cdot \ln(a)$

77.

$$y = 7\log_2(x^4 - 2)$$

formula

$$y = \log_2(f(x))$$

$$y' = \frac{f'(x)}{f(x)\ln(2)}$$

$$y' = 7\,\frac{(x^4-2)'}{(x^4-2)\cdot\ln(2)}$$

$$y' = \frac{7\,(4x^3 - 0)}{(x^4-2)\ln(2)}$$

$$y' = \frac{7\,(4x^3)}{(x^4-2)\ln(2)}$$

$$y' = \frac{28x^3}{(x^4-2)\ln(2)}$$

OR

$$\frac{dy}{dx} = \frac{28x^3}{(x^4-2)\ln(2)}$$

78 $y = \log_3(x)$

$$y' = \frac{(x)'}{(x)\ln(3)}$$

$$y' = \frac{1}{(x)\ln(3)}$$

OR

$$\frac{dy}{dx} = \frac{1}{(x)\ln(3)}$$

formula

$$y = \log_3(f(x))$$

$$y' = \frac{f'(x)}{f(x)\ln(3)}$$

79 use logarithmic differentiation

$$\boxed{f(x) = \frac{(x+4)^{12}}{(4x-12)^{10}}}$$

formula logs

$\ln \frac{f(x)}{g(x)} = \ln f(x) - \ln(g(x))$

$\cancel{\ln x^n = n\ln x}$

$$\ln(f(x)) = \ln \frac{(x+4)^{12}}{(4x-12)^{10}}$$

$$\ln(f(x)) = \ln(x+4)^{12} - \ln(4x-12)^{10}$$ rewrite

$$\ln(f(x)) = 12\ln(x+4) - 10\ln(4x-12)$$

$$\frac{f'(x)}{f(x)} = 12\,\frac{(x+4)'}{(x+4)} - 10\,\frac{(4x-12)'}{4x-12}$$

$$\frac{f'(x)}{f(x)} = 12\,\frac{(1+0)}{(x+4)} - 10\,\frac{(4-0)}{(4x-12)}$$

$$\frac{f'(x)}{f(x)} = \frac{12\,(1)}{x+4} - \frac{10\,(4)}{4x-12}$$

$$\frac{f'(x)}{f(x)} = \frac{12}{x+4} - \frac{40}{4x-12}$$

$$\cancel{f(x)\cdot}\frac{f'(x)}{\cancel{f(x)}} = f(x)\left[\frac{12}{x+4} - \frac{40}{4x-12}\right]$$

$$f'(x) = \frac{(x+4)^{12}}{(4x-12)^{10}}\left[\frac{12}{x+4} - \frac{4(10)}{4(x-3)}\right]$$

Subst

$$f'(x) = \frac{(x+4)^{12}}{(4x-12)^{10}}\left[\frac{12}{x+4} - \frac{10}{x-3}\right]$$

80

Formula

$$y = \sin^{-1}(x)$$

$$y' = \frac{(x)'}{\sqrt{1-(x)^2}}$$

$$y' = \frac{1}{\sqrt{1-x^2}}$$

$$y = \sin^{-1}(f(x))$$

$$y' = \frac{f'(x)}{\sqrt{1-(f(x))^2}}$$

$$y = \tan^{-1}(f(x))$$

$$y' = \frac{f'(x)}{1+(f(x))^2}$$

$$y = \tan^{-1}(x)$$

$$y' = \frac{(x)'}{1+(x)^2}$$

$$y' = \frac{1}{1+x^2}$$

$$y = \sec^{-1}(x)$$

$$y' = \frac{(x)'}{|x|\sqrt{(x)^2-1}}$$

$$y' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$y = \sec^{-1}(f(x))$$

$$y' = \frac{f'(x)}{|f(x)|\sqrt{(f(x))^2-1}}$$

81. $f(x) = \sin^{-1}(6x^4)$

$$f'(x) = \frac{(6x^4)'}{\sqrt{1-(6x^4)^2}}$$

$$f'(x) = \frac{24x^3}{\sqrt{1-(6x^4)(6x^4)}}$$

$$f'(x) = \frac{24x^3}{\sqrt{1-36x^{4+4}}}$$

$$f'(x) = \frac{24x^3}{\sqrt{1-36x^8}}$$

Formula

$$y = \sin^{-1}(f(x))$$

$$y' = \frac{f'(x)}{\sqrt{1-(f(x))^2}}$$

82 $y = 7\tan^{-1}(3x)$

$$y' = 7\frac{(3x)'}{1+(3x)^2}$$

$$y' = 7\frac{(3)}{1+(3x)^2}$$

$$y' = \frac{21}{1+(3x)(3x)}$$

$$y' = \frac{21}{1+9x^2}$$

OR

$$y' = \frac{21}{1+(3x)^2}$$

OR

$$\frac{dy}{dx} = \frac{21}{1+(3x)^2}$$

formula

$$y = \tan^{-1}(f(x))$$

$$y' = \frac{f'(x)}{1+(f(x))^2}$$

83

$$f(st) = \cot^{-1}(e^{st})$$

$$f'(st) = -\frac{(e^{st})'}{1+(e^{st})^2}$$

$$f'(st) = -\frac{(e^{st})(st)'}{1+(e^{st})(e^{st})}$$

$$f'(st) = -\frac{(e^{st})(1)}{1+e^{st+st}}$$

$$f'(st) = -\frac{e^{st}}{1+e^{2st}}$$

Formula

$$y = \cot^{-1}(f(x))$$

$$y' = -\frac{f'(x)}{1+(f(x))^2}$$

$$y = e^{f(x)}$$

$$y' = e^{f(x)} \cdot f'(x)$$

84. The sides of a square increase in length at a rate of 4 m/sec.

a. At what rate is the area of the square changing when the sides are 20 m long?
b. At what rate is the area of the square changing when the sides are 25 m long?

a. Write an equation relating the area of a square, A, and the side length of the square, s.

[ ] → $A = s^2$

Differentiate both sides of the equation with respect to t.

$\frac{dA}{dt} = (\boxed{2s})\frac{ds}{dt}$

$\frac{dA}{dt} = 2(20)(4) = 160 \ m^2/s$

The area of the square is changing at a rate of $\boxed{160}$ (1) $\boxed{m^2/s}$ when the sides are 20 m long.

b. The area of the square is changing at a rate of $\boxed{200}$ (2) $\boxed{m^2/s}$ when the sides are 25 m long.

(1) ○ m^2/s ○ m/s ○ m ○ m^3/s

(2) ○ m ○ m^2/s ○ m/s ○ m^3/s

$\frac{dA}{dt} = 2(25)(4) = 200 \ m^2/s$

Answers $A = s^2$

2s

160

(1) m^2/s

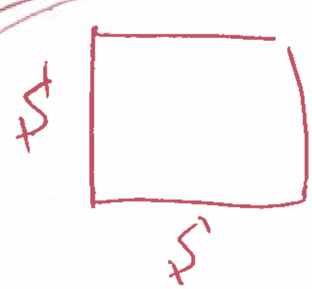
200

(2) m^2/s

~~area of~~ Square

$A = s^2$

$\frac{dA}{dt} = 2s \cdot \frac{ds}{dt}$



Square

85. The area of a circle increases at a rate of 2 cm^2 / s.

a. How fast is the radius changing when the radius is 4 cm?
b. How fast is the radius changing when the circumference is 6 cm?

a. Write an equation relating the area of a circle, A, and the radius of the circle, r.

[ ] $\rightarrow A = \pi r^2$

(Type an exact answer, using π as needed.)

Differentiate both sides of the equation with respect to t.

$\frac{dA}{dt} = (\boxed{2\pi r}) \frac{dr}{dt}$

(Type an exact answer, using π as needed.)

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$2 = 2\pi(4) \frac{dr}{dt}$

$2 = 8\pi \frac{dr}{dt}$

$\frac{2}{8\pi} = \frac{\cancel{8\pi} \frac{dr}{dt}}{8\pi}$

$\frac{2(1)}{2(4\pi)} = \frac{dr}{dt}$

$\frac{1}{4\pi} = \frac{dr}{dt}$

When the radius is 4 cm, the radius is changing at a rate of $\boxed{\frac{1}{4\pi}}$ (1) $\boxed{cm/s}$

(Type an exact answer, using π as needed.)

b. When the circumference is 6 cm, the radius is changing at a rate of $\boxed{1/3}$ (2) $\boxed{cm/s}$

(Type an exact answer, using π as needed.)

area of Circle

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

(1) ○ cm^2 / s.
○ cm^3 / s.
○ cm.
○ cm / s.

(2) ○ cm^3 / s.
○ cm^2 / s.
○ cm / s.
○ cm.

$C = 2\pi r$

$6 = 2\pi r$

$\frac{6}{2\pi} = \frac{2\pi r}{2\pi}$

$\frac{2(3)}{2(\pi)} = r$

$\frac{3}{\pi} = r$

Answers $A = \pi r^2$

$2\pi r$

$\frac{1}{4\pi}$

(1) cm / s.

$\frac{1}{3}$

(2) cm / s.

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

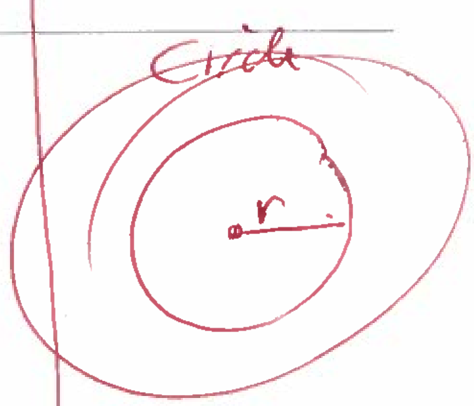
$2 = 2\cancel{\pi}\left(\frac{3}{\cancel{\pi}}\right) \frac{dr}{dt}$

$2 = 6 \frac{dr}{dt}$

$\frac{2}{6} = \frac{6 \frac{dr}{dt}}{6}$

$\frac{2(1)}{2(3)} = \frac{dr}{dt}$

$\frac{1}{3} = \frac{dr}{dt}$



86. A rope passing through a capstan on a dock is attached to a boat offshore. The rope is pulled in at a constant rate of 3 ft / s, and the capstan is 4 ft vertically above the water. How fast is the boat traveling when it is 7 ft from the dock?

Let x be the horizontal distance from the boat to the dock and z be the length of the rope. Write an equation relating x and z.

[ ] → $x^2+4^2=z^2$

Differentiate both sides of the equation with respect to t.

$(2x)\frac{dx}{dt} = (2z)\frac{dz}{dt}$

When the boat is 7 ft from the dock, it is traveling at about 3.46 (1) ft/s
(Round to two decimal places as needed.)

(1)
- ○ ft^3 / s.
- ○ ft^2 / s.
- ○ ~~ft.~~
- ✓ ft / s.

Answers $x^2+4^2=z^2$

2x

2z

3.46

(1) ft / s.

$x^2+4^2=z^2$

$2x\frac{dx}{dt}+0=2z\frac{dz}{dt}$

$2x\frac{dx}{dt}=2z\frac{dz}{dz}$

87. A 13-foot ladder is leaning against a vertical wall (see figure) when Jack begins pulling the foot of the ladder away from the wall at a rate of 0.7 ft / s. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 5 ft from the wall?

$a^2+b^2=c^2$

$x^2+y^2=(13)^2$

$x^2+y^2=169$

Let x be the distance from the foot of the ladder to the wall and let y be the distance from the top of the ladder to the ground. Write an equation relating x and y.

$x^2+y^2=169$

Differentiate both sides of the equation with respect to t.

$(2x)\frac{dx}{dt}+(2y)\frac{dy}{dt}=0$

When the foot of the ladder is 5 ft from the wall, the top of the ladder is sliding down the wall at a rate of 0.29

(1) ft/s

(Round to two decimal places as needed.)

(1) ○ ft^2 / s.
○ ft^3 / s.
○ ft.
⊘ ft / s.

$x^2+y^2=169$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

Answers $x^2+y^2=169$

2x

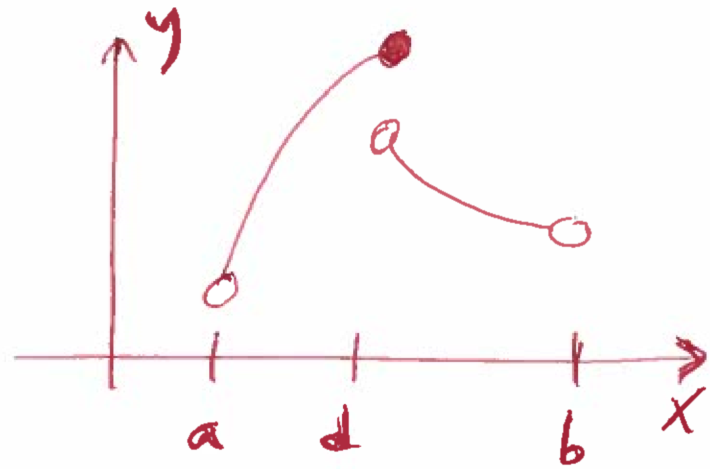
2y

0

0.29

(1) ft / s.

(88) determine from the graph whether the function has any absolute extreme value on



the absolute maximum occurs at $x=d$ and there is no absolute minimum on $[a, b]$

(89) find Critical Points

$f(x) = 2x^2 + 3x + 1$

$f'(x) = 4x + 3 + 0$

$f'(x) = 4x + 3$

set $4x + 3 = 0$

$4x + \cancel{3} - \cancel{3} = 0 - 3$

$4x = -3$

$\frac{4x}{4} = \frac{-3}{4}$

$x = -3/4$ Critical Point

(90) Find the critical points

$f(x) = 2x^2 + 5x + 1$

$f'(x) = 4x + 5 + 0$

$f'(x) = 4x + 5$

set $4x + 5 = 0$

$4x + \cancel{5} - \cancel{5} = 0 - 5$

$4x = -5$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{-5}{4}$

$x = -5/4$ Critical point

$y = x^n$

$y' = nx^{n-1}$

(9) find critical points

$$f(x) = -\frac{x^3}{3} + 144x$$

$$f'(x) = -\cancel{\tfrac{1}{3}}(\cancel{3}x^2) + 144$$

$$f'(x) = -1x^2 + 144$$

set $-1x^2 + 144 = 0$

$$-1x^2 = -144$$

$$\frac{-1x^2}{-1} = \frac{-144}{-1}$$

$$x^2 = 144$$

$$\sqrt{x^2} = \pm\sqrt{144}$$

$$x = \pm 12$$

$y = x^n$

$y' = nx^{n-1}$

critical points

$x = -12$ OR $x = 12$

(92) determine the location and value of the absolute extreme values of f on the ~~st~~ given interval, if they exist.

$f(x) = x^2 - 11$ on $[-3, 4]$

x_1 4_1

$f'(x) = 2x - 0$

$f'(x) = 2x$

set $2x = 0$

$\frac{2x}{2} = \frac{0}{2}$

$x = 0$ critical value

$f(x) = x^2 - 11$

$f(-3) = (-3)^2 - 11$

$f(-3) = (-3)(-3) - 11$

$f(-3) = 9 - 11$

$f(-3) = -2$

$f(x) = x^2 - 11$

$f(4) = (4)^2 - 11$

$f(4) = (4)(4) - 11$

$f(4) = 16 - 11$

$f(4) = 5$

$f(0) = (0)^2 - 11$

$f(0) = (0)(0) - 11$

$f(0) = 0 - 11$

$f(0) = -11$

absolute max $f(4) = 5$

absolute min $f(0) = -11$

(94) find max

$s = -16t^2 + 32t + 48$

$s' = -16(2t) + 32(1) + 0$

$s' = -32t + 32 + 0$

$s' = -32t + 32$

set $-32t + 32 = 0$

$-32t = -32$

$\frac{-32t}{-32} = \frac{-32}{-32}$

$t = 1$ Critical point

$s'(t) = -32t + 32$

$s'(0) = -32(0) + 32$

$s'(0) = 0 + 32$

$s'(0) = 32 > 0$ increasing $(-\infty, 1)$

$s'(2) = -32(2) + 32$

$s'(2) = -64 + 32$

$s'(2) = -32 < 0$ decreasing $(1, \infty)$

Max at $t = 1$

$s(1) = -16 + 32 + 48$

$s(1) = 64$

$s(1) = -16(1)^2 + 32(1) + 48$

$s(1) = -16(1)(1) + 32(1) + 48$

max → $(1, 64)$

95. Suppose a tour guide has a bus that holds a maximum of 82 people. Assume his profit (in dollars) for taking n people on a city tour is $P(n) = n(41 - 0.5n) - 82$. (Although P is defined only for positive integers, treat it as a continuous function.)

a. How many people should the guide take on a tour to maximize the profit?
b. Suppose the bus holds a maximum of 34 people. How many people should be taken on a tour to maximize the profit?

a. Find the derivative of the given function P(n).

$P'(n) =$ [ ] $P'(n) = 41 - n$

If the bus holds a maximum of 82 people, the guide should take [41] people on a tour to maximize the profit.

b. If the bus holds a maximum of 34 people, the guide should take [34] people on a tour to maximize the profit.

Answers $-n + 41$

41

34

$P(n) = n(41 - 0.5n) - 82$

$P(n) = 41n - 0.5n^2 - 82$

$P'(n) = 41 - 0.5(2n) - 0$

$P'(n) = 41 - 1n$

$P'(n) = 41 - n$

set $41 - n = 0$

$\cancel{41} - n \cancel{- 41} = 0 - 41$

$-n = -41$

$\frac{-n}{-1} = \frac{-41}{-1}$

$n = 41$

$P'(40) = 41 - 40 = 1 > 0$ increasing $(-\infty, 41)$

$P'(42) = 41 - 42 = -1 < 0$ decreasing $(41, \infty)$

Max at $n = 41$

If the bus holds a maximum of 34 people ~~the~~ the guide should take 34 people to maximize

96. At what points c does the conclusion of the Mean Value Theorem hold for $f(x) = x^3$ on the interval $[-6,6]$?

The conclusion of the Mean Value Theorem holds for c = ____.
(Use a comma to separate answers as needed. Type an exact answer, using radicals as needed.)

Answer: $2\sqrt{3}, -2\sqrt{3}$

$f(x) = x^3$

$f'(x) = 3x^2$

mean value theorem

$[-6, 6]$
$a \quad b$

$$\boxed{\frac{f(b) - f(a)}{b - a} = f'(c)}$$

find c

Set

$$\frac{f(6) - f(-6)}{(6) - (-6)} =$$

$$\frac{(6)^3 - (-6)^3}{6+6} =$$

$$\frac{(6)(6)(6) - (-6)(-6)(-6)}{6+6} =$$

$$\frac{216 + 216}{12} =$$

$$\frac{432}{12} =$$

$36 =$

$f'(x) = 3x^2 = 36$

$3x^2 = 36$

$\frac{\cancel{3}x^2}{\cancel{3}} = \cancel{\frac{36}{3}}$

$x^2 = 12$

$\sqrt{x^2} = \pm\sqrt{12}$

$x = \pm\sqrt{12}$

$x = \pm\sqrt{4 \cdot 3}$

$x = \pm\sqrt{4}\sqrt{3}$

$x = \pm 2\sqrt{3}$

Prime
2, 3, 5, 7 ...

$2\,|\,12$
$2\,|\,6$
$3\,|\,3$
1

$x = 2\sqrt{3}$ OR $x = -2\sqrt{3}$

97. **a.** Determine whether the Mean Value Theorem applies to the function $f(x) = -6 + x^2$ on the interval $[-2,1]$.
b. If so, find the point(s) that are guaranteed to exist by the Mean Value Theorem.

a. Choose the correct answer below.

- **A.** Yes, because the function is continuous on the interval $[-2,1]$ and differentiable on the interval $(-2,1)$.
- **B.** No, because the function is not continuous on the interval $[-2,1]$, and is not differentiable on the interval $(-2,1)$.
- **C.** No, because the function is differentiable on the interval $(-2,1)$, but is not continuous on the interval $[-2,1]$.
- **D.** No, because the function is continuous on the interval $[-2,1]$, but is not differentiable on the interval $(-2,1)$.

b. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The point(s) is/are x = ________.
(Simplify your answer. Use a comma to separate answers as needed.)
- **B.** The Mean Value Theorem does not apply in this case.

Answers A. Yes, because the function is continuous on the interval $[-2,1]$ and differentiable on the interval $(-2,1)$.

A. The point(s) is/are $x = \boxed{-\frac{1}{2}}$.

(Simplify your answer. Use a comma to separate answers as needed.)

$f(x) = -6 + x^2$

$f'(x) = 0 + 2x$

$f'(x) = 2x$

$\frac{f(b) - f(a)}{b-a} = f'(c)$

Mean Value Theorem

$\cancel{(-6+(1)^2)}$

$\frac{f(1) - f(-2)}{(1)-(-2)} =$

$\frac{(-6+(1)^2) - (-6+(-2)^2)}{1+2} =$

$\frac{(-6+(1)(1)) - (-6+(-2)(-2))}{1+2} =$

$\frac{(-6+1) - (-6+4)}{1+2} =$

$\frac{(-5)-(-2)}{3} =$

$\frac{-5+2}{3} =$

$\frac{-3}{3} =$

$-1 =$

$[-2, 1]$
a b

let

$f'(x) = 2x = -1$

$2x = -1$

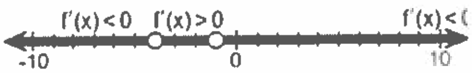
$\frac{2x}{2} = \frac{-1}{2}$

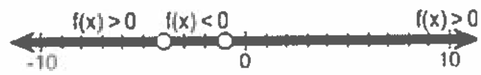
$x = -\frac{1}{2}$

98. Sketch a function that is continuous on $(-\infty,\infty)$ and has the following properties. Use a number line to summarize information about the function.

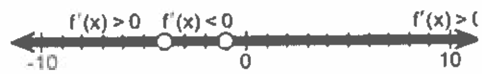
$f'(x) < 0$ on $(-\infty,-4)$; $f'(x) > 0$ on $(-4,-1)$; $f'(x) < 0$ on $(-1,\infty)$.

Which number line summarizes the information about the function?

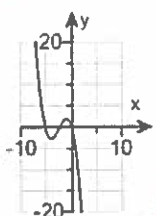
A. 

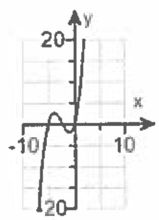
B. 

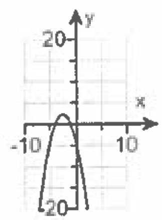
C. 

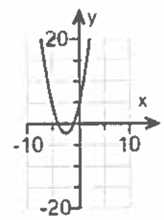
D. 

Which of the following graphs matches the description of the given properties?

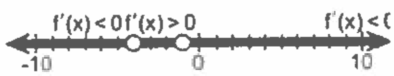
A. 

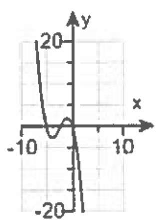
B. 

C. 

D. 

Answers

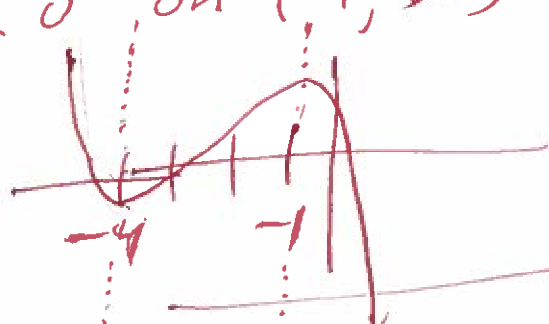
A. 

A. 

$f'(x) < 0$ on $(-\infty, -4)$ f(x) is decreasing

$f'(x) > 0$ on $(-4, -1)$ f(x) is increasing

$f'(x) < 0$ on $(-1, \infty)$ f(x) is decreasing



99. Find the intervals on which f is increasing and the intervals on which it is decreasing.

$f(x) = 7 - x^2$

$f(x) = 7 - x^2$
$f'(x) = 0 - 2x$
$f'(x) = -2x$
let $-2x = 0$
$\frac{-2x}{-2} = \frac{0}{-2}$
critical point $x = 0$
check
$-1 \quad 0 \quad 1$

Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- A. The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.
  (Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)
- B. The function is decreasing on ______. The function is never increasing.
  (Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- C. The function is increasing on ______. The function is never decreasing.
  (Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- D. The function is never increasing nor decreasing.

$f'(-1) = -2(-1) = 2 > 0$ increasing
$f'(1) = -2(1) = -2 < 0$ decreasing

Answer: A. The function is increasing on $(-\infty,0)$ and decreasing on $(0,\infty)$.
(Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)

---

100. Find the intervals on which f is increasing and the intervals on which it is decreasing.

$f(x) = -6 + x - x^2$

Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- A. The function is increasing on $(-\infty, \frac{1}{2})$ and decreasing on $(\frac{1}{2}, \infty)$.
  (Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)
- B. The function is decreasing on ______. The function is never increasing.
  (Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- C. The function is increasing on ______. The function is never decreasing.
  (Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- D. The function is never increasing nor decreasing.

Answer: A. The function is increasing on $\left(-\infty,\frac{1}{2}\right)$ and decreasing on $\left(\frac{1}{2},\infty\right)$.
(Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)

$f(x) = -6 + x - x^2$
$f'(x) = 0 + 1 - 2x$
$f'(x) = 1 - 2x$
let $1 - 2x = 0$
$\cancel{1} - 2x \cancel{- 1} = 0 - 1$
$-2x = -1$
$\frac{-2x}{-2} = \frac{-1}{-2}$
$x = \frac{1}{2}$
critical point
$0 \quad \frac{1}{2} \quad 1$

$f'(0) = 1 - 2(0) = 1 - 0 = 1 > 0$ increasing
$f'(1) = 1 - 2(1) = 1 - 2 = -1 < 0$
decreasing

101. Locate the critical points of the following function. Then use the Second Derivative Test to determine whether they correspond to local maxima, local minima, or neither.

$f(x) = 7 - 4x^2$

What is(are) the critical point(s) of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The critical point(s) is(are) x = 0 .
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- B. There are no critical points for f.

What is/are the local maximum/maxima of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The local maximum/maxima of f is/are at x = 0 . Max at x=0 OR (0, 7)
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- B. There is no local maximum of f.

What is/are the local minimum/minima of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The local minimum/minima of f is/are at x = ______ .
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- B. There is no local minimum of f.

Answers A. The critical point(s) is(are) x = 0 .
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

A. The local maximum/maxima of f is/are at x = 0 .
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

B. There is no local minimum of f.

$f(x) = 7 - 4x^2$

$f'(x) = 0 - 8x$

$f'(x) = -8x$

set $-8x = 0$

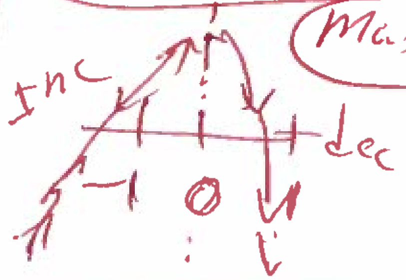
$\frac{-8x}{-8} = \frac{0}{-8}$

$x = 0$

$f'(-1) = -8(-1) = 8 > 0$ increasing

$f'(1) = -8(1) = -8 < 0$ decreasing

Max at $x = 0$



$f''(x) = -8$

concave down OR Max

$f(x) = 7 - 4x^2$

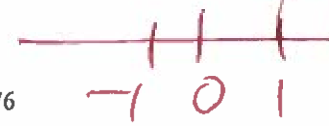
$f(0) = 7 - 4(0)^2$

$f(0) = 7 - 4(0)$

$f(0) = 7 - 0$

$f(0) = 7$

Max at $x = 0$



102. Locate the critical points of the following function. Then use the Second Derivative Test to determine whether they correspond to local maxima, local minima, or neither.

$f(x) = -3x^3 - 18x^2 + 12$ → $f'(x) = -9x^2 - 36x + 0 = -9x^2 - 36x$

What is(are) the critical point(s) of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

Let $-9x^2 - 36x = 0$

$-9x(x+4) = 0$

$-9x = 0$ or $x + 4 = 0$

$\frac{-9x}{-9} = \frac{0}{-9}$ or $x + 4 - 4 = 0 - 4$

$x = 0$ or $x = -4$ critical point

- A. The critical point(s) is(are) x = 0, −4.
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- B. There are no critical points for f.

What is/are the local maximum/maxima of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The local maximum/maxima of f is/are at x = 0 .
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- B. There is no local maximum of f.

What is/are the local minimum/minima of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The local minimum/minima of f is/are at x = −4 .
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- B. There is no local minimum of f.

$f''(x) = -18x - 36$

$f''(-4) = -18(-4) - 36$

$f''(-4) = 72 - 36$

$f''(-4) = 36 > 0$

concave up

min at $x = -4$

$f''(0) = -18(0) - 36$

$f''(0) = 0 - 36$

$f''(0) = -36 < 0$

concave down max at $x = 0$

Answers A. The critical point(s) is(are) x = 0, −4 .
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

A. The local maximum/maxima of f is/are at x = 0 .
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

A. The local minimum/minima of f is/are at x = −4 .
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

103. Fill in the blanks: The goal of an optimization problem is to find the maximum or minimum value of the _______ function subject to the _______.

The goal of an optimization problem is to find the maximum or minimum value of the (1) objective function function subject to the (2) constraints

(1)
- optimization function
- objective function
- constraint function
- subjective function

(2)
- extreme values.
- variables.
- constraints.
- optimizations.

Answers (1) objective function

(2) constraints.

10 Use a linear approximation to estimate the following quantity. Choose a value to produce a small error.

$\ln(1.05)$

$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

$a = 1$

$L(x) = f(a) + f'(a)(x - a)$

$L(1.05) = f(1) + f'(1)(1.05 - 1)$

$L(1.05) = \ln(1) + \frac{1}{(1)}(1.05 - 1)$

$L(1.05) = 0 + 1(1.05 - 1)$

$L(1.05) = 0 + 1(.05)$

$L(1.05) = 0 + .05$

$L(1.05) = 0.05$

(105) Consider the following function and express the relationship between a small change in x and the corresponding change in y in the form $dy = f'(x)\,dx$

formula

$y = e^{f(x)}$

$y' = e^{f(x)} \cdot f'(x)$

$f(x) = e^{11x}$

$f'(x) = e^{11x}(11x)'$

$f'(x) = e^{11x}(11)$

$y = e^{11x}$

$\frac{dy}{dx} = e^{11x} \cdot (11x)'$

$\frac{dy}{dx} = e^{11x}(11)$

$\frac{dy}{dx} = 11e^{11x}$

$\frac{dy}{\cancel{dx}}(\cancel{dx}) = 11e^{11x}(dx)$

$dy = 11e^{11x}\,dx$

(106)

find $dy = f'(x)\,dx$

$f(x) = 2x^3 - 3x$

$y = 2x^3 - 3x$

$\frac{dy}{dx} = 2(3x^{3-1}) - 3(1)$

$\frac{dy}{dx} = 6x^2 - 3$

$\frac{dy}{\cancel{dx}}(\cancel{dx}) = (6x^2 - 3)(dx)$

$dy = (6x^2 - 3)\,dx$

formula

$y = x^n$

$y' = nx^{n-1}$

$y = ax$

$y' = a$

107

find $dy = f'(x)\,dx$

Formula
$y = \cot(f(x))$
$y' = -\csc^2(f(x)) \cdot f'(x)$

$f(x) = \cot(3x)$

$y = \cot(3x)$

$\frac{dy}{dx} = -\csc^2(3x) \cdot (3x)'$

$\frac{dy}{dx} = -\csc^2(3x)\ (3)$

$\frac{dy}{dx} = -3\csc^2(3x)$

$\frac{dy}{\cancel{dx}}(\cancel{dx}) = \left(-3\csc^2(3x)\right) dx$

$dy = \left(-3\csc^2(3x)\right) dx$

(108) evaluate use L'Hopital Rule formula

$$\lim_{x \to a} \frac{f(x)}{g(x)} =$$

$$\lim_{x \to a} \frac{f'(x)}{g'(x)} =$$

$$\lim_{x \to 0} \frac{9\sin(6x)}{5x} =$$

$$\lim_{x \to 0} \frac{9\cos(6x)(6x)'}{5} =$$

$$\lim_{x \to 0} \frac{9\cos(6x)(6)}{5} =$$

$$\lim_{x \to 0} \frac{54\cos(6x)}{5} =$$

$$\frac{54\cos(6(0))}{5} =$$

$$\frac{54\cos(0)}{5} =$$

$$\frac{54(1)}{5} =$$

$$\frac{54}{5} =$$

109 evaluate the limit use L Hopital Rule

$$\lim_{u \to \frac{\pi}{4}} \frac{\tan(u) - \cot(u)}{4u - \pi} =$$

$$\lim_{u \to \frac{\pi}{4}} \frac{\sec^2(u) + \csc^2(u)}{4} =$$

$$\frac{\sec^2(\frac{\pi}{4}) + \csc^2(\frac{\pi}{4})}{4} =$$

$$\frac{\frac{1}{\cos^2(\frac{\pi}{4})} + \frac{1}{\sin^2(\frac{\pi}{4})}}{4} =$$

$$\frac{\frac{1}{(\frac{\sqrt{2}}{2})^2} + \frac{1}{(\frac{\sqrt{2}}{2})^2}}{4} =$$

$$\frac{\frac{1}{\frac{(\sqrt{2})^2}{(2)^2}} + \frac{1}{\frac{(\sqrt{2})^2}{(2)^2}}}{4} =$$

$$\frac{\frac{1}{\frac{2}{4}} + \frac{1}{\frac{2}{4}}}{4} =$$

$$\frac{\frac{1}{\frac{1}{2}} + \frac{1}{\frac{1}{2}}}{4} =$$

$$\frac{\frac{2}{1}\cdot\left(\frac{1}{1}\right) + \frac{2}{1}\left(\frac{1}{1}\right)}{4} =$$

$$\frac{\frac{2}{1} + \frac{2}{1}}{4} =$$

$$\frac{2+2}{4} =$$

$$\frac{4}{4} =$$

$$1 =$$

~~formulas~~ formulas

$$\lim_{x \to a} \frac{f(x)}{g(x)} =$$

$$\lim_{x \to a} \frac{f'(x)}{g'(x)} =$$

$$y = \tan(f(x))$$

$$y' = \tan^2(f(x)) \cdot f'(x)$$

$$y = \cot(f(x))$$

$$y' = \csc^2(f(x)) \cdot f'(x)$$

Use Newtons Method

110. Use a calculator or program to compute the first 10 iterations of Newton's method when they are applied to the following function with the given initial approximation.

$f(x) = x^2 - 23;\ x_0 = 5$ → $f'(x) = 2x$

$x_0 = 5$ $f(x) = x^2 - 23$

k	x_k	k	x_k
0	5.000000	6	4.795832
1	4.800000	7	4.795832
2	4.795833	8	4.795832
3	4.795832	9	4.795832
4	4.795832	10	4.795832
5	4.795832		

(Round to six decimal places as needed.)

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 5 - \frac{f(5)}{f'(5)}$$

$$x_1 = 5 - \frac{(5)^2 - 23}{2(5)}$$

$$x_1 = 5 - \frac{(5)(5) - 23}{10}$$

$$x_1 = 5 - \frac{25 - 23}{10}$$

$$x_1 = 5 - \frac{2}{10}$$

$$x_1 = 5 - .2$$

$$x_1 = 4.800000$$

Answers 5.000000

4.795832

4.800000

4.795832

4.795833

4.795832

4.795832

4.795832

4.795832

4.795832

4.795832

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Use a graphing calculator

$$x_2 = 4.800000 - \frac{f(4.800000)}{f'(4.800000)}$$

$$x_2 = 4.795833$$

111. Use a calculator or program to compute the first 10 iterations of Newton's method for the given function and initial approximation.

$f(x) = 2\sin x + x + 1$, $x_0 = 1.1$

$f'(x) = 2\cos(x) + 1 + 0 = 2\cos(x) + 1$

$x_0 = 1.1$

Complete the table.
(Do not round until the final answer. Then round to six decimal places as needed.)

k	x_k	k	x_k
1	−0.935670	6	−0.337584
2	−0.228777	7	−0.337584
3	−0.336532	8	−0.337584
4	−0.337584	9	−0.337584
5	−0.337584	10	−0.337584

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 1.1 - \frac{f(1.1)}{f'(1.1)}$

Answers − 0.935670

− 0.337584

− 0.228777

− 0.337584

− 0.336532

− 0.337584

− 0.337584

− 0.337584

− 0.337584

− 0.337584

$x_1 = 1.1 - \frac{2\sin(1.1) + (1.1) + 1}{2\cos(1.1) + 1}$

$x_1 = 1.1 - \frac{3.88241472}{1.90719224 3}$

$x_1 = -.93567036$

$x_1 = 1.1 - 2.03567036$

112. Determine the following indefinite integral. Check your work by differentiation.

$$\int \left(7x^{13} - 11x^{21}\right) dx$$

$\int \left(7x^{13} - 11x^{21}\right) dx =$ ______ (Use C as the arbitrary constant.)

Answer: $\frac{x^{14}}{2} - \frac{x^{22}}{2} + C$

formula
$\int x^n dx$
$\frac{x^{n+1}}{n+1} + C$

$\int(7x^{13} - 11x^{21})dx =$

$\frac{7x^{13+1}}{13+1} - \frac{11x^{21+1}}{21+1} + C =$

$\frac{7x^{14}}{14} - \frac{11x^{22}}{22} + C =$

$\frac{7x^{14}}{7(2)} - \frac{11x^{21}}{11(2)} + C =$

$\frac{x^{14}}{2} - \frac{x^{22}}{2} + C =$

(113) $\int \left(\frac{8}{\sqrt{x}} + 8\sqrt{x} \right) dx =$

$$\int \left(\frac{8}{x^{1/2}} + 8x^{1/2} \right) dx =$$

$$\int \left(8x^{-1/2} + 8x^{1/2} \right) dx =$$

$$\frac{8x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{8x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C =$$

$$\frac{8x^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}} + \frac{8x^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C =$$

$$\frac{8x^{\frac{-1+2}{2}}}{\frac{-1+2}{2}} + \frac{8x^{\frac{1+2}{2}}}{\frac{1+2}{2}} + C =$$

$$\frac{8x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{8x^{\frac{3}{2}}}{\frac{3}{2}} + C =$$

$$\frac{2}{1}\left(8x^{1/2}\right) + \frac{2}{3}\left(8x^{3/2}\right) + C =$$

$$16x^{1/2} + \frac{16}{3}x^{3/2} + C =$$

$$16\sqrt{x} + \frac{16}{3}x^{3/2} + C$$

Formula

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

(114) $\int \left(\frac{5}{w^2} - 6w^6\right) dw =$

$\int (5w^{-2} - 6w^6)\, dw =$

$\frac{5w^{-2+1}}{-2+1} - \frac{6w^{6+1}}{6+1} + C =$

$\frac{5w^{-1}}{-1} - \frac{6w^7}{7} + C =$

$-5w^{-1} - \frac{6w^7}{7} + C$

OR

$\frac{-5}{w} - \frac{6w^7}{7} + C =$

formula

$\int x^n dx =$

$\frac{x^{n+1}}{n+1} + C =$

115 $\int (6x+5)^2\,dx =$

$$\frac{1}{6}\int (6x+5)^2 (6)\,dx =$$

$$\frac{1}{6}\left(\frac{(6x+5)^{2+1}}{2+1}\right) + C =$$

$$\frac{1}{6}\;\frac{(6x+5)^3}{3} + C =$$

$$\frac{1}{18}(6x+5)^3 + C =$$

OR

$$\int (6x+5)(6x+5)\,dx =$$

$$\int 36x^2 + 30x + 30x + 25\,dx =$$

$$\int 36x^2 + 60x + 25\,dx =$$

$$\frac{36x^{2+1}}{2+1} + \frac{60x^{1+1}}{1+1} + 25x + C =$$

$$\frac{36x^3}{3} + \frac{6x^2}{2} + 25x + C =$$

$$12x^3 + 3x^2 + 25x + C$$

Formula

$$\int (f(x))^N \cdot f'(x)\,dx = \frac{(f(x))^{N+1}}{N+1} + C =$$

(116) $\int 4m(5m^2-7m)\,dm =$

$\int 20m^3 - 28m^2\,dm =$

$\frac{20m^{3+1}}{3+1} - \frac{28m^{2+1}}{2+1} + C =$

$\frac{20m^4}{4} - \frac{28m^3}{3} + C =$

$5m^4 - \frac{28m^3}{3} + C =$

formula

$\int x^n\,dx =$

$\frac{x^{n+1}}{n+1} + C =$

$\int a\,dx =$

$ax + C =$

(117) $\int (4x^{1/3} + 2x^{-1/3} + 6)\,dx =$

formula $\int x^n dx$ $\frac{x^{n+1}}{n+1} + C$

$\int a\,dx =$ $ax + C =$

$\frac{4x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + \frac{2x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + 6x + C =$

$\frac{4x^{\frac{1}{3}+\frac{3}{3}}}{\frac{1}{3}+\frac{3}{3}} + \frac{2x^{-\frac{1}{3}+\frac{3}{3}}}{-\frac{1}{3}+\frac{3}{3}} + 6x + C =$

$\frac{4x^{\frac{1+3}{3}}}{\frac{1+3}{3}} + \frac{2x^{\frac{-1+3}{3}}}{\frac{-1+3}{3}} + 6x + C =$

$\frac{4x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} + 6x + C =$

$\frac{3}{\cancel{4}}(\cancel{4}x^{4/3}) + \frac{3}{\cancel{2}}(\cancel{2}x^{2/3}) + 6x + C =$

$3x^{4/3} + 3x^{2/3} + 6x + C =$

118

$$\int 4\sqrt[5]{x}\,dx =$$

$$\int 4x^{1/5}\,dx =$$

$$\frac{4x^{1/5+1}}{\frac{1}{5}+1} + C =$$

$$\frac{4x^{\frac{1}{5}+\frac{5}{5}}}{\frac{1}{5}+\frac{5}{5}} + C =$$

$$\frac{4x^{\frac{1+5}{5}}}{\frac{1+5}{5}} + C =$$

$$\frac{4x^{\frac{6}{5}}}{\frac{6}{5}} + C =$$

$$\frac{5}{6}\left(4x^{6/5}\right) + C =$$

$$\frac{5}{\cancel{(2)}(3)}\cancel{(2)}(2)\,x^{6/5} + C$$

$$\frac{10}{3}x^{6/5} + C =$$

formula

$$\int x^{n}\,dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

(119) $\int (4x+5)(3-x)\,dx =$

$\int (12x - 4x^2 + 15 - 5x)\,dx =$

$\int (-4x^2 + 7x + 15)\,dx =$

$\frac{-4x^{2+1}}{2+1} + \frac{7x^{1+1}}{1+1} + 15x + C =$

$\frac{-4x^3}{3} + \frac{7x^2}{2} + 15x + C =$

formula

$\int x^n\,dx =$

$\frac{x^{n+1}}{n+1} + C =$

$\int a\,dx =$

$ax + C =$

(120) $\int \left(\frac{6}{x^4} + 2 - \frac{8}{x^2}\right) dx =$

$\int \left(6x^{-4} + 2 - 8x^{-2}\right) dx =$

$\frac{6x^{-4+1}}{-4+1} + 2x - \frac{8x^{-2+1}}{-2+1} + C =$

$\frac{6x^{-3}}{-3} + 2x - \frac{8x^{-1}}{-1} + C =$

$-2x^{-3} + 2x + 8x^{-1} + C =$

$\frac{-2}{x^3} + 2x + \frac{8}{x} + C =$

formula

$\int x^n dx =$

$\frac{x^{n+1}}{n+1} + C =$

$\int a\, dx =$

$ax + C =$

121. $\int \frac{3x^5-6x^4}{x^3}\,dx$

$$\int \frac{3x^5}{x^3}-\frac{6x^4}{x^3}\,dx =$$

$$\int 3x^{5-3}-6x^{4-3}\,dx =$$

$$\int 3x^2-6x^1\,dx =$$

$$\int 3x^2-6x\,dx =$$

$$\frac{3x^{2+1}}{2+1}-\frac{6x^{1+1}}{1+1}+C =$$

$$\frac{3x^3}{3}-\frac{6x^2}{2}+C =$$

$$x^3-3x^2+C =$$

Formula

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C =$$

$$\int a\,dx = ax+C$$

(122) $\int (\csc^2(2\theta) + 8)\, d\theta =$

$\frac{1}{2}\int (\csc^2(2\theta)(2)\, d\theta + \int 8\, d\theta =$

$-\frac{1}{2}\cot(2\theta) + 8\theta + C =$

Formula

$\int \csc^2(f(x)) \cdot f'(x)\, dx =$

$-\cot(f(x)) + C =$

$\int a\, dx =$

$ax + C =$

128

$$\int (\sec^2(x) - 4)\, dx =$$

$$\int (\sec^2(x) \cdot (1) - 4)\, dx =$$

$$\tan(x) - 4x + C =$$

formula

$$\int \sec^2(f(x)) \cdot f'(x)\, dx =$$

$$\tan(f(x)) + C =$$

$$\int a\, dx$$

$$ax + C$$

(124) for function f find the antiderivative F that satisfies the given condition

$$f(x) = 8x^3 + 3\sin x \qquad F(0) = 2$$

$$\int f(x)\,dx = \int 8x^3 + 3\sin x\,dx$$

$$F(x) = \frac{8x^{3+1}}{3+1} - 3\cos(x) + C$$

$$F(x) = \frac{8x^4}{4} - 3\cos(x) + C$$

$$F(x) = 2x^4 - 3\cos(x) + C$$

$$F(0) = 2(0)^4 - 3\cos(0) + C = 2$$

$$2(0)(0)(0)(0) - 3(1) + C = 2$$

$$0 - 3 + C = 2$$

$$-3 + C = 2$$

$$\cancel{-3} + C \cancel{+3} = 2 + 3$$

$$C = 5$$

$$F(x) = 2x^4 - 3\cos(x) + C$$

$$F(x) = 2x^4 - 3\cos(x) + 5$$

formula

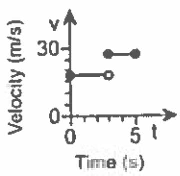
$$\int \sin(x)\,dx = -\cos(x) + C$$

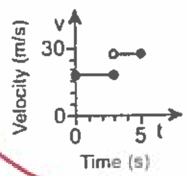
$$y = \cos(x)$$

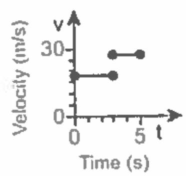
$$y' = -\sin(x)$$

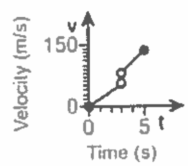
125. Suppose an object moves along a line at 18 m/s for $0 \le t \le 3$ s and at 27 m/s for $3 < t \le 5$ s. Sketch the graph of the velocity function and find the displacement of the object for $0 \le t \le 5$.

Sketch the graph of the velocity function. Choose the correct graph below.

A. 

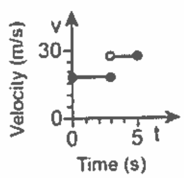
B. 

C. 

D. 

The displacement of the object for $0 \le t \le 5$ is 108 m. (Simplify your answer.)

Answers

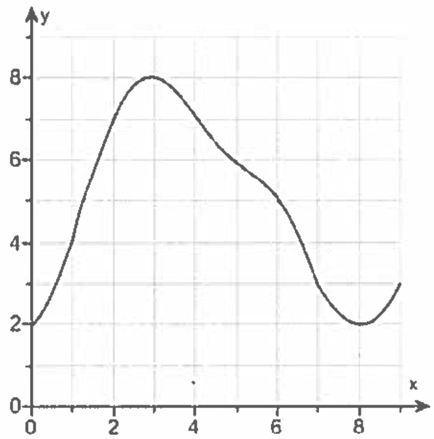


B.

108

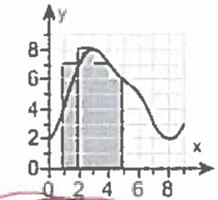
126.

Approximate the area of the region bounded by the graph of f(x) (shown below) and the x-axis by dividing the interval [1,5] into n = 4 subintervals. Use a left and right Riemann sum to obtain two different approximations. Draw the approximating rectangles.

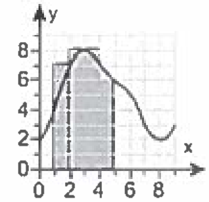


In which graph below are the selected points the left endpoints of the 4 approximating rectangles?

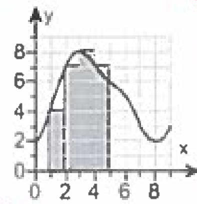
A.



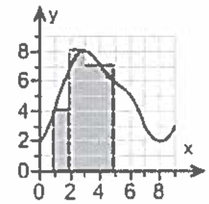
B.



C.



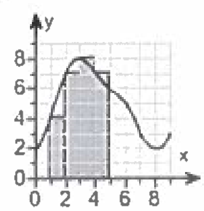
D.



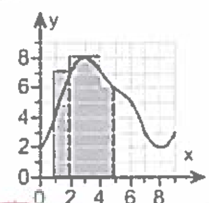
Using the specified rectangles, approximate the area.

In which graph below are the selected points the right endpoints of the 4 approximating rectangles?

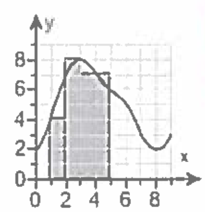
A.



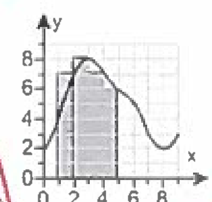
B.



C.

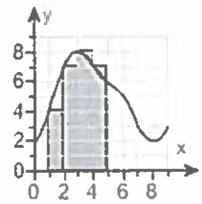


D.

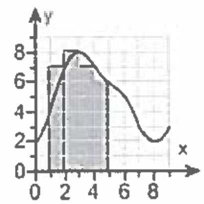


Using the specified rectangles, approximate the area.

Answers

C. 

26

D. 

28

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127. Does the right Riemann sum underestimate or overestimate the area of the region under the graph of a positive decreasing function? Explain.

Choose the correct answer below.

- A. Overestimate; the rectangles do not fit under the curve.
- B. Overestimate; the rectangles all fit under the curve.
- C. Underestimate; the rectangles do not fit under the curve.
- D. Underestimate; the rectangles all fit under the curve.

Answer: D. Underestimate; the rectangles all fit under the curve.

128. The following function is negative on the given interval.

Part 1

$f(x) = -5 - x^2$; [3,7]

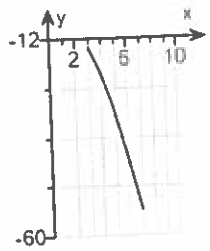
a. Sketch the function on the given interval.

b. Approximate the net area bounded by the graph of f and the x-axis on the interval using a left, right, and midpoint Riemann sum with n = 4.

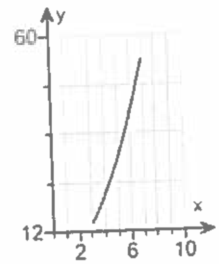
a. Choose the correct graph below.

A.

B.



C.

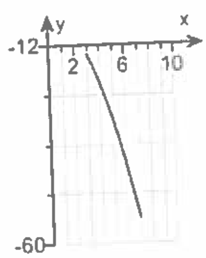


b. The approximate net area using a left Riemann sum is -106.
(Type an integer or a decimal.)

The approximate net area using a midpoint Riemann sum is -125.
(Type an integer or a decimal.)

The approximate net area using a right Riemann sum is -146.
(Type an integer or a decimal.)

Answers



B.

-106

-125

-146

128 $\int_3^7 (-5-x^2)\,dx$ Part 2

formula
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C =$

$\int a\,dx = ax + C =$

$\left(-5x - \frac{x^{2+1}}{2+1}\right)\Big|_3^7 =$

$\left(-5x - \frac{x^3}{3}\right)\Big|_3^7 =$

$\left(-5(7) - \frac{(7)^3}{3}\right) - \left(-5(3) - \frac{(3)^3}{3}\right) =$

$\left(-35 - \frac{(7)(7)(7)}{3}\right) - \left(-15 - \frac{(3)(3)(3)}{3}\right) =$

$\left(-35 - \frac{343}{3}\right) - \left(-15 - \frac{27}{3}\right) =$

$\left(\frac{-35}{1}\left(\frac{3}{3}\right) - \frac{343}{3}\right) - \left(\frac{-15}{7}\left(\frac{3}{3}\right) - \frac{27}{3}\right) =$

$\left(\frac{-105}{3} - \frac{343}{3}\right) - \left(\frac{-45}{3} - \frac{27}{3}\right) =$

$\left(\frac{-105-343}{3}\right) - \left(\frac{-45}{3} - \frac{27}{3}\right) =$

$\left(\frac{-448}{3}\right) - \left(\frac{-72}{3}\right) =$

$\frac{-448}{3} + \frac{72}{3} =$

$\frac{-376}{3} =$ -125.333333

129.

$$\int_0^3 (3x+2)\,dx =$$

formula

$$\int x^n\,dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

$$\int a\,dx =$$

$$ax + C =$$

$$\left(\frac{3x^{1+1}}{1+1} + 2x\right)\Big/_0^3 =$$

$$\left(\frac{3x^2}{2} + 2x\right)\Big/_0^3 =$$

$$\left(\frac{3(3)^2}{2} + 2(3)\right) - \left(\frac{3(0)^2}{2} + 2(0)\right) =$$

$$\left(\frac{3(3)(3)}{2} + 2(3)\right) - \left(\frac{3(0)(0)}{2} + 2(0)\right) =$$

$$\left(\frac{27}{2} + 6\right) - \left(\frac{0}{2} + 0\right) =$$

$$\left(\frac{27}{2} + \frac{6}{1}\left(\frac{2}{2}\right)\right) - (0 + 0) =$$

$$\left(\frac{27}{2} + \frac{12}{2}\right) - (0) =$$

$$\left(\frac{27+12}{2}\right) - (0) =$$

$$\frac{39}{2} - 0 =$$

$$\frac{39}{2} =$$

$$19.5 =$$

(130)

$$\frac{d}{dx}\int_a^x f(t)\,dt =$$

where a and b are constants

$$f(x) =$$

$$\frac{d}{dx}\int_a^b f(t)\,dt =$$

where a and b are constants

$$0 =$$

(131) $\int_0^1 (x^2-3x+5)\,dx =$

$$\left(\frac{x^{2+1}}{2+1} - \frac{3x^{1+1}}{1+1} + 5x\right)\Big|_0^1 =$$

$$\left(\frac{x^3}{3} - \frac{3x^2}{2} + 5x\right)\Big|_0^1 =$$

$$\left(\frac{(1)^3}{3} - \frac{3(1)^2}{2} + 5(1)\right) - \left(\frac{(0)^3}{3} - \frac{3(0)^2}{2} + 5(0)\right) =$$

$$\left(\frac{(1)(1)(1)}{3} - \frac{3(1)(1)}{2} + 5(1)\right) - \left(\frac{(0)(0)(0)}{3} - \frac{3(0)(0)}{2} + 5(0)\right) =$$

$$\left(\frac{1}{3} - \frac{3}{2} + \frac{5}{1}\right) - \left(\frac{0}{3} - \frac{0}{2} + 0\right) =$$

$$\left(\frac{1}{3}\left(\frac{2}{2}\right) - \frac{3}{2}\left(\frac{3}{3}\right) + \frac{5}{1}\left(\frac{6}{6}\right)\right) - (0 - 0 + 0) =$$

$$\left(\frac{2}{6} - \frac{9}{6} + \frac{30}{6}\right) - (0) =$$

$$\left(\frac{2-9+30}{6}\right) - (0) =$$

$$\left(\frac{23}{6}\right) - (0) =$$

$$\frac{23}{6} - 0 =$$

$$\frac{23}{6} =$$

(13c) $\displaystyle\int_{-2}^{5} (x^2-3x-10)\,dx$

formula

$\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C =$

$\displaystyle\int a\,dx = ax$

$$\left(\frac{x^{2+1}}{2+1} - \frac{3x^{1+1}}{1+1} - 10x\right)\Big|_{-2}^{5} =$$

$$\left(\frac{x^3}{3} - \frac{3x^2}{2} - 10x\right)\Big|_{-2}^{5} =$$

$$\left(\frac{(5)^3}{3} - \frac{3(5)^2}{2} - 10(5)\right) - \left(\frac{(-2)^3}{3} - \frac{3(-2)^2}{2} - 10(-2)\right) =$$

$$\left(\frac{(5)(5)(5)}{3} - \frac{3(5)(5)}{2} - 10(5)\right) - \left(\frac{(-2)(-2)(-2)}{3} - \frac{3(-2)(-2)}{2} - 10(-2)\right) =$$

$$\left(\frac{125}{3} - \frac{75}{2} - 50\right) - \left(\frac{-8}{3} - \frac{12}{2} + 20\right) =$$

$$\left(\frac{125}{3}\left(\frac{2}{2}\right) - \frac{75}{2}\left(\frac{3}{3}\right) - \frac{50}{1}\left(\frac{6}{6}\right)\right) - \left(\frac{-8}{3}\left(\frac{2}{2}\right) - \frac{12}{2}\left(\frac{3}{3}\right) + \frac{20}{1}\left(\frac{6}{6}\right)\right) =$$

$$\left(\frac{250}{6} - \frac{225}{6} - \frac{300}{6}\right) - \left(\frac{-16}{6} - \frac{36}{6} + \frac{120}{6}\right) =$$

$$\left(\frac{250 - 225 - 300}{6}\right) - \left(\frac{-16-36+120}{6}\right) =$$

$$\left(\frac{-275}{6}\right) - \left(\frac{68}{6}\right) =$$

$$\frac{-275}{6} - \frac{68}{6} =$$

$$\frac{-275-68}{6} =$$

$$-\frac{343}{6} =$$

(133) $\int_{-8}^{8} (64 - x^2)\, dx$

Formula

$\int a\, dx = ax + C =$

$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C =$

$\left(64x - \frac{x^{2+1}}{2+1}\right)\Big|_{-8}^{8}$

$\left(64x - \frac{x^3}{3}\right)\Big|_{-8}^{8}$

$\left(64(8) - \frac{(8)^3}{3}\right) - \left(64(-8) - \frac{(-8)^3}{3}\right) =$

$\left(64(8) - \frac{(8)(8)(8)}{3}\right) - \left(64(-8) - \frac{(-8)(-8)(-8)}{3}\right) =$

$\left(512 - \frac{512}{3}\right) - \left(-512 - \frac{-512}{3}\right) =$

$\left(512 - \frac{512}{3}\right) - \left(-512 + \frac{512}{3}\right) =$

$\left(\frac{512(3)}{(3)} - \frac{512}{3}\right) - \left(\frac{-512(3)}{(3)} + \frac{512}{3}\right) =$

$\left(\frac{1536}{3} - \frac{512}{3}\right) - \left(\frac{-1536}{3} + \frac{512}{3}\right)$

$\left(\frac{1536 - 512}{3}\right) - \left(\frac{-1536 + 512}{3}\right)$

$\left(\frac{1024}{3}\right) - \left(\frac{-1024}{3}\right)$

$\frac{1024}{3} + \frac{1024}{3} =$

$\frac{1024 + 1024}{3} =$

$\frac{2048}{3} =$

134. Is x^{37} an even or odd function?

x^{37} is an odd function

is $\cos^3(x)$ an even or odd function?

$\cos^3(x)$ is an even function

135 On which derivative rule is the Substitution Rule based?

the Chain Rule

(136) $\int 2x\,(x^2+20)^{12}\,dx =$

$\int (x^2+20)^{12}(2x)\,dx =$

$\dfrac{(x^2+20)^{12+1}}{12+1} + C =$

$\dfrac{(x^2+20)^{13}}{13} + C =$

$\frac{1}{13}(x^2+20)^{13} + C =$

formula

$\int (f(x))^N \cdot f'(x)\,dx =$

$\dfrac{(f(x))^{N+1}}{N+1} + C =$

(137) $\int 10x \cos(5x^2+1)\,dx =$

$\int \cos(5x^2+1)\cdot(10x)\,dx =$

$\sin(5x^2+1) + C =$

formule

$\int \cos(f(x))\cdot f'(x)\,dx =$

$\sin(f(x)) + C =$

138.

$$\int (4x+5)\sqrt{2x^2+5x}\,dx$$

$$\int \sqrt{2x^2+5x}\,(4x+5)\,dx$$

$$\int (2x^2+5x)^{1/2}(4x+5)\,dx$$

$$\frac{(2x^2+5x)^{1/2+1}}{\frac{1}{2}+1}+C=$$

$$\frac{(2x^2+5x)^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}}+C=$$

$$\frac{(2x^2+5x)^{\frac{1+2}{2}}}{\frac{1+2}{2}}+C=$$

$$\frac{(2x^2+5x)^{\frac{3}{2}}}{3/2}+C=$$

$$\frac{2}{3}(2x^2+5x)^{3/2}+C$$

formula

$$\int (f(x))^{N} f'(x)\,dx =$$

$$\frac{(f(x))^{N+1}}{N+1}+C=$$

139 $\int (7x^6+6)\sqrt{x^7+6x}\,dx =$

$$\int \sqrt{x^7+6x}\,(7x^6+6)\,dx =$$

$$\int (x^7+6x)^{1/2}(7x^6+6)\,dx =$$

$$\frac{(x^7+6x)^{1/2+1}}{\frac{1}{2}+1}+C=$$

$$\frac{(x^7+6x)^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}}+C=$$

$$\frac{(x^7+6x)^{\frac{1+2}{2}}}{\frac{1+2}{2}}+C=$$

$$\frac{(x^7+6x)^{3/2}}{3/2}+C=$$

$$\frac{2}{3}(x^7+6x)^{3/2}+C$$

formula

$$\int (f(x))^N \cdot f'(x)\,dx =$$

$$\frac{(f(x))^{N+1}}{N+1}+C=$$

14ii

$$\int e^{7x+3}\,dx =$$

$$\frac{1}{7}\int e^{7x+3}(7)\,dx =$$

$$\frac{1}{7}e^{7x+3}+c =$$

formula

$$\int e^{f(x)}\cdot f'(x)\,dx =$$

$$e^{f(x)}+c =$$

$y = e^{f(x)}$

$y' = e^{f(x)}\cdot f'(x)$

$y = 7x+3$

$y' = 7+0$

$y'' = 7$

(141) $\int x^7 e^{x^8} dx =$

$\int e^{x^8} \cdot x^7 dx =$

$\frac{1}{8} \int e^{x^8} \cdot (8x^7) dx =$

$\frac{1}{8} e^{x^8} + C =$

formula

$\int e^{f(x)} \cdot f'(x) dx$

$e^{f(x)} + C =$

$y = e^{f(x)}$

$y' = e^{f(x)} \cdot f'(x)$

$y = x^8$

$y' = 8x^{8-1}$

$y' = 8x^7$

142

$$\int (x^2+x)^8 (2x+1)\,dx =$$

$$\frac{(x^2+x)^{8+1}}{8+1} + C =$$

$$\frac{(x^2+x)^9}{9} + C =$$

formula

$$\int (f(x))^n \cdot f'(x)\,dx =$$

$$\frac{(f(x))^{n+1}}{n+1} + C$$

$y = ax$

$y' = a$

$y = x^n$

$y' = nx^{n-1}$

$y = x^2+x$

$y' = 2x+1$

$y' = 2x+1$

143

$$\int \frac{e^{8x}}{e^{8x}+5}\,dx =$$

$$\frac{1}{8}\int \frac{e^{8x}\cdot(8)\,dx}{e^{8x}+5} =$$

$$\frac{1}{8}\ln\left|e^{8x}+5\right| + C =$$

formula

$$\int \frac{f'(x)}{f(x)}\,dx =$$

$$\ln|f(x)| + C =$$

$$y = e^{f(x)}$$

$$y' = e^{f(x)}\cdot f'(x)$$

$$y = e^{8x}+5$$

$$y' = e^{8x}(8x)' + 0$$

$$y' = e^{8x}(8)$$

$$y' = e^{8x}(8)$$

(144) $\int_0^{\pi/15} \cos(5x)\,dx =$

$\frac{1}{5}\int_0^{\pi/15} \cos(5x)\cdot 5\,dx =$

$\frac{1}{5}\sin(5x)\Big|_0^{\pi/15} =$

$\left(\frac{1}{5}\sin\left(5\left(\frac{\pi}{15}\right)\right)\right) - \left(\frac{1}{5}\sin(5(0))\right) =$

$\left(\frac{1}{5}\sin\left(\frac{5\pi}{15}\right)\right) - \left(\frac{1}{5}\sin(0)\right) =$

$\left(\frac{1}{5}\sin\left(\frac{\pi}{3}\right)\right) - \left(\frac{1}{5}\sin(0)\right) =$

$\left(\frac{1}{5}\left(\frac{\sqrt{3}}{2}\right)\right) - \left(\frac{1}{5}(0)\right) =$

$\left(\frac{\sqrt{3}}{10}\right) - (0) =$

$\frac{\sqrt{3}}{10} - 0 =$

$\frac{\sqrt{3}}{10} =$

Formula

$\int \cos(f(x))\cdot f'(x)\,dx =$

$\sin(f(x)) + C =$

145 $\int_0^2 12e^{3x}\,dx =$

$12\int_0^2 e^{3x}\,dx =$

$(12)\left(\frac{1}{3}\right)\int_0^2 e^{3x}(3)\,dx =$

$\frac{12}{3}\int_0^2 e^{3x}(3)\,dx =$

$4\int_0^2 e^{3x}(3)\,dx =$

$4e^{3x}\Big|_0^2 =$

$(4e^{3(2)}) - (4e^{3(0)}) =$

$(4e^{6}) - (4e^{0}) =$

$(4e^{6}) - (4(1)) =$

$4e^{6} - 4 =$

formula

$\int e^{f(x)} f'(x)\,dx =$

$e^{f(x)} + C =$

$y = e^{3x}$

$y' = e^{3x} \cdot (3x)'$

$y' = e^{3x}(3)$

146

$$\int_0^1 \frac{2x}{(x^2+2)^2}\,dx$$

$$\int_0^1 (x^2+2)^{-2}(2x)\,dx$$

$$\left.\frac{(x^2+2)^{-2+1}}{-2+1}\right|_0^1 =$$

$$\left.\frac{(x^2+2)^{-1}}{-1}\right|_0^1 =$$

$$\left.-\frac{1}{(x^2+2)^1}\right|_0^1$$

$$\left(-\frac{1}{((1)^2+2)}\right)-\left(-\frac{1}{((0)^2+2)}\right)=$$

$$\left(-\frac{1}{(1+2)}\right)-\left(-\frac{1}{((0)+2)}\right)=$$

$$\left(\frac{-1}{3}\right)-\left(\frac{-1}{2}\right)=$$

$$-\frac{1}{3}+\frac{1}{2}=$$

$$-\frac{1}{3}\left(\frac{2}{2}\right)+\frac{1}{2}\left(\frac{3}{3}\right)=$$

$$-\frac{2}{6}+\frac{3}{6}=$$

$$\frac{-2+3}{6}=$$

$$\frac{1}{6}=$$

formula

$$\int (f(x))^N \cdot f'(x)\,dx =$$

$$\frac{(f(x))^{N+1}}{N+1}+C=$$

$$y=x^2+2$$

$$y'=2x+0$$

$$y'=2x$$

(147) $\int \frac{dx}{x^2-2x+10} =$

$\int \frac{dx}{x^2-2x+10+(\frac{1}{2}(-2))^2-(\frac{1}{2}(-2))^2} =$ complete the square

$\int \frac{dx}{x^2-2x+10+(-1)^2-(-1)^2} =$

$\int \frac{dx}{x^2-2x+10+(1)-(1)} =$

$\int \frac{dx}{x^2-2x+1+10-1} =$ rewrite

$\int \frac{dx}{x^2-2x+1+9} =$

$\int \frac{dx}{(x-1)(x-1)+9} =$

$\int \frac{dx}{(x-1)^2+(3)^2} =$

$\int \frac{1}{(x-1)^2+(3)^2}\, dx =$

$\int \frac{1}{(3)^2+(x-1)^2}\, dx =$

$\frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right)+C =$

formula

$\int \frac{f'(x)}{a^2+(f(x))^2} =$

$\frac{1}{a}\tan^{-1}\left(\frac{f(x)}{a}\right)+C =$

(148) evaluate the following integral using integration by parts.

$$\int 17t \cdot e^t \, dt$$

	derivative D	integrate I
+	$17t$	e^t
−	17	e^t

$$17te^t - \int 17e^t \, dt =$$

$$17te^t - 17e^t + C =$$

Formula

$y = at$

$y' = a$

$y = t^n$

$y' = nt^{n-1}$

$$\int e^{f(x)} \cdot f'(x) \, dx =$$

$$e^{f(x)} + C =$$

149 evaluate the following integral using integration by parts

$$\int 8x \ln(6x)\, dx$$

	Derivative D	integrate I
+	$\ln(6x)$	$8x$
−	$\frac{1}{x}$	$4x^2$

$$(\ln(6x))(4x^2) - \int \frac{1}{x} \cdot 4x^2 dx =$$

$$4x^2 \ln(6x) - \int \frac{4x^2}{x} dx =$$

$$4x^2 \ln(6x) - \int 4x \, dx =$$

$$4x^2 \ln(6x) - \frac{4x^{1+1}}{1+1} + C =$$

$$4x^2 \ln(6x) - \frac{4x^2}{2} + C =$$

$$4x^2 \ln(6x) - 2x^2 + C =$$

Formula

$$y = \ln(ax)$$
$$y' = \frac{(ax)'}{ax}$$
$$y' = \frac{a}{ax}$$
$$y' = \frac{1}{x}$$

$$\int x^n dx =$$
$$\frac{x^{n+1}}{n+1} + C =$$

(50) If the general solution of a differential equation is $y(t) = Ce^{-3t} + 13$, what is the solution that satisfies the initial condition $y(0) = 10$?

$$y(t) = Ce^{-3t} + 13$$

$$y(0) = Ce^{-3(0)} + 13 = 10$$

$$Ce^{-3(0)} + 13 = 10$$

$$Ce^{0} + 13 = 10$$

$$C(1) + 13 = 10$$

$$C + 13 = 10$$

$$C + \cancel{13} - \cancel{13} = 10 - 13$$

$$C = -3$$

$$y(t) = Ce^{-3t} + 13$$

$$y(t) = -3e^{-3t} + 13$$