First order - Linear Differential Equation

Solve DEALWARE2

$y'(t) = ky + b$ $\quad k \neq 0$ and b are real numbers

09/8/17

$$\frac{y'(t)}{ky+b} = \frac{ky+b}{ky+b}$$

$$\frac{y'(t)}{ky+b} = 1$$

$$\frac{\frac{dy}{dt}}{ky+b} = 1$$

$$\frac{1}{ky+b}\frac{dy}{dt} = 1$$

$$\frac{1}{ky+b}\frac{dy}{dt}(dt) = 1\,(dt)$$

$$\frac{1}{ky+b}\,dy = 1\,dt$$

$$\int \frac{1}{ky+b}\,dy = \int 1\,dt$$

$$\frac{1}{k}\int \frac{(k)(1)}{ky+b}\,dy = \int 1\,dt$$

$$\frac{1}{k}\int \frac{k}{ky+b}\,dy = \int 1\,dt$$

$$\frac{1}{k}\ln|ky+b| + C_1 = t + C_2$$

2

$$k\left(\frac{1}{k}\right)\ln|ky+b| + kC_1 = k(t+C_2)$$

$$\ln|ky+b| + kC_1 = kt + kC_2$$

$$\ln|ky+b| + kC_1 - kC_1 = kt + kC_2 - kC_1$$

$$\ln|ky+b| = kt + kC_2 - kC_1$$

$$\ln|ky+b| = kt + k(C_2 - C_1)$$

let $C_3 = C_2 - C_1$

$$\ln|ky+b| = kt + kC_3$$

$$e^{\ln|ky+b|} = e^{kt+kC_3}$$

$$ky+b = e^{kt} \cdot e^{kC_3}$$

let $C_4 = e^{kC_3}$

$$ky+b = e^{kt} \cdot C_4$$

$$ky+b = C_4 e^{kt}$$

$$ky+b-b = C_4 e^{kt} - b$$

$$ky = C_4 e^{kt} - b$$

$$\frac{ky}{k} = \frac{C_4 e^{kt}}{k} - \frac{b}{k}$$

$$y = \frac{C_4}{k} e^{kt} - \frac{b}{k}$$

$y = Ce^{kt} - \frac{b}{k}$

$y(t) = Ce^{kt} - \frac{b}{k}$ OR $C = \frac{C_y}{k}$

3.

Check

$y(t) = Ce^{kt} - \frac{b}{k}$

$y'(t) = Ce^{kt}(k) - 0$

$y'(t) = Ce^{kt}(k)$

$y'(t) = Cke^{kt}$

Substitute

Original

$y'(t) = ky(t) + b$

$Cke^{kt} = k(Ce^{kt} - \frac{b}{k}) + b$

$Cke^{kt} = kCe^{kt} - k(\frac{b}{k}) + b$

$Cke^{kt} = kCe^{kt} - b + b$

$Cke^{kt} = kCe^{kt}$

$Cke^{kt} = Cke^{kt}$ ✓

use formula

If $y'(t) = ky + b$ $k \neq 0$ and b are real numbers

then $y(t) = Ce^{kt} - \frac{b}{k}$

(4)

Solve $y'(t) = -0.03y + 6$ Same form

$y(0) = 0$ $k = -0.03$ and $b = 6$

$$y(t) = Ce^{kt} - \frac{b}{k}$$

$$y(t) = Ce^{-0.03t} - \frac{6}{-0.03}$$

$$y(t) = Ce^{-0.03t} + 200$$

$$y(0) = Ce^{-0.03(0)} + 200 = 0$$

$$Ce^{0} + 200 = 0$$

$$C(1) + 200 = 0$$

$$C + 200 = 0$$

$$C + 200 - 200 = 0 - 200$$

$$C = -200$$

$$y(t) = -200e^{-0.03t} + 200$$

5.

$$y(t) = 200\left(-e^{-0.03t} + 1\right)$$

$$\boxed{y(t) = 200\left(1 - e^{-0.03t}\right)}$$

answer

If $y(t) = -200e^{-0.03t} + 200$

then $y'(t) = -0.03y + 6$

Ck

$$y(t) = -200e^{-0.03t} + 200$$

$$y'(t) = -200(-0.03)e^{-0.03t} + 0$$

$$y'(t) = 6e^{-0.03t}$$

Subst

$$y'(t) = -0.03y + 6$$

$$6e^{-0.03t} = -0.03\left(-200e^{-0.03t} + 200\right) + 6$$

$$6e^{-0.03t} = 6e^{-0.03t} - 6 + 6$$

$$6e^{-0.03t} = 6e^{-0.03t}$$